

Durability of Industrial Composites

This book offers quantitative solutions to unsolved durability problems of composite laminates under any loading or chemical exposure. The reader will herein find complete solutions to the following topics.

- Load-dependent durability under simultaneous cyclic and static loads.
- Load-independent chemical durability in corrosive chemicals.
- Strain-corrosion durability, especially of underground sanitation pipes.
- The anomalous and abrasive failures are also considered

The book contains two parts

Part 1 – Computation of the total strains

Chapters 1 through 7 – Total strains in commercial laminates

Part 2 – Computation of durability

Chapter 8 – The eight modes of failure
Chapter 9 – Effects of resin toughness, moisture and temperature
Chapter 10 – Chemical durability
Chapter 11 – Strain-corrosion durability
Chapter 12 – Abrasion life
Chapter 13 – Regression equations of plies and pipes
Chapter 14 – Weep failure and the classical HDB
Chapter 15 – Rupture failure
Chapter 16 – The unified equation
Chapter 17 – The unified equation applied to 15HR
Chapter 18 – Anomalous failure and impermeable pipes
Chapter 19 – Sudden-death and the regression equations
Chapter 20 – Strengths of UD55 oil pipes

Appendix 1 – The work of Regester

Appendix 2 – Paris law and the fatigue mechanism

The book presents a detailed explanation of the new concepts of failure thresholds and their pivotal role in the solution of chemical and load-dependent durability of industrial equipment. The failure thresholds and the new concepts introduced in this book should be of great interest to practicing engineers, researchers and students. All discussions are fully illustrated by well-designed numerical examples.

Cover photo: Water treatment plant, courtesy of Fibrav

Durability of Industrial Composites

Antonio Carvalho Filho

SECOND EDITION

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REVISED AND EXPANDED



Foreword

It ain't what you don't know that gets you into trouble, but what you know for sure and ain't so

Mark Twain

This book is divided in two parts. The first part, comprising seven chapters, describes a simple algorithm to compute the total strains (mechanical and residual) in plies of circular cylindrical laminates. The total ply strains are fundamental to compute the load-dependent durability of composite equipment. The reader familiar with the classical laminate theory may skip the part 1 and go directly to chapter 8 in part 2.

The part 2 of the book opens with the identification of the seven known modes of equipment failure. In spite of my efforts, I could not identify an eighth mode of failure. I suspect that it does not exist. The book is complete. The reader will find in it the models to compute the durability corresponding to the seven known modes of laminate failure.

The proposed models predict the durability of composite equipment in all possible scenarios, involving any aggressive chemical, moisture, temperature or mechanical loading. We separate the computation models in two groups, one dealing with environment-controlled and the other with load-controlled failures.

- *The models to solve the environmental durability problems are straightforward, requiring no outlandish hypothesis or new concepts.*
- *The models to solve the load-controlled durability problems, however, require a host of new hypothesis and assumptions, plus the rejection of a few old concepts.*

In this book, I introduce several new concepts and assumptions to compute the durability of laminates. I was uncertain to do so at first, as the ominous words of Mark Twain haunted me.

I am assuming things that I know for sure to be correct. What if it ain't so?

However, as time went on and as I delved deeper into the issues, my early doubts and concerns slowly dissipated. To the best of my knowledge, as I finish this writing, the several new assumptions and concepts introduced in this book are correct.

Consider, as examples, the following four bold proposals introduced in this book.

- *Except for weep failure, the load-controlled durability of any laminate is defined by its critical ply. This tremendous simplification reduces the computation of load-dependent laminate durability (except weep) to the analysis of plies.*
- *We dismiss as meaningless some old and cherished classical concepts, like the HDB, Sb and Sc.*
- *The pure cyclic regression equations of plies and laminates derive directly from their easy to measure short-term strengths and the constant fatigue threshold. This proposal eliminates the expensive experimental measurement of all pure cyclic equations.*
- *The unified equation computes the load-controlled durability of any laminate under any loading, using nothing more than the available pure cyclic and pure static regression equations of the*

critical ply. This tremendous simplification allows the computation of all load-controlled durability (except weep) of any laminate from known ply data.

The above are just a few of the many innovations introduced in this book. I have tried my very best to develop compelling arguments to support their correctness. Furthermore, I have introduced many easy and direct numerical examples to clarify and consolidate all new concepts.

The unified equation offers significant improvements over the classical Goodman diagrams currently used to compute load-dependent durability of laminates. There are three difficulties with the Goodman diagrams.

- *They are laminate and load specific, requiring new diagrams to accommodate changes in laminate configuration or loading.*
- *They ignore the time variable.*
- *They are currently restricted to rupture failures, leaving aside the far more important infiltration, weep and stiffness failures.*

Unlike the Goodman diagrams, the unified equation holds for all modes of load-dependent failure, all laminates all loadings, while taking into account the time variable. The unified equation is one of the highlights of this book.

The equations to compute the durability of industrial equipment are straightforward, intuitive and easy to use. No complex mathematical apparatus is required in their derivation and application. Furthermore, they have solid support from well-known published data. My role in writing this book consisted essentially in collecting, examining and reinterpreting the existing data. I see my contribution to the study of the durability of composites as that of James Clerk Maxwell to the electromagnetic theory. Like Maxwell, I did nothing more than collecting and reinterpreting scattered and apparently unrelated experimental data into meaningful equations. Like Maxwell, I did no experiment. I have simply collected and reinterpreted the existing data.

In writing this book, I have put considerable effort on clarity. The text is, perhaps, wordy and sometimes certainly repetitive. This I have done on purpose, for the sake of clarity. I have consciously sacrificed brevity and style to make sure the readers correctly understand the ideas and concepts as they are, with little risk of misinterpretation.

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December 11, 2024

Book review

The scanty literature on the durability of composites is fraught with misconceptions, glaring gaps and a multitude of obscure points. Any book proposing to eliminate these difficulties needs special attention.

The ambitious goal of the author is “*to quantify the durability of any laminate in any environment under any loading*”. This goal reaches far beyond anything in the current literature.

My job as a reviewer consists essentially in verifying the coverage and usefulness of the book, and checking the correctness of the proposed assumptions.

On the topic of usefulness, the author seems to have taken great pain in presenting detailed descriptions of the seven ageing processes of industrial laminates. The academic researcher will benefit from the clear descriptions of these ageing processes. The practicing engineer will appreciate the simplicity of the computation protocols. The young student will enjoy the opportunity to start a career in composites based on a comprehensive and clear platform.

As for coverage of the subject, the book appears to be complete and exhaustive, providing full solutions to the durability problem of all modes of industrial laminate failure.

In summary, the author provides a multitude of fundamental insights and solutions to important engineering problems, way beyond the current knowledge and methods.

Next, I will check the validity of the proposed hypotheses and their consistency.

I start with the unified equation, introduced to solve load-dependent durability problems. While the unified equation itself is not controversial, the unorthodox assumptions to derive its inputs may be. The two most controversial assumptions are:

- *The failure regression equations are independent of the working temperature and moisture. At first glance, such a sweeping and bold simplification seems highly suspicious.*
- *All pure cyclic regression lines, for any ply or laminate, converge on the same fatigue threshold. This implies in the same value for the fatigue threshold, regardless of the resin matrix, ply or laminate construction and the mode of failure. To begin with, there is no mention of a “fatigue threshold” in the published literature. Furthermore, how could it have the same value in all cases?*

The correctness of the load-dependent durability models developed in the book rests on the veracity of the above assumptions. In my opinion these assumptions, although at first glance hard to accept, are in fact reasonable.

Another point of interest is the assumption that failure of the critical ply defines the load-dependent laminate durability. This central simplification, similar to the classical first ply failure, shifts the difficult analysis of laminates to the simpler analysis of plies.

I would like to finish this review by highlighting the author’s efforts to present a clear and consistent set of descriptions of the ageing processes of composite materials. All readers, skeptic or otherwise, will marvel and greatly benefit from the many precious insights and explanations scattered throughout the text. Such powerful insights may well set the pathways for future research. Perhaps in time, as the readers and this

new technology mature, the hardest skeptics of today may accept the ideas herein proposed, if not in full, at least in part.

The enormous mass of insights and innovative solutions proposed by the author, while in some cases departing from the mainstream ideas, are nevertheless consistent with the accepted scientific methods. The proposed solutions promise new roads of development that may become paradigms for future research. Furthermore, the material in this book allows the systematic and accurate solution of durability problems outside the reach of the current knowledge. In this regard, this is a unique book, unmatched by any previous publications.

Taubate November 19, 2022

A handwritten signature in dark ink, appearing to read 'A. M. de O.', is positioned above a solid horizontal line.

Prof. Antonio Marmo de Oliveira.

Retired Full Professor of ITA (Instituto Tecnológico de Aeronáutica) S.P. Brasil

Comments from an unidentified reviewer on the first edition of this book

The author correctly limits the focus of this book to industrial composites operating in chemical environments, as the proposed approaches may not be relevant for other composites such as those found in aerospace, automotive, marine or civil structures.

The book deals with the durability problem of any laminate, application, loading and environment. It is not restricted to chemical service. Its relevance is obvious in applications involving aerospace, automotive, marine and civil structures.

Reviewer agrees with the author that the technical approaches described in the proposed book are different from those in other related books, but the proposal does not include evidence of validation of these different approaches. Accordingly, reviewer suggests that comparison of predictions from the proposed approaches with corresponding experimental results be included in the book.

All parameters used in the proposed computation models are experimental, taken from the published literature. The predictions from the proposed approaches, therefore, coincide with all available experimental results.

It is not clear whether the author intends the proposed book to be a textbook or a monograph, but the sample chapter has the appearance of a monograph (i.e., no worked out example problems or homework problems).

This is a book for professional engineers. The many carefully chosen and well-worked numerical examples are all pertinent to engineering practice. I have not included homework problems, as this is not a textbook. However, students would profit highly from reading this book.

In summary, the proposed book may contain useful practical information of interest to engineers who are concerned with industrial composites, but the book may not be as useful to students taking related courses.

I respectfully disagree. The initiating students will profit highly from this fresh, useful and practical approach.

Considering all of the existing and proposed books on the subject, reviewer is of the opinion that there is more of a need for a textbook on durability of composites than there is for another monograph on the subject.

This is not just another monograph. I have strived to develop and present a comprehensive set of innovative and revolutionary approaches to solve many until now unsolved problems.

The proposed book could be improved significantly by adding worked out example problems and homework problems so that it could be used as a textbook.

The book includes many worked out numerical examples. Homework problems will not be included.

Units

The quantities presented in this book have no reference to units.

All quantities, unless otherwise identified, enter the equations in the following units:

Quantity	unit
α – Coefficient of thermal expansion	/C
β – Coefficient of hydric expansion	/ Δm
E – Young's modulus	kg/cm ²
[Q] – Ply elastic matrix	kg/cm ²
Density	g/cm ³
t – Ply or laminate thickness	mm, cm
σ – Stress	kg/cm ²
[A] – Laminate stiffness matrix	kg/cm
[N ^T] – Laminate thermal resultant	kg/cm
[N ^H] – Laminate hydric resultant	kg/cm
[N ^M] – Mechanical resultant	kg/cm
P – Pressure	kg/cm ²
D – Diameter	mm, cm
RT – Room temperature	C
OT – Operating temperature	C
PT – Stress-free temperature	C
PPT – Unassisted peak process temperature	C
PCT – Post-cure temperature	C
HDT – Heat distortion temperature	C
t – Time	sec, month, hour, year
Δe – Eroded or penetrated depth	mm
Loading frequency	Hz

Acknowledgements

I am especially indebted to Carlos Marques, my intellectual partner of many years, for challenging my early false starts in the study of the durability of composites, and showing me the right direction. Carlos and I would often debate the durability of composites well into the night, trying to make sense of the available data and apparently conflicting reports. In doing this, we had no ambition or expectation of finding the solution ourselves. All we wanted and hoped for in those long gone days was to acquire a glimpse of the factors affecting the durability and the ageing processes of composite materials.

I wish Carlos were alive today, to see our early unpretentious discussions materialized in the present book.

I am also thankful to my colleagues Gabriel Gonzalez, Antonio Marmo de Oliveira, Francisco Xavier de Carvalho and Edouard Zurstrassen for their encouragement and guidance. Their expert criticism and suggestions made me rethink and rewrite entire chapters, leading to a better book.

Part 1

The laminates used in industrial service have five modes of failure, each leading to a different durability. Of those, the most important is certainly the resin deterioration from chemical exposure, followed by the damage accumulation from mechanical loads. The analysis of durability from chemical attack is simple, straightforward and easy to understand. By contrast, the mechanisms of load-induced deterioration are relatively complex, requiring several regression equations and the accurate knowledge of the total strains on the critical plies.

The part 1 of this book, through chapter 7, introduces a simple and easy protocol to compute the total strains on any ply embedded in circular cylindrical laminates. The regression equations are addressed in the part 2.

The reader familiar with the computation of total strains in embedded plies can skip the next seven chapters and go directly to the part 2 of the book, starting in chapter 8.

Chapter 1

Properties of standard plies

1 – Introduction: Plies are the smallest macroscopic structural units recognizable in composite laminates. On a microscopic level, the heterogeneous plies exhibit a clearly distinct and discontinuous fibrous phase embedded in a continuous resin matrix. However, on a macroscopic scale, it is usual to smear the small diameter fibers into the resin matrix to obtain essentially homogeneous plies. The reinforcing fibers may have several arrays in composite materials, like continuous and aligned to chopped and random. Furthermore, plies of resin matrix reinforced with granular sand or woven glass fabrics are also used. This book deals with plies of polyester or vinyl ester matrices reinforced with glass fibers.

Most practical composites applications involve laminates combining many distinct plies. This book discusses the durability of commercially important laminates combining several distinct plies. The orthotropic plies of woven or continuous UD fibers have two orthogonal principal directions identified by the digits 1 and 2. See figure 1. The axis 1 coincides with the fiber direction. The axis 2 is transverse to the fibers. The isotropic plies of sand, neat resin and chopped fibers have no preferential direction. The perpendicular axis 3, or z , is irrelevant to our discussions.

The properties of the macroscopically homogeneous plies vary with the fiber content, direction and arrangement, while having the same value everywhere.

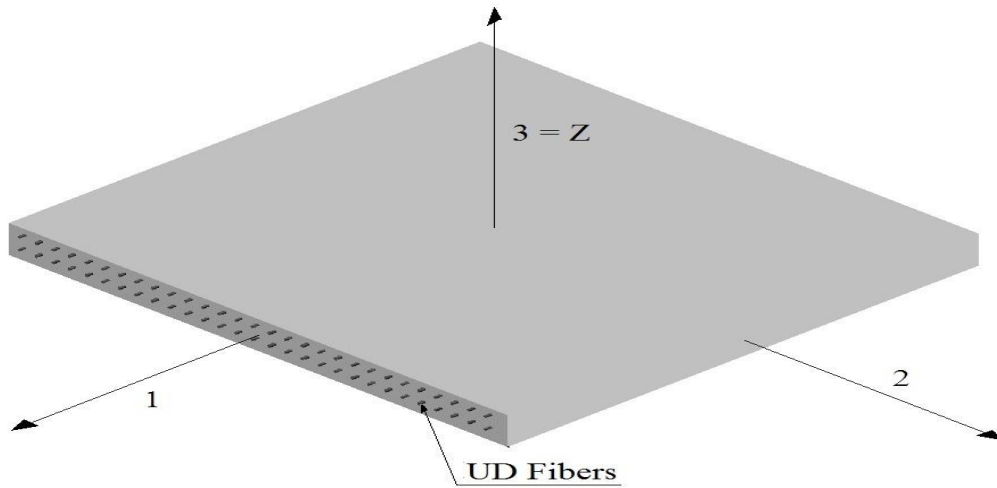


Figure 1

Unidirectional UD ply and the principal frame 1, 2, 3. The local axis 1 coincides with the fiber direction. The transverse axis 2 is perpendicular to the fibers. The properties of orthotropic UD plies vary with the direction. The isotropic plies of neat resin, or sand, or chopped fibers have the same properties in all directions.

Figure 2 shows the global axes “x” and “y” of a laminate composed of four plies. In the analysis of laminates, we need to know the ply positions and orientations. The angle “ α ” between the ply axis 1 and the global axis “x” defines the orientation. The ply position is the distance “z” from its middle to the laminate mid-surface. The distance “z” and the angle “ α ” completely define the position and orientation of orthotropic plies embedded in laminates.

In cylindrical laminates, we define the global frame with the “x” axis in the longitudinal and the “y” axis in the hoop direction. The plies can be laid up with their local axes 1 – 2 taking any orientation with respect to the global reference frame x – y.

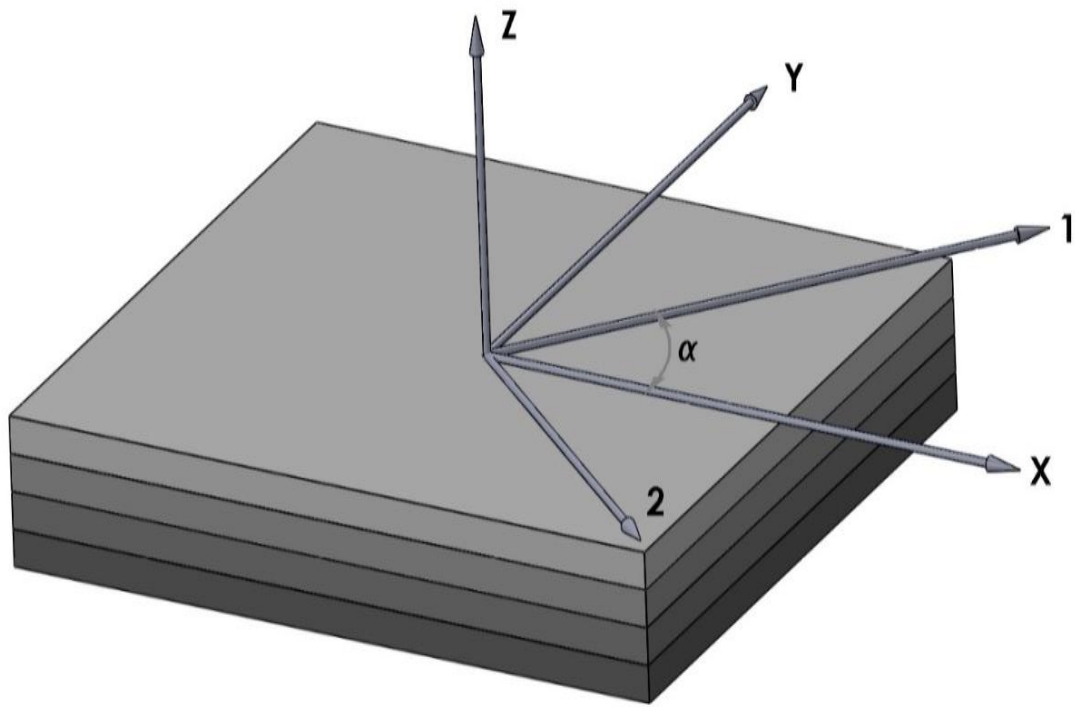


Figure 2

The ply position z is the distance from the laminate mid-surface to the ply mid-point. The angle " α " between the global " x " and local " 1 " axes defines the ply orientation. The commercial laminates may combine several plies in any position or orientation. The plies can take any arbitrary orientation, producing anisotropic laminates with different properties in every direction. In practice, all commercial laminates are balanced and orthotropic. Truly anisotropic laminates are not common in engineering applications.

The laminate properties derive from those of their constituent plies. These properties vary with the glass content, introducing considerable complications for designers. To eliminate this difficulty, we consider all laminate made of standard plies, with fixed glass contents. The properties of the standard laminates are not the same as those of the actual laminates, but this is not relevant. The concept of standard laminates made of standard plies is desirable for designers and manufacturers. For the designer it means no doubt as to the mechanical properties. For the manufacturer it means freedom to use any resin content that best suits his needs. Some manufactures prefer to use excess resin to facilitate their process, while others do the opposite to cut costs. The actual laminate turns out either thicker or thinner than the designed standard, while performing in essentially in the same way.

2 – The standard plies. The concept of standard ply is rather intuitive. Most engineers think in terms of standard plies even if not aware. This section brings out and emphasizes this intuitive knowledge. The standard plies are idealized entities, created especially for design purposes. To facilitate acceptance, their compositions are as close as possible to those of the real plies most commonly found in practice. This arbitrary choice makes the idealized standard plies very close to the real ones. By extension, the properties of standard and real laminates should differ just slightly. The designers to do their work using the known properties of standard laminates, instead of those of real ones. The error incurred in doing this is irrelevant, as long as the manufacturer uses the specified number of plies.

Before getting to the standard laminates, let us discuss the standard plies. The four commercial standard plies defined in this book have the following compositions:

- Neat resin

- 30% chopped glass fibers by weight
- 70% continuous UD glass fibers by weight
- 80% sand by weight

Table 1 lists the most important engineering properties required to characterize a standard ply. Note that all properties, including the failure thresholds, refer to the local ply frame 1 – 2. The failure thresholds are the familiar and well-known short-term ply strengths, referred to here by a different and fancier name. There is a failure threshold for every load-dependent mode of failure. The part 2 of this book gives a detailed discussion of this topic.

There are two ways to determine the mechanical properties of standard plies. The first, and most common, is by lab tests. As an alternative, we may use a variety of micromechanical models freely available in the literature. The lab tests work well in all cases. The micromechanical models apply to some ply properties, like the moduli and the coefficients of expansion, but cannot predict the failure thresholds. The failure thresholds are experimental.

Our discussion moves next to the measurement – or computation – of the standard ply properties. The first issue is the choice of the standard glass loading. This choice is arbitrary. In practice, we choose it to be as close as possible those most common in real plies. This arbitrary choice solves a long-standing issue of perennial discussion and disagreement between designers, fabricators and inspectors. The adoption of agreed upon standard plies, with fixed glass loadings and properties, eliminates this nightmare.

<i>Coefficients of thermal expansion</i>	<i>Coefficients of hydric expansion</i>
$[\alpha] = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ 0 \end{bmatrix}$	$[\beta] = \begin{bmatrix} \beta_1 \\ \beta_2 \\ 0 \end{bmatrix}$
<i>Elastic moduli</i>	<i>Poisson's ratios</i>
$[E] = \begin{bmatrix} E_1 \\ E_2 \\ G_{12} \end{bmatrix}$	$[\nu] = \begin{bmatrix} \nu_{12} \\ \nu_{21} \end{bmatrix}$
<i>Elastic matrix</i>	<i>Failure thresholds</i>
$[Q] = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}$	$\begin{bmatrix} T_i \\ T_w \\ T_s \\ T_r \\ T_0 \end{bmatrix} = \begin{bmatrix} \text{Infiltration} \\ \text{Weep} \\ \text{Stiffness} \\ \text{Rupture} \\ \text{Fatigue} \end{bmatrix}$
<i>Stiffness matrix</i>	
$[A] = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \times t$	

Table 1

Ply properties in the local frame 1 – 2. The listed values refer to measurements at room temperature in a dry atmosphere. The ply thickness is “t”.

The use of standard plies and laminates serve the interests of the fabricators, inspectors and designers. The designers would use non-controversial standard properties to specify the number of plies in any given laminate. The fabricator would use the specified number of plies with

no concern with the glass content. The inspector would check just the number of plies. The specification of the number of plies – with no reference to laminate thickness and glass loading – is enough to ascertain the desired performance. The actual glass loading, laminate thickness and mechanical properties are irrelevant as long as the number of plies are correct.

The concept of standard plies brings the following benefits:

- *The designer would do his work based on standard plies of known properties. He would compute and specify the required number, type and orientation of plies in the laminate. The standard laminate thickness and the glass weights in kg/cm² are easy to compute, if desired.*
- *The fabricator would execute the designer's specification with his mind set on the number of plies and unconcerned with the glass loading. The concept of standard plies facilitates the communication, understanding and execution at all levels in design, fabrication and inspection.*
- *The inspector would be pleased to check for nothing more than the presence of the specified plies. The use of standard plies eliminates the nagging discussions about laminate thickness, glass loadings and mechanical properties.*

This chapter tabulates the properties of commercial standard plies. To minimize discrepancies between computations and reality, the glass loadings of the standard plies agree as much as possible with their “natural” or “most easily achievable” values. For example, the glass loadings of UD plies varies in practice from 65% to 75%, with the most common and easily achievable average value of 70%. Therefore, the glass content of standard UD plies is taken as (UD) = 70%. The plies of randomly distributed chopped fibers have a natural glass loading of 30% by weight. Therefore, the glass loading for standard chopped plies is (chop) = 30%. And so on. The fiber arrangement and the properties of the constituent materials determine the properties of the standard plies. The next section discusses the constituent materials.

3 – The constituent materials: The ply properties derive from the constituents listed in table 2. The glass fibers have fixed composition and properties. Unlike the fibers, the resin matrices have widely different compositions and properties. However, in the presence of high modulus fibers, the resin differences smooth out, allowing the specification of the ply properties for a “representative” resin. The only ply properties affected by resin matrices are (a) the water absorption Δm and (b) the failure thresholds. The type of resin does not significantly affect the other properties, like the coefficients of thermal expansion, Poisson ratios, modulus of elasticity, regression slopes, etc.

<i>Property</i>	<i>Glass fibers</i>	<i>Resin matrix</i>
<i>Elastic modulus (kg/cm²)</i>	<i>750 000</i>	<i>30 000</i>
<i>Poisson's ratio</i>	<i>0.20</i>	<i>0.30</i>
<i>Coefficient of thermal expansion (1/C)</i>	<i>5.5×10^{-6}</i>	<i>60.0×10^{-6}</i>
<i>Density (g/cm³)</i>	<i>2.62</i>	<i>1.20</i>
<i>Elongation at break (%)</i>	<i>-----</i>	<i>1.0 – 5.0</i>
<i>Water saturation (%)</i>	<i>0.0</i>	<i>1.0</i>

Table 2

Typical resin and fiber properties. The elongation at break of the resin has a strong effect on the failure thresholds.

4 – Properties of standard plies: As just discussed, the properties of the standard plies – except the thresholds – are resin-independent. This simplification facilitates the analysis with negligible effect on the accuracy. The effect of different resins will emerge in the part 2 of this book, with the introduction and discussion of the failure thresholds.

As an additional simplification, we assume the invariance of the standard ply properties with respect to the operating temperature OT and moisture. These are reasonable assumptions, as long as the operating temperature OT is below the resin HDT and the water pick up Δm is small. The water saturation of commercial resins is indeed small (around 1.0%) and do not affect the ply properties much. For a detailed justification of this assumption, see chapter 10.

As mentioned earlier, there are two ways to measure the standard ply properties. We will develop a few simple numerical examples to illustrate both methods. The derivation from the constituents is straightforward, using the micromechanical equations available in the published literature. The derivation from lab tests requires a simple correction discussed as a numerical example in the next chapter.

Example 1: Derive the tensile modulus, the Poisson's ratio and the coefficient of thermal expansion of standard UD plies in the fiber direction 1.

Since all fibers are parallel and aligned in the principal ply direction 1, the tensile modulus derives from the simple law of mixtures

$$E_1 = (\text{glass})_{vol} \times E_{glass} + (\text{resin})_{vol} \times E_{resin} \quad (1)$$

The glass and resin loadings by volume derive from the densities and standard weight fractions of the constituents. The standard weight fraction of glass in UD plies is $(UD) = 0.70$. Table 2 lists the glass and resin densities. The following values apply to standard UD plies.

$$(\text{glass})_{vol} = \frac{\frac{0.70}{2.62}}{\frac{0.70}{2.62} + \frac{0.30}{1.20}} = 0.52$$

$$(\text{resin})_{vol} = \frac{\frac{0.30}{1.20}}{\frac{0.70}{2.62} + \frac{0.30}{1.20}} = 0.48$$

Entering the above in equation (1), we obtain

$$E_1 = 0.52 \times E_{glass} + 0.48 \times E_{resin}$$

Table 2 lists the elastic moduli of the glass and resin.

$$E_1 = 0.52 \times 750000 + 0.48 \times 30000 \approx 400000 \text{ kg/cm}^2$$

The same reasoning applies to the Poisson's ratio

$$\nu_{12} = (\text{glass})_{vol} \times \nu_{glass} + (\text{resin})_{vol} \times \nu_{resin}$$

$$\nu_{12} = 0.52 \times 0.20 + 0.48 \times 0.30 = 0.25$$

The equation to compute the coefficient of thermal expansion is slightly more elaborate

$$\alpha_1 = \frac{\alpha_{glass} \times glass_{vol} \times E_{glass} + \alpha_{resin} \times resin_{vol} \times E_{resin}}{E_1}$$

$$\alpha_1 = \frac{5.5 \times 10^{-6} \times 0.52 \times 750000 + 60 \times 10^{-6} \times 0.48 \times 30000}{400000} = 7.5 \times 10^{-6}$$

Similar micromechanical equations exist to compute other ply properties.

Example 2: Derive the expressions linking the glass weight in kg/m² to the thicknesses of the standard ply in mm.

The applicable formulas are

$$\left[\text{Glass weight in } \frac{kg}{m^2} \right] = (glass)_{weight} \times area \times density \times thickness$$

$$density = \frac{1}{\frac{(glass)_{weight}}{2.62} + \frac{(resin)_{weight}}{1.20}}$$

For standard UD plies, we have

$$(glass)_{weight} = 0.70$$

$$(resin)_{weight} = 0.30$$

Entering these in the above equations, we obtain

$$\left[\text{glass weight in } \frac{kg}{m^2} \right]_{UD} = 0.70 \times 1 \times \frac{1}{\frac{0.70}{2.62} + \frac{0.30}{1.20}} \times t_{UD}$$

$$\left[\text{glass weight in } \frac{kg}{m^2} \right]_{UD} = 1.5 \times t_{UD} \text{ (} t_{UD} \text{ in mm)}$$

For standard plies of chopped glass, we have

$$(glass)_{weight} = 0.30$$

$$(resin)_{weight} = 0.70$$

$$\left[\text{glass weight in } \frac{kg}{m^2} \right]_{chop} = 0.30 \times 1 \times \frac{1}{\frac{0.30}{2.62} + \frac{0.70}{1.20}} \times t_{chop}$$

$$\left[\frac{kg}{m^2} \right]_{chop} = 0.43 \times t_{chop} \quad (t_{chop} \text{ in mm})$$

For standard plies of sand we have

$$(sand)_{weight} = 0.80$$

$$(resin)_{weight} = 0.20$$

$$\left[\text{sand weight in } \frac{kg}{m^2} \right]_{sand} = 0.80 \times 1 \times \frac{1}{\frac{0.80}{2.62} + \frac{0.20}{1.20}} \times t_{sand}$$

$$\left[\frac{kg}{m^2} \right]_{sand} = 1.69 \times t_{sand} \quad (t_{sand} \text{ in mm})$$

Table 3 summarizes the above results

Standard ply	Glass weight and ply thickness (mm)
70% by weight UD	$\left[\frac{kg}{m^2}\right]_{UD} = 1.35 \times t_{UD}$
30% by weight chopped fibers	$\left[\frac{kg}{m^2}\right]_{chop} = 0.43 \times t_{chop}$
80% by weight sand	$\left[\frac{kg}{m^2}\right]_{sand} = 1.69 \times t_{sand}$

Table 3

Glass weight and ply thickness for standard plies. The ply thicknesses are in mm.

Example 3: Compute the standard thickness of a laminate with the following weight composition.

UD fibers: 5.0 kg/m^2 ; Chopped fibers: 1.2 kg/m^2 ; Sand: 10.5 kg/m^2

The laminate standard thickness is

$$t = t_{UD} + t_{chop} + t_{sand}$$

Using the relations from table 3 we have

$$t = \frac{5.0}{1.35} + \frac{1.2}{0.43} + \frac{10.5}{1.69} = 12.71 \text{ mm}$$

The above is the standard laminate thickness. The true thickness, measured on the actual laminate, is different.

Example 4: A thickness measurement prior to burnout of the laminate discussed in example 3 revealed a thickness of 15.3 mm. This is $15.3 - 12.71 = 2.59 \text{ mm}$ more than the standard value. Explain the discrepancy.

The excess measured thickness $15.3 - 12.71 = 2.59 \text{ mm}$ corresponds to a fictitious ply of neat resin distributed throughout the actual laminate. The actual laminate is therefore equivalent to a standard laminate with the following standard ply construction.

- Standard UD ply of thickness $\frac{5.0}{1.35} = 3.70 \text{ mm}$
- Standard chopped ply of thickness $\frac{1.2}{0.43} = 2.79 \text{ mm}$
- Standard core ply of thickness $\frac{10.5}{1.69} = 6.21 \text{ mm}$
- Ply of neat resin of thickness 2.59 mm .

The excess resin in this fictitious ply is spread in the laminate thickness.

5 – The elastic matrix [Q]: The previous section illustrated the use of micromechanical equations to derive the engineering properties of standard plies. In this section, we will derive the elastic matrices [Q] of standard plies from those engineering properties. We do this because it is easier to work with ply matrices [Q] instead of engineering properties. The standard elastic [Q] matrices tabulated in this chapter are very accurate.

The constitutive equation of plies under plane stress referred to the principal axes 1 – 2 is

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \times \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \quad (2)$$

Equation (2) defines the ply elastic matrix [Q] in the local ply frame 1 – 2.

$$[Q]_{12} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}$$

The ply elastic matrix in terms of the engineering properties is

$$[Q]_{12} = \begin{bmatrix} \frac{E_1}{1-\nu_{12}\nu_{21}} & \frac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}} & 0 \\ \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} & \frac{E_2}{1-\nu_{12}\nu_{21}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \quad (2A)$$

The reader will note the ply constitutive equation (2) expressed in terms of stresses and moduli, as homogeneous materials. The concepts of stresses and moduli are valid for macroscopic homogeneous plies. The concepts of stresses and moduli are not applicable to non-homogeneous laminates, combining different plies.

The following tables present the properties of a few select standard commercial plies. The data are valid for both dry and wet plies, at any temperature below the resin HDT. The next chapter discusses the laminates and their properties.

Example 5: Compute the stresses in the principal directions of standard UD plies subjected to the following strains.

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 0.015 \\ 0.002 \\ 0 \end{bmatrix}$$

The ply stresses derive from the constitutive equation (2)

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \times \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}$$

Table 4 lists the standard elastic matrix [Q] of UD plies

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} 400000 & 30000 & 0 \\ 30000 & 100000 & 0 \\ 0 & 0 & 35000 \end{bmatrix} \times \begin{bmatrix} 0.015 \\ 0.002 \\ 0 \end{bmatrix}$$

Developing the above, we have

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} 6060 \\ 650 \\ 0 \end{bmatrix} \frac{kg}{cm^2}$$

Coefficients of thermal expansion	Coefficients of hydric expansion

$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_{12} \end{bmatrix} = \begin{bmatrix} 60.0 \\ 60.0 \\ 0 \end{bmatrix} \times 10^{-6} / ^\circ C$	$\begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_{12} \end{bmatrix} = \begin{bmatrix} 0.40 \\ 0.40 \\ 0 \end{bmatrix} l/\Delta m$
<i>Modulus of elasticity</i>	<i>Poisson's ratio</i>
$\begin{bmatrix} E_1 \\ E_2 \\ G_{12} \end{bmatrix} = \begin{bmatrix} 30000 \\ 30000 \\ 11500 \end{bmatrix} kg/cm^2$	$\begin{bmatrix} \nu_{12} \\ \nu_{21} \end{bmatrix} = \begin{bmatrix} 0.30 \\ 0.30 \end{bmatrix}$
<i>Elastic matrix</i>	<i>Failure thresholds</i>
$\begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} = \begin{bmatrix} 33000 & 10000 & 0 \\ 10000 & 33000 & 0 \\ 0 & 0 & 11500 \end{bmatrix} kg/cm^2$	$\begin{bmatrix} T_i \\ T_w \\ T_s \\ T_r \\ T_0 \end{bmatrix} = \begin{bmatrix} - & - \\ - & - \\ - & - \\ - & - \\ - & - \end{bmatrix}$
<i>Stiffness matrix</i>	
$[A] = \begin{bmatrix} 33000 & 10000 & 0 \\ 10000 & 33000 & 0 \\ 0 & 0 & 11500 \end{bmatrix} \times t \text{ kg/cm}$	

Table 4

Standard ply of neat resin in the local frame. The plies of neat resin are not critical for load-dependent failures and therefore have no failure thresholds.

<i>Coefficients of thermal expansion</i>	<i>Coefficients of hydric expansion</i>
$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_{12} \end{bmatrix} = \begin{bmatrix} 25.0 \\ 25.0 \\ 0 \end{bmatrix} \times 10^{-6} / ^\circ C$	$\begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_{12} \end{bmatrix} = \begin{bmatrix} 0.19 \\ 0.19 \\ 0 \end{bmatrix}$
<i>Modulus of elasticity</i>	<i>Poisson's ratio</i>
$\begin{bmatrix} E_1 \\ E_2 \\ G_{12} \end{bmatrix} = \begin{bmatrix} 70000 \\ 70000 \\ 27000 \end{bmatrix} kg/cm^2$	$\begin{bmatrix} \nu_{12} \\ \nu_{21} \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.25 \end{bmatrix}$
<i>Elastic matrix</i>	<i>Failure thresholds</i>
$\begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} = \begin{bmatrix} 77000 & 23000 & 0 \\ 23000 & 77000 & 0 \\ 0 & 0 & 27000 \end{bmatrix} kg/cm^2$	$\begin{bmatrix} T_i \\ T_w \\ T_s \\ T_r \\ T_0 \end{bmatrix} = \begin{bmatrix} 0.30\% - 0.50\% \\ 0.80\% - 1.10\% \\ - & - \\ 1.50\% - 2.50\% \\ 0.05\% \end{bmatrix}$
<i>Stiffness matrix</i>	
$[A] = \begin{bmatrix} 77000 & 23000 & 0 \\ 23000 & 77000 & 0 \\ 0 & 0 & 27000 \end{bmatrix} \times t \text{ kg/cm}$	

Table 5

Standard ply of chopped glass in the local frame. The isotropic failure thresholds vary with the resin toughness.

<i>Coefficients of thermal expansion</i>	<i>Coefficients of hydric expansion</i>

$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_{12} \end{bmatrix} = \begin{bmatrix} 7.5 \\ 45.0 \\ 0 \end{bmatrix} \times 10^{-6} / ^\circ C$	$\begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_{12} \end{bmatrix} = \begin{bmatrix} 0,04 \\ 0,18 \\ 0 \end{bmatrix}$
<i>Modulus of elasticity</i>	<i>Poisson's ratio</i>
$\begin{bmatrix} E_1 \\ E_2 \\ G_{12} \end{bmatrix} = \begin{bmatrix} 400000 \\ 100000 \\ 35000 \end{bmatrix} kg/cm^2$	$\begin{bmatrix} \nu_{12} \\ \nu_{21} \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.07 \end{bmatrix}$
<i>Elastic matrix</i>	<i>Thresholds</i>
$\begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} = \begin{bmatrix} 400000 & 30000 & 0 \\ 30000 & 100000 & 0 \\ 0 & 0 & 35000 \end{bmatrix} kg/cm^2$	$\begin{bmatrix} T_i \\ T_w \\ T_s \\ T_r \\ T_0 \end{bmatrix} = \begin{bmatrix} 0.25\% - 0.50\% \\ 0.25\% - 0.55\% \\ 0.40\% - 1.00\% \\ 0.40\% - 1.00\% \\ 0.05\% \end{bmatrix}$
<i>Stiffness matrix</i>	
$[A] = \begin{bmatrix} 400000 & 30000 & 0 \\ 30000 & 10000 & 0 \\ 0 & 0 & 35000 \end{bmatrix} \times t kg/cm$	

Table 6

Standard UD ply in the local frame. The transverse failure thresholds vary with the resin toughness.

<i>Coefficients of thermal expansion</i>	<i>Coefficients of hydric expansion</i>
$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_{12} \end{bmatrix} = \begin{bmatrix} 13.0 \\ 13.0 \\ 0 \end{bmatrix} \times 10^{-6} / ^\circ C$	$\begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_{12} \end{bmatrix} = \begin{bmatrix} 0,15 \\ 0,15 \\ 0 \end{bmatrix}$
<i>Modulus of elasticity</i>	<i>Poisson's ratio</i>
$\begin{bmatrix} E_1 \\ E_2 \\ G_{12} \end{bmatrix} = \begin{bmatrix} 60000 \\ 60000 \\ 23000 \end{bmatrix} \frac{kg}{cm^2}$	$\begin{bmatrix} \nu_{12} \\ \nu_{21} \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.25 \end{bmatrix}$
<i>Elastic matrix</i>	<i>Thresholds</i>
$\begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} = \begin{bmatrix} 66000 & 20000 & 0 \\ 20000 & 66000 & 0 \\ 0 & 0 & 23000 \end{bmatrix} \frac{kg}{cm^2}$	$\begin{bmatrix} T_i \\ T_w \\ T_s \\ T_r \\ T_0 \end{bmatrix} = \begin{bmatrix} \text{Not applicable} \\ \text{Not applicable} \\ \text{Not applicable} \\ \text{Not applicable} \\ \text{Not applicable} \end{bmatrix}$
<i>Stiffness matrix</i>	
$[A] = \begin{bmatrix} 66000 & 20000 & 0 \\ 20000 & 66000 & 0 \\ 0 & 0 & 23000 \end{bmatrix} \times t kg/cm$	

Table 7

Standard sand-filled ply in the local frame. The sand plies are not critical for load-dependent failures and therefore have no failure thresholds.

Example 6: Use equation (2A) to derive the elastic matrix $[Q]$ of UD plies.

$$[Q]_{12} = \begin{bmatrix} \frac{E_1}{1 - \nu_{12}\nu_{21}} & \frac{\nu_{21}E_1}{1 - \nu_{12}\nu_{21}} & 0 \\ \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} & \frac{E_2}{1 - \nu_{12}\nu_{21}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix}$$

Entering the data from table 6 in the above, we have

$$[Q]_{12} = \begin{bmatrix} \frac{400000}{1 - 0.07 \times 0.25} & \frac{0.07 \times 400000}{1 - 0.07 \times 0.25} & 0 \\ \frac{0.30 \times 100000}{1 - 0.07 \times 0.25} & \frac{100000}{1 - 0.07 \times 0.25} & 0 \\ 0 & 0 & 35000 \end{bmatrix}$$

$$[Q]_{12} = \begin{bmatrix} 400000 & 30000 & 0 \\ 30000 & 100000 & 0 \\ 0 & 0 & 35000 \end{bmatrix}$$

6 – Ply properties in the global frame. –To compute the laminate standard matrices, all ply properties must be referred to the global frame x – y. This section describes the protocols to rotate the matrices [Q], [α] and [β] of UD plies from their local frames 1 – 2 to the global frame x – y.

The elastic matrix [Q]₁₂ of UD plies in the local system 1 – 2 is

$$[Q]_{12} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}$$

When rotated to the global system x – y, this matrix becomes

$$[Q]_{xy} = \begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{yx} & Q_{yy} & Q_{ys} \\ Q_{sx} & Q_{sy} & Q_{ss} \end{bmatrix}$$

The equations to compute the elements of the rotated matrix [Q]_{xy} in the global system are.

$$\begin{aligned} Q_{xx} &= m^4 Q_{11} + n^4 Q_{22} + 2m^2 n^2 Q_{12} + 4m^2 n^2 Q_{66} \\ Q_{xy} &= Q_{yx} = m^2 n^2 Q_{11} + m^2 n^2 Q_{22} + (m^4 + n^4) Q_{12} - 4m^2 n^2 Q_{66} \\ Q_{xs} &= Q_{sx} = m^3 n Q_{11} - mn^3 Q_{22} + (mn^3 - m^3 n) Q_{12} + 2(mn^3 - m^3 n) Q_{66} \\ Q_{yy} &= n^4 Q_{11} + m^4 Q_{22} + 2m^2 n^2 Q_{12} + 4m^2 n^2 Q_{66} \\ Q_{ys} &= Q_{sy} = mn^3 Q_{11} - m^3 n Q_{22} + (m^3 n - mn^3) Q_{12} + 2(m^3 n - mn^3) Q_{66} \\ Q_{ss} &= m^2 n^2 Q_{11} + m^2 n^2 Q_{22} - 2m^2 n^2 Q_{12} + (m^2 - n^2)^2 Q_{66} \end{aligned} \quad (3)$$

Where “m” is the cosine and “n” is the sine of the angle α formed by the local ply axis 1 and the global laminate axis “x”.

The equations to rotate the hydric and thermal expansion coefficients of UD plies from the local to the global frame are.

$$\begin{aligned} \alpha_x &= m^2 \alpha_1 + n^2 \alpha_2 \\ \alpha_y &= n^2 \alpha_1 + m^2 \alpha_2 \\ \alpha_{xy} &= 2mn(\alpha_2 - \alpha_1) \end{aligned} \quad (4)$$

$$\begin{aligned} \beta_x &= m^2 \beta_1 + n^2 \beta_2 \\ \beta_y &= n^2 \beta_1 + m^2 \beta_2 \\ \beta_{xy} &= 2mn(\beta_2 - \beta_1) \end{aligned} \quad (5)$$

Where again “m” is the cosine and “n” is the sine of the angle α formed by the local ply axis 1 and the longitudinal global axis “x”. This completes the rotation protocol for UD plies. We next apply these equations to UD55, UD70 and UD90 plies.

6.1 – Matrices of UD55 plies in the global frame. The elastic matrix $[Q_{12}]$ of standard UD plies in the local frame is

$$\begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} = \begin{bmatrix} 400000 & 30000 & 0 \\ 30000 & 100000 & 0 \\ 0 & 0 & 35000 \end{bmatrix}$$

The elements of the ply elastic matrix $[Q_{xy}]$ in the global frame are computed as indicated in equation (3) for $\alpha = 55$ degrees.

$$\begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{yx} & Q_{yy} & Q_{ys} \\ Q_{sx} & Q_{sy} & Q_{ss} \end{bmatrix} = \begin{bmatrix} 132470 & 96220 & \pm 46370 \\ 96220 & 235070 & \pm 94580 \\ \pm 46370 & \pm 94580 & 101220 \end{bmatrix}$$

This is the rotated elastic matrix of UD55 plies, where the $\pm$ signs refer to the ply orientation $+ 55$ or $- 55$ degrees. The stiffness matrix of the ply is the product of its elastic matrix and thickness

$$[A] = \begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{yx} & Q_{yy} & Q_{ys} \\ Q_{sx} & Q_{sy} & Q_{ss} \end{bmatrix} \times t = \begin{bmatrix} 132470 & 96220 & \pm 46370 \\ 96220 & 235070 & \pm 94580 \\ \pm 46370 & \pm 94580 & 101220 \end{bmatrix} \times t$$

Since all commercial laminates are balanced, the $\pm$ terms cancel out when combined in pairs. Therefore, the above stiffness matrix reduces to

$$[A]_{55} = \begin{bmatrix} 132470 & 96220 & 0 \\ 96220 & 235070 & 0 \\ 0 & 0 & 101220 \end{bmatrix} \times t$$

In the above, “t” is the thickness of the UD55 plies when combined in pairs, as is always the case in balanced laminates.

The coefficients of hydic expansion of the individual UD55 plies in the global frame are

$$\begin{aligned} \beta_x &= m^2\beta_1 + n^2\beta_2 \\ \beta_y &= n^2\beta_1 + m^2\beta_2 \\ \beta_{xy} &= 2mn(\beta_2 - \beta_1) \end{aligned}$$

$$\begin{aligned} \beta_x &= 0.33 \times 0.04 + 0.67 \times 0.18 = 0.13 \\ \beta_y &= 0.67 \times 0.04 + 0.33 \times 0.18 = 0.09 \\ \beta_{xy} &= \pm 2 \times 0.67 \times 0.33(0.18 - 0.04) = \pm 0.06 \end{aligned}$$

The hydic force resultants of UD55 plies in the global frame are

$$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = \begin{bmatrix} Q_{xx} \times \beta_x + Q_{xy} \times \beta_y + Q_{xs} \times \beta_{xy} \\ Q_{yx} \times \beta_x + Q_{yy} \times \beta_y + Q_{ys} \times \beta_{xy} \\ 0 \end{bmatrix} \times t \times \Delta m$$

$$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = \begin{bmatrix} 132470 \times 0.13 + 96220 \times 0.09 + 46370 \times 0.06 \\ 96220 \times 0.13 + 235070 \times 0.09 + 94580 \times 0.06 \\ 0 \end{bmatrix} \times t \times \Delta m$$

$$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = \begin{bmatrix} 28663 \\ 39340 \\ 0 \end{bmatrix} \times t \times \Delta m$$

Where “t” is the thickness of the UD55 plies. These resultants are directly proportional to the ply thickness “t” and the water pick up “Δm”.

The derivation of the thermal resultants follows the same procedure. The coefficients of thermal expansion of the UD plies in the global system come from (5) by taking $m = \cos(\pm 55) = 0.573$ and $n = \sin(\pm 55) = \pm 0.819$.

$$\alpha_x = (0.573)^2 \times 7.5 \times 10^{-6} + (0.819)^2 \times 45.0 \times 10^{-6} = 32.0 \times 10^{-6}$$

$$\alpha_y = (0.819)^2 \times 7.5 \times 10^{-6} + (0.573)^2 \times 45.0 \times 10^{-6} = 19.9 \times 10^{-6}$$

$$\alpha_{xy} = \pm 2 \times 0.573 \times 0.819 \times (45.0 - 7.5) \times 10^{-6} = \pm 35.2 \times 10^{-6}$$

The thermal force resultants of UD plies referred to the global axes are

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} Q_{xx} \times \alpha_x + Q_{xy} \times \alpha_y + Q_{xs} \times \alpha_{xy} \\ Q_{yx} \times \alpha_x + Q_{yy} \times \alpha_y + Q_{ys} \times \alpha_{xy} \\ 0 \end{bmatrix} \times t \times (OT - PT)$$

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} 132470 \times 32.0 + 96220 \times 19.9 + 46370 \times 35.2 \\ 96220 \times 32.0 + 235070 \times 19.9 + 94580 \times 35.2 \\ 0 \end{bmatrix} \times 10^{-6} \times t \times (OT - PT)$$

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} 7.86 \\ 11.14 \\ 0 \end{bmatrix} \times t \times (OT - PT)$$

Where t is the ply thickness. The temperatures enter the equation in degrees Celcius. The thermal resultants are usually negative since in most applications $PT > OT$. The thermal resultants are proportional to the ply thickness “t” and to the temperature drop $\Delta T = PT - OT$. Table 8 shows the matrices of UD55 plies in the global system.

$[Q] = \begin{bmatrix} 132470 & 96220 & \pm 46370 \\ 96220 & 235070 & \pm 94580 \\ \pm 46370 & \pm 94580 & 101220 \end{bmatrix}$
$[A]_{55} = \begin{bmatrix} 132470 & 96220 & 0 \\ 96220 & 235070 & 0 \\ 0 & 0 & 101220 \end{bmatrix} \times t$
$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} 7.86 \\ 11.14 \\ 0 \end{bmatrix} \times t \times (OT - PT)$
$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = \begin{bmatrix} 28663 \\ 39340 \\ 0 \end{bmatrix} \times t \times \Delta m$

Table 8

Matrices of standard UD55 plies in the global system.

6.2 – Matrices of UD70 plies in the global frame. The elastic matrix $[Q]_{xy}$ of UD70 plies in the global system is computed taking $m = \cos(\pm 70) = 0.34$ and $n = \sin(\pm 70) = \pm 0.94$ in equation (1).

$$\begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{yx} & Q_{yy} & Q_{ys} \\ Q_{sx} & Q_{sy} & Q_{ss} \end{bmatrix} = \begin{bmatrix} 104000 & 61000 & \pm 11300 \\ 61000 & 334000 & \pm 85140 \\ \pm 11300 & \pm 85140 & 66000 \end{bmatrix}$$

The stiffness matrix of UD70 plies in the global frame is

$$[A] = \begin{bmatrix} 104000 & 61000 & 0 \\ 61000 & 334000 & 0 \\ 0 & 0 & 66000 \end{bmatrix} \times t$$

In the above, t is the thickness of the UD70 plies when used in pairs, as is always the case in balanced laminates.

The thermal and hydric coefficients of the UD ply in the global system are.

$$\alpha_x = (\cos 70)^\circ (7.5 \times 10^{-6}) + (\sin 70)^\circ (45.0 \times 10^{-6}) = 40.61 \times 10^{-6}$$

$$\alpha_y = (\sin 70)^\circ (7.5 \times 10^{-6}) + (\cos 70)^\circ (45.0 \times 10^{-6}) = 11.89 \times 10^{-6}$$

$$\alpha_{xy} = 2 \times (\cos 70)(\sin 70)(45.0 - 7.5) \times 10^{-6} = \pm 24.10 \times 10^{-6}$$

$$\beta_x = 0.12 \times 0.04 + 0.88 \times 0.18 = 0.16$$

$$\beta_y = 0.88 \times 0.04 + 0.12 \times 0.18 = 0.06$$

$$\beta_{xy} = \pm 2 \times 0.88 \times 0.12 \times (0.18 - 0.04) = \pm 0.03$$

The hydric force resultants are

$$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = \begin{bmatrix} Q_{xx} \times \beta_x + Q_{xy} \times \beta_y + Q_{xs} \times \beta_{xy} \\ Q_{yx} \times \beta_x + Q_{yy} \times \beta_y + Q_{ys} \times \beta_{xy} \\ 0 \end{bmatrix} \times t \times \Delta m$$

$$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = \begin{bmatrix} 104000 \times 0.16 + 61000 \times 0.06 + 11300 \times 0.03 \\ 61000 \times 0.16 + 334000 \times 0.06 + 85140 \times 0.03 \\ 0 \end{bmatrix} \times t \times \Delta m$$

$$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = \begin{bmatrix} 20640 \\ 32350 \\ 0 \end{bmatrix} \times t \times \Delta m$$

The thermal force resultants are

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} Q_{xx} \times \alpha_x + Q_{xy} \times \alpha_y + Q_{xs} \times \alpha_{xy} \\ Q_{yx} \times \alpha_x + Q_{yy} \times \alpha_y + Q_{ys} \times \alpha_{xy} \\ 0 \end{bmatrix} \times t \times (OT - PT)$$

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} 104000 \times 40.61 + 61000 \times 11.89 + 11300 \times 24.10 \\ 61000 \times 40.61 + 334000 \times 11.89 + 85140 \times 24.10 \\ 0 \end{bmatrix} \times 10^{-6} \times t \times (OT - PT)$$

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} 5.22 \\ 8.50 \\ 0 \end{bmatrix} \times t \times (OT - PT)$$

Table 9 summarizes all of the above.

$[Q] = \begin{bmatrix} 104000 & 61000 & \pm 11300 \\ 61000 & 334000 & \pm 85140 \\ \pm 11300 & \pm 85140 & 66000 \end{bmatrix}$
$[A] = \begin{bmatrix} 104000 & 61000 & 0 \\ 61000 & 334000 & 0 \\ 0 & 0 & 66000 \end{bmatrix} \times t$
$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} 5.22 \\ 8.50 \\ 0 \end{bmatrix} \times t \times (OT - PT)$
$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = \begin{bmatrix} 20640 \\ 32350 \\ 0 \end{bmatrix} \times t \times \Delta m$

Table 9

Standard matrices of UD70 plies in the global frame.

6.3 – Matrices of UD90 plies in the global frame. The UD90 plies are laid up with the fibers in the hoop direction, at an angle $\alpha = 90$ degrees to the longitudinal axis. From equation (1), and taking into account that $m = \cos(90) = 0$ and $n = \sin(90) = 1$, the elastic matrix $[Q]_{xy}$ for the UD90 ply in the global system is

$$\begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{yx} & Q_{yy} & Q_{ys} \\ Q_{sx} & Q_{sy} & Q_{ss} \end{bmatrix} = \begin{bmatrix} 100000 & 30000 & 0 \\ 30000 & 400000 & 0 \\ 0 & 0 & 35000 \end{bmatrix}$$

The UD coefficients of thermal expansion of the UD plies in the global system are

$$\alpha_x = (\cos 90)^2 (7.5 \times 10^{-6}) + (\sin 90)^2 (45.0 \times 10^{-6}) = 45.00 \times 10^{-6}$$

$$\alpha_y = (\sin 90)^2 (7.5 \times 10^{-6}) + (\cos 90)^2 (45.0 \times 10^{-6}) = 7.50 \times 10^{-6}$$

$$\alpha_{xy} = 0$$

In a like manner, the global hydric expansion coefficients of UD plies are

$$\beta_x = (\cos 90)^2 \times 0.04 + (\sin 90)^2 \times 0.18 = 0.18$$

$$\beta_y = (\sin 90)^2 \times 0.04 + (\cos 90)^2 \times 0.18 = 0.04$$

$$\beta_{xy} = 2mn(\beta_2 - \beta_1) = 0.0$$

The stiffness matrix $[A]$ of the UD90 plies in the global frame is

$$[A] = \begin{bmatrix} 100000 & 30000 & 0 \\ 30000 & 400000 & 0 \\ 0 & 0 & 35000 \end{bmatrix} \times t$$

The thermal and hydric force resultants are

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} 4.73 \\ 4.35 \\ 0 \end{bmatrix} \times t \times (OT - PT)$$

$$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = \begin{bmatrix} 19200 \\ 21400 \\ 0 \end{bmatrix} \times t \times \Delta m$$

The computed results are in table 10.

$[Q] = \begin{bmatrix} 100000 & 30000 & 0 \\ 30000 & 400000 & 0 \\ 0 & 0 & 35000 \end{bmatrix}$
$[A] = \begin{bmatrix} 100000 & 30000 & 0 \\ 30000 & 400000 & 0 \\ 0 & 0 & 35000 \end{bmatrix} \times t$
$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} 4.73 \\ 4.35 \\ 0 \end{bmatrix} \times t \times (OT - PT)$
$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = \begin{bmatrix} 19200 \\ 21400 \\ 0 \end{bmatrix} \times t \times \Delta m$

Table 10

Matrices of UD90 plies in the global frame.

6.4 – Matrices of chopped fibers ply in the global system. The chopped fibers plies are isotropic and do not require rotation.

The stiffness matrix of chopped fibers plies in the global frame is

$$[A] = \begin{bmatrix} 77000 & 23000 & 0 \\ 23000 & 77000 & 0 \\ 0 & 0 & 27000 \end{bmatrix} \times t$$

In the above, t is the thicknesses of the chopped fibers ply

The thermal and hydric expansion coefficients of chopped fiber plies in the global system are.

$$\begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix} = \begin{bmatrix} 25.0 \\ 25.0 \\ 0 \end{bmatrix} \times 10^{-6}$$

$$\begin{bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{bmatrix} = \begin{bmatrix} 0.19 \\ 0.19 \\ 0 \end{bmatrix}$$

The hydric force resultants of chopped fibers plies in the global frame are

$$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = \begin{bmatrix} Q_{xx} \times \beta_x + Q_{xy} \times \beta_y + Q_{xs} \times \beta_{xy} \\ Q_{yx} \times \beta_x + Q_{yy} \times \beta_y + Q_{ys} \times \beta_{xy} \\ 0 \end{bmatrix} \times t \times \Delta m$$

$$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = \begin{bmatrix} 77000 \times 0.19 + 23000 \times 0.19 \\ 77000 \times 0.19 + 23000 \times 0.19 \\ 0 \end{bmatrix} \times t \times \Delta m$$

$$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = \begin{bmatrix} 19000 \\ 19000 \\ 0 \end{bmatrix} \times t \times \Delta m$$

The thermal force resultants are

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} Q_{xx} \times \alpha_x + Q_{xy} \times \alpha_y + Q_{xs} \times \alpha_{xy} \\ Q_{yx} \times \alpha_x + Q_{yy} \times \alpha_y + Q_{ys} \times \alpha_{xy} \\ 0 \end{bmatrix} \times t \times (OT - PT)$$

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} 77000 \times 25 + 23000 \times 25 \\ 77000 \times 25 + 23000 \times 25 \\ 0 \end{bmatrix} \times 10^{-6} \times t \times (OT - PT)$$

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} 2.5 \\ 2.5 \\ 0 \end{bmatrix} \times t \times (OT - PT)$$

Table 11 summarizes all of the above.

$[Q] = \begin{bmatrix} 77000 & 23000 & 0 \\ 23000 & 77000 & 0 \\ 0 & 0 & 27000 \end{bmatrix}$
$[A] = \begin{bmatrix} 77000 & 23000 & 0 \\ 23000 & 77000 & 0 \\ 0 & 0 & 27000 \end{bmatrix} \times t$
$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} 2.5 \\ 2.5 \\ 0 \end{bmatrix} \times t \times (OT - PT)$
$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = \begin{bmatrix} 19000 \\ 19000 \\ 0 \end{bmatrix} \times t \times \Delta m$

Table 11
Matrices of chopped fibers plies

6.5 – Matrices of sand-filled plies in the global frame. The sand-filled plies are isotropic and do not require rotation.

The stiffness matrix of sand-filled plies in the global frame is

$$[A] = \begin{bmatrix} 66000 & 20000 & 0 \\ 20000 & 66000 & 0 \\ 0 & 0 & 23000 \end{bmatrix} \times t$$

In the above, t is the thicknesses of the ply

The thermal and hydric expansion coefficients of sand-filled plies in the global system are.

$$\begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix} = \begin{bmatrix} 13.0 \\ 13.0 \\ 0 \end{bmatrix} \times 10^{-6}$$

$$\begin{bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{bmatrix} = \begin{bmatrix} 0.15 \\ 0.15 \\ 0 \end{bmatrix}$$

The hydric force resultants in the global frame are

$$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = \begin{bmatrix} Q_{xx} \times \beta_x + Q_{xy} \times \beta_y + Q_{xs} \times \beta_{xy} \\ Q_{yx} \times \beta_x + Q_{yy} \times \beta_y + Q_{ys} \times \beta_{xy} \\ 0 \end{bmatrix} \times t \times \Delta m$$

$$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = \begin{bmatrix} 66000 \times 0.15 + 20000 \times 0.15 \\ 66000 \times 0.15 + 20000 \times 0.15 \\ 0 \end{bmatrix} \times t \times \Delta m$$

$$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = \begin{bmatrix} 12800 \\ 12800 \\ 0 \end{bmatrix} \times t \times \Delta m$$

The thermal force resultants are

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} Q_{xx} \times \alpha_x + Q_{xy} \times \alpha_y + Q_{xs} \times \alpha_{xy} \\ Q_{yx} \times \alpha_x + Q_{yy} \times \alpha_y + Q_{ys} \times \alpha_{xy} \\ 0 \end{bmatrix} \times t \times (OT - PT)$$

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} 66000 \times 13 + 20000 \times 13 \\ 66000 \times 13 + 20000 \times 13 \\ 0 \end{bmatrix} \times 10^{-6} \times t \times (OT - PT)$$

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} 1.12 \\ 1.12 \\ 0 \end{bmatrix} \times t \times (OT - PT)$$

Table 12 summarizes all of the above.

$[Q] = \begin{bmatrix} 66000 & 20000 & 0 \\ 20000 & 66000 & 0 \\ 0 & 0 & 23000 \end{bmatrix}$
$[A] = \begin{bmatrix} 66000 & 20000 & 0 \\ 20000 & 66000 & 0 \\ 0 & 0 & 23000 \end{bmatrix} \times t$
$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} 1.12 \\ 1.12 \\ 0 \end{bmatrix} \times t \times (OT - PT)$
$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = \begin{bmatrix} 12800 \\ 12800 \\ 0 \end{bmatrix} \times t \times \Delta m$

Table 12

Matrices of sand-filled plies.

6.6 – Matrices of neat resin in the global system. The plies of neat resin are isotropic and do not require rotation.

The stiffness matrix of sand-filled plies in the global frame is

$$[A] = \begin{bmatrix} 33000 & 10000 & 0 \\ 10000 & 33000 & 0 \\ 0 & 0 & 11500 \end{bmatrix} \times t$$

In the above, t is the thicknesses of the ply

The thermal and hydric coefficients of neat resin plies in the global system are.

$$\begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix} = \begin{bmatrix} 60.0 \\ 60.0 \\ 0 \end{bmatrix} \times 10^{-6}$$

$$\begin{bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{bmatrix} = \begin{bmatrix} 0.40 \\ 0.40 \\ 0 \end{bmatrix}$$

The hydric force resultants on the global frame are

$$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = \begin{bmatrix} Q_{xx} \times \beta_x + Q_{xy} \times \beta_y + Q_{xs} \times \beta_{xy} \\ Q_{yx} \times \beta_x + Q_{yy} \times \beta_y + Q_{ys} \times \beta_{xy} \\ 0 \end{bmatrix} \times t \times \Delta m$$

$$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = \begin{bmatrix} 33000 \times 0.40 + 10000 \times 0.40 \\ 33000 \times 0.40 + 10000 \times 0.40 \\ 0 \end{bmatrix} \times t \times \Delta m$$

$$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = \begin{bmatrix} 17200 \\ 17200 \\ 0 \end{bmatrix} \times t \times \Delta m$$

The thermal force resultants are

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} Q_{xx} \times \alpha_x + Q_{xy} \times \alpha_y + Q_{xs} \times \alpha_{xy} \\ Q_{yx} \times \alpha_x + Q_{yy} \times \alpha_y + Q_{ys} \times \alpha_{xy} \\ 0 \end{bmatrix} \times t \times (OT - PT)$$

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} 33000 \times 60 + 10000 \times 60 \\ 33000 \times 60 + 10000 \times 60 \\ 0 \end{bmatrix} \times 10^{-6} \times t \times (OT - PT)$$

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} 2.58 \\ 2.58 \\ 0 \end{bmatrix} \times t \times (OT - PT)$$

Table 13 summarizes all of the above.

$[Q] =$	$\begin{bmatrix} 33000 & 10000 & 0 \\ 10000 & 33000 & 0 \\ 0 & 0 & 11500 \end{bmatrix}$
$[A] =$	$\begin{bmatrix} 33000 & 10000 & 0 \\ 10000 & 33000 & 0 \\ 0 & 0 & 11500 \end{bmatrix} \times t$
	$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} 2.58 \\ 2.58 \\ 0 \end{bmatrix} \times t \times (OT - PT)$
	$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = \begin{bmatrix} 17200 \\ 17200 \\ 0 \end{bmatrix} \times t \times \Delta m$

Table 13

Matrices of plies of neat resin.

Chapter 2

Laminate circularity

1 – Introduction: The previous chapter presented the relevant properties of commercial standard plies. The commercial laminates consist of several of those plies, stacked together with their own set of properties. When embedded in laminates, the perfectly bonded plies deform without slipping, developing strains determined by their position and orientation in the laminate. The fundamental importance of the ply strains is fundamental in the study of the load-dependent durability of laminates. Our goal in the part 1 of this book is the computation of such strains.

The embedded ply may occupy any position and have any orientation in the laminate. It can be placed near the laminate inner or outer surfaces, or near the mid-surface, with their local axes 1 – 2 taking any orientation with respect to the global frame x – y. The laminate matrices result from the addition of ply matrices, taking into account their orientation and position relative to the mid-surface. Equation (1) shows the complete constitutive equation of laminates.

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} & A_{xs} & B_{xx} & B_{xy} & B_{xs} \\ A_{yx} & A_{yy} & A_{ys} & B_{yx} & B_{yy} & B_{ys} \\ A_{sx} & A_{sy} & A_{ss} & B_{sx} & B_{sy} & B_{ss} \\ B_{xx} & B_{xy} & B_{xs} & D_{xx} & D_{xy} & D_{xs} \\ B_{yx} & B_{yy} & B_{ys} & D_{yx} & D_{yy} & D_{ys} \\ B_{sx} & B_{sy} & B_{ss} & D_{sx} & D_{sy} & D_{ss} \end{bmatrix} \times \begin{bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} \quad (1)$$

The ply constitutive equation (1A) is

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} & A_{xs} \\ A_{yx} & A_{yy} & A_{ys} \\ A_{sx} & A_{sy} & A_{ss} \end{bmatrix} \times \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} \quad (1A)$$

Comparing the two equations, we see the following differences:

- The laminate equation (1) uses force $[N]$ and moment $[M]$ resultants in place of stresses.
- The laminate matrices $[A]$, $[B]$ and $[D]$ replace the ply elastic matrix $[Q]$.
- The matrix $[\epsilon^0]$ represents the membrane strains of the laminate mid-surface.
- The matrix $[k]$ represents the changes in curvature of the mid-surface.

A full appreciation of equation (1) requires familiarity with the fundamentals of classic laminate theory. The readers not familiar with this theory can go to any of the many textbooks available in the literature. See ref. 1 and 2. We will not dwell in the fundamental details. We proceed directly with our discussions, lest we lose sight of the object of this book, which is to evaluate the durability of composites.

Equation (1) is general and holds for any laminate and loading. The designer uses the known external force and moment resultants $[N]$ and $[M]$, as well as the laminate matrices $[A]$, $[B]$ and $[D]$ to compute the unknown strains and changes in curvature. This book is not about laminate matrices and force resultants. The brief discussion of such topics will be just enough to allow the computation of the total ply strains in circular cylinders. Our goal in this book is the study of the durability of composites, not of their technology.

***Example 1.** Most industrial applications of composites involve circular cylindrical laminates under internal pressure that maintain their circularity and develop the same strain in all plies. Unlike the strains, the stresses vary from ply to ply, being too complex for analysis. The analysis of laminates uses strain computed from the constitutive equation using external force and moment resultants, instead of stresses.*

In applications where the laminate does not preserve the circularity, as in underground pipes deflected by vertical soil loads, the plies develop bending strains that depend on their distance “ z ” from the laminate mid-surface. In this book, we will ignore the bending strains that complicates the analysis while throwing no light on the subject of durability. The preservation of circularity allows the analysis of asymmetrical laminates as symmetrical, which is an important and welcome simplification.

In the part 1 of this book, we will develop a protocol to compute the total strains of plies embedded in circular laminates. The computation protocol is easy to understand, simple to use and very accurate. Its drawback is the exclusive and limited applicability to circular cylindrical laminates. In spite of this limitation, however, the model herein presented is of great help in conveying the essential ideas of total strain computation.

2 – Thermal loads: When embedded in laminates, the plies are no longer free to expand or contract when subjected to changes in temperature. Instead, they take the equilibrium thermal position imposed by the laminate. At the equilibrium position, some of the plies end up in tension and others in compression. The residual strains come from the plies forced to comply with the laminate deformation, in spite of their own preferences. The force resultant obtained by adding the residual thermal forces in all plies is of course zero. The thermal resultants, however, are not zero. We will return to this point shortly.

The heat released or externally supplied in the cure of the laminate raises its temperature to a peak value P_T . The resin crosslinks in a stress-free condition at the peak process temperature

PT. The crosslinking congeals and fix the ply positions in the laminate, forcing their subsequent expansion/contraction as a single body. The residual thermal strains develop in the plies as the laminate temperature changes from PT to the operating value OT. The laminate may shrink or expand in passing from PT to OT. If PT is higher than OT, the laminate cools off and shrinks. Otherwise, it heats up and expands. For example, a laminate cured at room temperature, with a low stress-free temperature $PT = 50^\circ\text{C}$, will expand when operating at $OT = 80^\circ\text{C}$.

The laminate thermal equilibrium position stabilizes at the operating temperature OT. Equation (1) computes the mid-surface strains and the changes in curvature at this position, with the thermal resultants defined as

$$[N^T] = [A] \times [\alpha] \times (OT - PT)$$

$$[M^T] = [B] \times [\alpha] \times (OT - PT)$$

Expanding the above, we have

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} \sum (Q_{xx}t\alpha_x + Q_{xy}t\alpha_y + Q_{xs}t\alpha_{xy}) \\ \sum (Q_{yx}t\alpha_x + Q_{yy}t\alpha_y + Q_{ys}t\alpha_{xy}) \\ \sum (Q_{sx}t\alpha_x + Q_{sy}t\alpha_y + Q_{ss}t\alpha_{xy}) \end{bmatrix} \times (OT - PT) \quad (2)$$

$$\begin{bmatrix} M_x^T \\ M_y^T \\ M_{xy}^T \end{bmatrix} = \begin{bmatrix} \sum (Q_{xx}tz\alpha_x + Q_{xy}tz\alpha_y + Q_{xs}tz\alpha_{xy}) \\ \sum (Q_{yx}tz\alpha_x + Q_{yy}tz\alpha_y + Q_{ys}tz\alpha_{xy}) \\ \sum (Q_{sx}tz\alpha_x + Q_{sy}tz\alpha_y + Q_{ss}tz\alpha_{xy}) \end{bmatrix} \times (OT - PT) \quad (3)$$

The temperatures PT and OT are assumed the same in all plies of thin-walled laminates. In equations (2) and (3) the “alphas” represent the coefficients of thermal expansion of each ply in the global system $x - y$, and “z” is the ply distance from the mid-surface. The thermal resultants depend on the ply properties, positions, thicknesses and orientations in the laminate. All temperature changes take as reference the stress-free peak temperature PT. This is an important detail. The reference temperature to compute the thermal resultants is the stress-free peak temperature PT, not the room temperature RT or the operating temperature OT.

At equilibrium, the strains and changes in curvature of the laminate mid-surface are computed by entering the thermal loadings from equations (2) and (3) in the general equation (1).

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \\ M_x^T \\ M_y^T \\ M_{xy}^T \end{bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} & A_{xs} & B_{xx} & B_{xy} & B_{xs} \\ A_{yx} & A_{yy} & A_{ys} & B_{yx} & B_{yy} & B_{ys} \\ A_{sx} & A_{sy} & A_{ss} & B_{sx} & B_{sy} & B_{ss} \\ B_{xx} & B_{xy} & B_{xs} & D_{xx} & D_{xy} & D_{xs} \\ B_{yx} & B_{yy} & B_{ys} & D_{yx} & D_{yy} & D_{ys} \\ B_{sx} & B_{sy} & B_{ss} & D_{sx} & D_{sy} & D_{ss} \end{bmatrix} \times \begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \\ \kappa_x^e \\ \kappa_y^e \\ \kappa_{xy}^e \end{bmatrix} \quad (4)$$

The reader will note the superscript “e” designating “equilibrium” in the strain and curvature matrices. The designer knows the laminate matrices $[A]$, $[B]$ and $[D]$, as well as the thermal resultants. Inversion of equation (4) gives the strains and changes in curvature at equilibrium.

$$\begin{bmatrix} \varepsilon_x^e \\ \varepsilon_y^e \\ \gamma_{xy}^e \\ \kappa_x^e \\ \kappa_y^e \\ \kappa_{xy}^e \end{bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} & A_{xs} & B_{xx} & B_{xy} & B_{xs} \\ A_{yx} & A_{yy} & A_{ys} & B_{yx} & B_{yy} & B_{ys} \\ A_{sx} & A_{sy} & A_{ss} & B_{sx} & B_{sy} & B_{ss} \\ B_{xx} & B_{xy} & B_{xs} & D_{xx} & D_{xy} & D_{xs} \\ B_{yx} & B_{yy} & B_{ys} & D_{yx} & D_{yy} & D_{ys} \\ B_{sx} & B_{sy} & B_{ss} & D_{sx} & D_{sy} & D_{ss} \end{bmatrix}^{-1} \times \begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \\ M_x^T \\ M_y^T \\ M_{xy}^T \end{bmatrix} \quad (5)$$

Equation (5) computes the equilibrium strains and changes in curvature of the mid-surface of any laminate subjected to a temperature change from PT to OT. If OT is lower than PT, the thermal forces and moments are negative. Otherwise, they are positive.

We next simplify equation (5) for the special case of circular cylindrical laminates.

3 – Thermal loads and laminate circularity: Equations (2), (3) and (4) are applicable to any laminate and thermal loading. They compute the laminate warping and distortion that occur in response to changes in temperature. However, as explained, the cylindrical laminates of circular cross sections preserve their round shape and do not warp. They maintain their curvatures at any temperature. Mathematically, we describe the preservation of circularity as

$$\begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (6)$$

The preservation of circularity develops internal reactive moments computed by entering equation (6) in the general equation (4).

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \\ M_x^T + M_x^R \\ M_y^T + M_y^R \\ M_{xy}^T + M_{xy}^R \end{bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} & A_{xs} & B_{xx} & B_{xy} & B_{xs} \\ A_{yx} & A_{yy} & A_{ys} & B_{yx} & B_{yy} & B_{ys} \\ A_{sx} & A_{sy} & A_{ss} & B_{sx} & B_{sy} & B_{ss} \\ B_{xx} & B_{xy} & B_{xs} & D_{xx} & D_{xy} & D_{xs} \\ B_{yx} & B_{yy} & B_{ys} & D_{yx} & D_{yy} & D_{ys} \\ B_{sx} & B_{sy} & B_{ss} & D_{sx} & D_{sy} & D_{ss} \end{bmatrix} \times \begin{bmatrix} \varepsilon_x^e \\ \varepsilon_y^e \\ \gamma_{xy}^e \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (7)$$

In the above, we identify the reactive moments with the superscript “R”. The inclusion of the reactive moments in equation (7) eliminates the changes in curvature and assures the preservation of circularity. Reactive moments develop in all cylindrical laminates of circular cross section at temperatures other than PT.

In the manufacturing process, the plies congeal in a stress-free condition at the peak temperature PT. As the temperature changes from PT to OT, the cylinder takes its equilibrium position while preserving its circular cross section. The preservation of circularity

gives rise to the reactive moments just described and decouples the force and moment resultants, as seen by expanding equation (7)

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} & A_{xs} \\ A_{yx} & A_{yy} & A_{ys} \\ A_{sx} & A_{sy} & A_{ss} \end{bmatrix} \begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix} \quad (8)$$

$$\begin{bmatrix} M_x^T + M_x^R \\ M_y^T + M_y^R \\ M_{xy}^T + M_{xy}^R \end{bmatrix} = \begin{bmatrix} B_{xx} & B_{xy} & B_{xs} \\ B_{yx} & B_{yy} & B_{ys} \\ B_{sx} & B_{sy} & B_{ss} \end{bmatrix} \begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix} \quad (9)$$

$$\begin{bmatrix} M_x^R \\ M_y^R \\ M_{xy}^R \end{bmatrix} = \begin{bmatrix} B_{xx} & B_{xy} & B_{xs} \\ B_{yx} & B_{yy} & B_{ys} \\ B_{sx} & B_{sy} & B_{ss} \end{bmatrix} \begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix} - \begin{bmatrix} M_x^T \\ M_y^T \\ M_{xy}^T \end{bmatrix} \quad (9A)$$

In the special cases of symmetrical laminates, the coupling matrix $[B] = 0$ and the reactive moments are

$$\begin{bmatrix} M_x^R \\ M_y^R \\ M_{xy}^R \end{bmatrix} = - \begin{bmatrix} M_x^T \\ M_y^T \\ M_{xy}^T \end{bmatrix} \quad (9B)$$

Equation (8) computes the laminate equilibrium strains using the matrix $[A]$ alone, regardless of the matrix $[B]$. This is a very important simplification. The preservation of circularity allows the analysis of asymmetric laminates as symmetric, i.e., by ignoring the matrix $[B]$.

Note: If desired, we can use equation (9A) to compute the reactive moments from the thermal moment resultants, the coupling matrix $[B]$ and the membrane strains. These moments, however, are not relevant.

4 – Hydric loads: The foregoing analysis applies also to hydric loadings, taking into account that laminates expand on absorbing water. In both hydric and thermal cases, the laminate constraint prevents the embedded plies from taking their natural equilibrium dimensions. In the thermal case, the plies cannot freely contract from PT to OT. In the hydric case, they cannot freely expand when the resin saturates with water.

The computation of the resultant hydric forces and moments is similar to the thermal case

$$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = \begin{bmatrix} \sum (Q_{xx}t\beta_x + Q_{xy}t\beta_y + Q_{xs}t\beta_{xy}) \\ \sum (Q_{yx}t\beta_x + Q_{yy}t\beta_y + Q_{ys}t\beta_{xy}) \\ \sum (Q_{sx}t\beta_x + Q_{sy}t\beta_y + Q_{ss}t\beta_{xy}) \end{bmatrix} \times (\Delta m) \quad (10)$$

$$\begin{bmatrix} M_x^H \\ M_y^H \\ M_{xy}^H \end{bmatrix} = \begin{bmatrix} \sum (Q_{xx}tz\beta_x + Q_{xy}tz\beta_y + Q_{xs}tz\beta_{xy}) \\ \sum (Q_{yx}tz\beta_x + Q_{yy}tz\beta_y + Q_{ys}tz\beta_{xy}) \\ \sum (Q_{sx}tz\beta_x + Q_{sy}tz\beta_y + Q_{ss}tz\beta_{xy}) \end{bmatrix} \times (\Delta m) \quad (10A)$$

Equations (10) and (10A) show the hydric resultants computed in the same way as in the thermal case. Note the water pick up $-\Delta m$ introduced in place of the temperature change $\Delta T = OT - PT$, and the hydric expansion coefficients “betas” replacing the thermal coefficients “alphas”.

The amount of water picked up by the resin, Δm , is assumed the same for all plies. As in the thermal case, the hydric gradient across the wall thickness is assumed equal to zero.

Developing the equations as before, we obtain

$$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} & A_{xs} \\ A_{yx} & A_{yy} & A_{ys} \\ A_{sx} & A_{sy} & A_{ss} \end{bmatrix} \begin{bmatrix} \varepsilon_x^e \\ \varepsilon_y^e \\ \gamma_{xy}^e \end{bmatrix}_{HID} \quad (11)$$

$$\begin{bmatrix} M_x^H + M_x^R \\ M_y^H + M_y^R \\ M_{xy}^H + M_{xy}^R \end{bmatrix} = \begin{bmatrix} B_{xx} & B_{xy} & B_{xs} \\ B_{yx} & B_{yy} & B_{ys} \\ B_{sx} & B_{sy} & B_{ss} \end{bmatrix} \begin{bmatrix} \varepsilon_x^e \\ \varepsilon_y^e \\ \gamma_{xy}^e \end{bmatrix}_{HID} \quad (12)$$

$$\begin{bmatrix} M_x^R \\ M_y^R \\ M_{xy}^R \end{bmatrix}_{HID} = \begin{bmatrix} B_{xx} & B_{xy} & B_{xs} \\ B_{yx} & B_{yy} & B_{ys} \\ B_{sx} & B_{sy} & B_{ss} \end{bmatrix} \begin{bmatrix} \varepsilon_x^e \\ \varepsilon_y^e \\ \gamma_{xy}^e \end{bmatrix}_{HID} - \begin{bmatrix} M_x^H \\ M_y^H \\ M_{xy}^H \end{bmatrix} \quad (12A)$$

In symmetric laminates, the coupling matrix $[B] = 0$ and the reactive moments are

$$\begin{bmatrix} M_x^R \\ M_y^R \\ M_{xy}^R \end{bmatrix}_{HID} = - \begin{bmatrix} M_x^H \\ M_y^H \\ M_{xy}^H \end{bmatrix}_{HID} \quad (12B)$$

As in the thermal case, the preservation of circularity allows the computation of the laminate equilibrium hydric strains from equation (11), ignoring the coupling matrix $[B]$.

5 – Mechanical loads: The constitutive equation of circular cylindrical laminates under mechanical loads derives from the general equation (1) and the condition of circularity. The analysis is almost identical to those for the thermal and hydric cases. The difference lies in the arbitrary nature of the external forces, moments and torques. Unlike the thermal and hydric resultants, which are determined by the laminate construction, the applied external mechanical loading is independent of the laminate.

The force resultants $[N]$ generate membrane strains that preserve the circularity. The moment resultants $[M]$ generate bending strains that disturb the circularity. Let us take a closer look at the membrane forces $[N]$.

5.1 – Circular cylinders under membrane forces: The axial resultant N_x , the circumferential resultant N_y and the torque resultant N_{xy} , are membrane forces that preserve the original circular shape of the cylinder. The only non-membrane loads acting on the

laminates are the internal reactive moments required to maintain the circularity. The laminate curvatures do not change and the general equation (1) reduces to

$$\begin{bmatrix} N_x^M \\ N_y^M \\ N_{xy}^M \\ 0 + M_x^R \\ 0 + M_y^R \\ 0 + M_{xy}^R \end{bmatrix}_{MEC} = \begin{bmatrix} A_{xx} & A_{xy} & A_{xs} & B_{xx} & B_{xy} & B_{xs} \\ A_{yx} & A_{yy} & A_{ys} & B_{yx} & B_{yy} & B_{ys} \\ A_{sx} & A_{sy} & A_{ss} & B_{sx} & B_{sy} & B_{ss} \\ B_{xx} & B_{xy} & B_{xs} & D_{xx} & D_{xy} & D_{xs} \\ B_{yx} & B_{yy} & B_{ys} & D_{yx} & D_{yy} & D_{ys} \\ B_{sx} & B_{sy} & B_{ss} & D_{sx} & D_{sy} & D_{ss} \end{bmatrix} \times \begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \\ 0 \\ 0 \\ 0 \end{bmatrix}_{MEC} \quad (13)$$

Equation (13) emphasizes the absence of external bending moments by writing “zeros” in their places. Equation (13) includes the reactive moments required to preserve the circular shape, as discussed earlier.

Expanding equation (13), we obtain the mechanical strains at equilibrium

$$\begin{bmatrix} N_x^M \\ N_y^M \\ N_{xy}^M \end{bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} & A_{xs} \\ A_{yx} & A_{yy} & A_{ys} \\ A_{sx} & A_{sy} & A_{ss} \end{bmatrix} \times \begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix}_{MEC} \quad (14)$$

The reactive mechanical moments are:

$$\begin{bmatrix} M_x^R \\ M_y^R \\ M_{xy}^R \end{bmatrix}_{MEC} = \begin{bmatrix} B_{xx} & B_{xy} & B_{xs} \\ B_{yx} & B_{yy} & B_{ys} \\ B_{sx} & B_{sy} & B_{ss} \end{bmatrix} \times \begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix}_{MEC} \quad (15)$$

The preservation of circularity requires the presence of reactive mechanical moments that, like all reactive quantities, have the exact magnitudes to neutralize their cause. In symmetric laminates, the coupling matrix $[B] = 0$, and the reactive mechanical moments are zero.

$$\begin{bmatrix} M_x^R \\ M_y^R \\ M_{xy}^R \end{bmatrix}_{MEC} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The preceding analysis produces two interesting results.

- *First, the preservation of circularity eliminates the coupling matrix $[B]$ from the computation of the membrane strains. This is a welcome simplification.*
- *Second, the membrane strains are independent of the reactive bending moments.*

These apparently surprising results are in fact obvious if we give them some thought. It should be obvious that the coupling matrix $[B]$ cannot affect the membrane strains when the circular shape is preserved. The matrix $[B]$ is not relevant in the analysis of circular cylinders that maintain circularity. Furthermore, the preservation of circularity automatically takes care of the reactive moments.

5.2 – Circular cylinders deflected by lateral loads. The foregoing analysis holds for circular cylinders subjected to loads that preserve the circular shape. There are loadings, however, that change the cylinder curvature. Examples of such loadings are the vertical

soil weight on underground pipes and the bending at the knuckle of vertical storage tanks. The analysis of such cases require the general equation (1).

The matrix [B] couples the force and moment resultants in equation (1), creating a big complication to the analyst. To avoid this, most designers specify symmetrical or quasi-symmetrical laminates, that have matrices [B] = 0. In fully symmetrical laminates, the coupling matrix [B] = 0 and equation (1) reduces to

$$\begin{bmatrix} N_x^M \\ N_y^M \\ N_{xy}^M \end{bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} & A_{xs} \\ A_{yx} & A_{yy} & A_{ys} \\ A_{sx} & A_{sy} & A_{ss} \end{bmatrix} \times \begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix}$$

$$\begin{bmatrix} M_x^M \\ M_y^M \\ M_{xy}^M \end{bmatrix} = \begin{bmatrix} D_{xx} & D_{xy} & D_{xs} \\ D_{yx} & D_{yy} & D_{ys} \\ D_{sx} & D_{sy} & D_{ss} \end{bmatrix} \times \begin{bmatrix} \kappa_x^e \\ \kappa_y^e \\ \kappa_{xy}^e \end{bmatrix}$$

The last of the above equations compute the changes in curvature [k], which together with the ply position in the laminate, z, allow the computation of the bending strains.

$$\begin{bmatrix} \epsilon_x^b \\ \epsilon_y^b \\ \gamma_{xy}^b \end{bmatrix} = \begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix} + \begin{bmatrix} k_x^e \\ k_y^e \\ k_{xy}^e \end{bmatrix} \times z$$

The simplicity of the preceding equations provides a strong incentive to specify symmetrical laminates in the presence of external bending moments. This book ignores the bending strains, since their inclusion would complicate the analysis while bringing no benefit to the understanding of the durability problem.

Chapter 3

Laminate matrices and engineering properties

1 – Introduction: – This chapter derives the matrices of circular cylindrical laminates for the three loadings that appear in engineering practice, namely the hydric, the thermal and the mechanical loadings. The development assumes the following:

- 1. The laminate is stress-free when dry and at the peak temperature PT*
- 2. The cylindrical laminate preserves its original circular shape*
- 3. All plies operate at the same temperature OT . There is no thermal gradient*
- 4. The resin in all plies pick up the same amount of water Δm . There is no hydric gradient*
- 5. The laminate properties do not change with temperature or water pick up.*
- 6. The operating temperature OT is less than the resin HDT .*

Knowledge of the laminate matrices is essential to compute the total ply strains required in the analysis of the load-dependent durability. The laminate matrices are also useful to compute the engineering constants.

2 – Laminate construction – Figure 1 shows the three typical laminate constructions used in industrial service. What follows is a description of the function of each ply in such laminates.

- **Liner:** *The liner is a thin (0.3 mm) resin-rich inner barrier thought by many to provide water tightness and chemical durability to pipes and tanks. In chemical and sanitation service, the liner consists of a special lightweight veil saturated with resin. In oil pipes, the liner consists simply of a thin layer of neat resin, with no veil. As discussed later in this book, the resin-rich liners have a small effect on the durability of composite equipment. They do not play a critical role and have no effect on the stiffness, weep or infiltration thresholds. The resin-rich liners are not critical plies.*
- **Corrosion barrier:** *The industrial laminates designed for chemical or sanitation service, have at least one inner protective ply of chopped glass placed immediately on top of the liner. In non-aggressive sanitation service, this inner ply is the “weep barrier”. In chemical service, it is the “corrosion barrier”. The inner ply of chopped fibers is critical for infiltration and weep failures.*
- **Structural plies:** *The structural wall of the laminate consists of several plies of continuous unidirectional UD fibers to provide the high modulus required to accommodate the internal pressure and other loads. The UD plies in the structural wall are critical in all long-term failures involving loss of stiffness and rupture. In oil pipes, which have no barrier of chopped fibers, the UD plies are also critical for weep failures.*
- **Sand core:** *The underground pipes used in sanitation service usually have a core of sand-filled resin to increase their ring stiffness. The brittle sand core cracks easily on impact, placing stringent demands on the handling of sanitation pipes. The sand core is the critical ply for the anomalous failure described in chapter 18. Sometimes, as explained in the example 7 of chapter 7, the sand core is also the critical ply for weep failure.*

- **Topcoat:** The topcoat is a thin (0.3 mm) resin-rich outer ply used for protection against spillage and UV radiation. The topcoat is not critical.

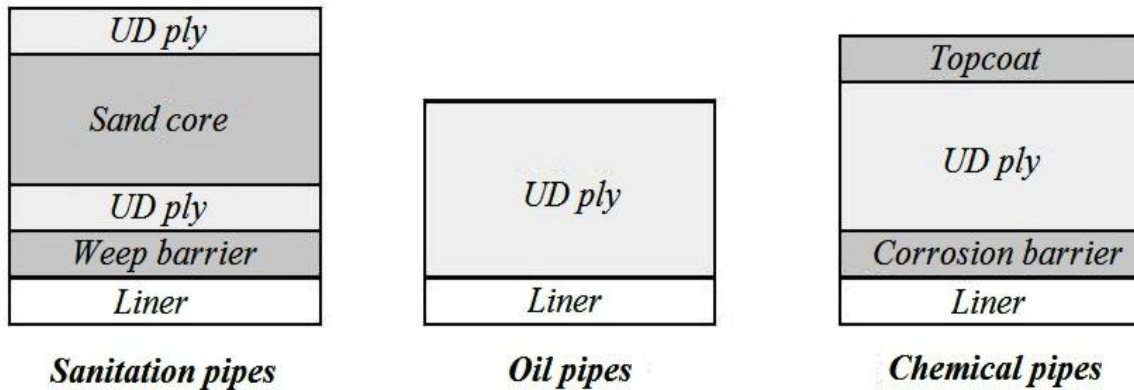


Figure 1
Typical laminate constructions for sanitation, chemical and oil service. The corrosion barrier is the critical ply for infiltration failure in chemical service. The weep barrier is the critical ply for weep failure in sanitation pipes. The UD plies are critical for rupture and stiffness failures.

The foregoing discussion identified the UD and the chopped fibers as the two critical plies for industrial service. The study of the durability of any industrial equipment boils down to the analysis of these two plies. The relevant durability parameters associated with them – the regression coefficients G_s and G_c , as well as the failure thresholds – must be carefully measured.

3 – Laminate matrices – In this chapter we will develop the protocols to compute the laminate matrices $[A]$, $[N^T]$ and $[N^H]$, from the known matrices $[Q]$, $[\alpha]$ and $[\beta]$ of standard plies, rotated to the global frame $x - y$. We start with a general approach that we later simplify for commercial laminates. The three laminate matrices of interest are.

- The stiffness matrix $[A]$
- The matrix of thermal force resultants $[N^T]$
- The matrix of hydric force resultants $[N^H]$

To obtain laminate matrices, we simply add up those of their constituent plies.

The laminate stiffness matrix is

$$[A] = \begin{bmatrix} \sum Q_{xx}t & \sum Q_{xy}t & \sum Q_{xs}t \\ \sum Q_{yx}t & \sum Q_{yy}t & \sum Q_{ys}t \\ \sum Q_{sx}t & \sum Q_{sy}t & \sum Q_{ss}t \end{bmatrix} \quad (1)$$

In equation (1) “t” is the thickness and [Q] is the elastic matrix of the plies referred to the global frame x – y.

The laminate hydric $[N^H]$ and thermal $[N^T]$ force resultants are

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} \sum (Q_{xx}t\alpha_x + Q_{xy}t\alpha_y + Q_{xs}t\alpha_{xy}) \\ \sum (Q_{yx}t\alpha_x + Q_{yy}t\alpha_y + Q_{ys}t\alpha_{xy}) \\ \sum (Q_{sx}t\alpha_x + Q_{sy}t\alpha_y + Q_{ss}t\alpha_{xy}) \end{bmatrix} \times (OT - PT) \quad (2)$$

$$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = \begin{bmatrix} \sum (Q_{xx}t\beta_x + Q_{xy}t\beta_y + Q_{xs}t\beta_{xy}) \\ \sum (Q_{yx}t\beta_x + Q_{yy}t\beta_y + Q_{ys}t\beta_{xy}) \\ \sum (Q_{sx}t\beta_x + Q_{sy}t\beta_y + Q_{ss}t\beta_{xy}) \end{bmatrix} \times (\Delta m) \quad (3)$$

The thermal resultants depend on the temperature change $\Delta T = OT - PT$. The hydric resultants depend on the amount of water Δm picked up by the resin. In the above equations, “t” is the ply thickness and [Q] is the ply elastic matrix in the global frame. The “alphas” and “betas” are the expansion coefficients of each ply, also in the global system. The thermal resultants take as reference the stress-free temperature PT. The hydric resultants take as reference the stress-free dry condition. The small thickness of composite laminates justifies the assumption of same operating temperature OT and water pick up Δm for all plies.

3.1 – The stiffness matrix [A]. The stiffness matrix [A] of the laminate is

$$[A] = [Q]_{chop} \times t_{chop} + [Q]_{UD} \times t_{UD} + [Q]_{sand} \times t_{sand} + [Q]_r \times t_r$$

Where

$[Q]_{chop} \times t_{chop}$ is the stiffness matrix of the plies of chopped fibers

$[Q]_{UD} \times t_{UD}$ is the stiffness matrix of the UD plies

$[Q]_{sand} \times t_{sand}$ is the stiffness matrix of the sand ply

$[Q]_r \times t_r$ is the stiffness matrix of the resin ply

In the above, the stiffness matrices of the standard plies in the global system are available from chapter 1, and the ply thicknesses are those specified by the designer.

Example 1. Compute the stiffness matrix of a laminate comprising the following plies.

$$t_{chop} = 5.5 \text{ mm and } t_r = 0.9 \text{ mm}$$

The laminate has only two plies, namely of chopped fibers and neat resin. Therefore, the stiffness matrix [A] is

$$[A] = [A]_{chop} + [A]_r$$

$$[A] = [Q]_{chop} \times t_{chop} + [Q]_r \times t_r$$

Entering the tabulated stiffness matrices from chapter 1, we have

$$\begin{bmatrix} A_{xx} & A_{xy} & A_{xs} \\ A_{yx} & A_{yy} & A_{ys} \\ A_{sx} & A_{sy} & A_{ss} \end{bmatrix} = \begin{bmatrix} 77000 & 23000 & 0 \\ 23000 & 77000 & 0 \\ 0 & 0 & 27000 \end{bmatrix} \times t_{chop} + \begin{bmatrix} 33\,000 & 10\,000 & 0 \\ 10\,000 & 33\,000 & 0 \\ 0 & 0 & 11\,500 \end{bmatrix} \times t_r$$

$$\begin{bmatrix} A_{xx} & A_{xy} & 0 \\ A_{yx} & A_{yy} & 0 \\ 0 & 0 & A_{ss} \end{bmatrix} = \begin{bmatrix} 77000 & 23000 & 0 \\ 23000 & 77000 & 0 \\ 0 & 0 & 27000 \end{bmatrix} \times 0.55 + \begin{bmatrix} 33\,000 & 10\,000 & 0 \\ 10\,000 & 33\,000 & 0 \\ 0 & 0 & 11\,500 \end{bmatrix} \times 0.09$$

$$\begin{bmatrix} A_{xx} & A_{xy} & 0 \\ A_{yx} & A_{yy} & 0 \\ 0 & 0 & A_{ss} \end{bmatrix} = \begin{bmatrix} 42647 & 13550 & 0 \\ 13550 & 42647 & 0 \\ 0 & 0 & 15885 \end{bmatrix} \text{ kg/cm}$$

This is the standard stiffness matrix $[A]$ of the proposed laminate. The reader will note the ply thicknesses entering the formula in cm.

Example 2. Compute the stiffness matrix $[A]$ of a UD55 laminate with the following ply thicknesses

$$t_{chop} = 2.1 \text{ mm}, t_{UD} = 5.1 \text{ mm and } t_r = 0.9 \text{ mm}$$

The laminate stiffness matrix is

$$\begin{bmatrix} A_{xx} & A_{xy} & 0 \\ A_{yx} & A_{yy} & 0 \\ 0 & 0 & A_{ss} \end{bmatrix} = \begin{bmatrix} 77000 & 23000 & 0 \\ 23000 & 77000 & 0 \\ 0 & 0 & 27000 \end{bmatrix} \times 0.21 + \begin{bmatrix} 132470 & 92220 & 0 \\ 92220 & 235070 & 0 \\ 0 & 0 & 101220 \end{bmatrix} \times 0.51$$

$$+ \begin{bmatrix} 33\,000 & 10\,000 & 0 \\ 10\,000 & 33\,000 & 0 \\ 0 & 0 & 11\,500 \end{bmatrix} \times 0.09$$

$$\begin{bmatrix} A_{xx} & A_{xy} & 0 \\ A_{yx} & A_{yy} & 0 \\ 0 & 0 & A_{ss} \end{bmatrix} = \begin{bmatrix} 86699 & 52762 & 0 \\ 52762 & 139025 & 0 \\ 0 & 0 & 58327 \end{bmatrix} \text{ kg/cm}$$

Example 3. Compute the stiffness matrix of a hoop-chop laminate with the following construction.

$$t_{chop} = 3.7 \text{ mm}, t_{UD} = 4.1 \text{ mm}, t_{sand} = 6.2 \text{ mm and } t_r = 0.9 \text{ mm}$$

The hoop-chop laminates include plies of neat resin, chopped fibers, sand and UD90. The standard stiffness matrices of all these plies are in chapter 1.

$$[A] = [A]_{chop} + [A]_{UD} + [A]_{sand} + [A]_r$$

$$\begin{bmatrix} A_{xx} & A_{xy} & 0 \\ A_{yx} & A_{yy} & 0 \\ 0 & 0 & A_{ss} \end{bmatrix} = \begin{bmatrix} 77000 & 23000 & 0 \\ 23000 & 77000 & 0 \\ 0 & 0 & 27000 \end{bmatrix} \times t_{chop} + \begin{bmatrix} 100000 & 30000 & 0 \\ 30000 & 400000 & 0 \\ 0 & 0 & 35000 \end{bmatrix} \times t_{UD}$$

$$\begin{bmatrix} 66\,000 & 20\,000 & 0 \\ 20\,000 & 66\,000 & 0 \\ 0 & 0 & 23\,000 \end{bmatrix} \times t_{sand} + \begin{bmatrix} 33\,000 & 10\,000 & 0 \\ 10\,000 & 33\,000 & 0 \\ 0 & 0 & 11\,500 \end{bmatrix} \times t_r$$

$$\begin{bmatrix} A_{xx} & A_{xy} & 0 \\ A_{yx} & A_{yy} & 0 \\ 0 & 0 & A_{ss} \end{bmatrix} = \begin{bmatrix} 77000 & 23000 & 0 \\ 23000 & 77000 & 0 \\ 0 & 0 & 27000 \end{bmatrix} \times 0.37 + \begin{bmatrix} 100000 & 30000 & 0 \\ 30000 & 400000 & 0 \\ 0 & 0 & 35000 \end{bmatrix} \times 0.41$$

$$\begin{bmatrix} 66\,000 & 20\,000 & 0 \\ 20\,000 & 66\,000 & 0 \\ 0 & 0 & 23\,000 \end{bmatrix} \times 0.62 + \begin{bmatrix} 33\,000 & 10\,000 & 0 \\ 10\,000 & 33\,000 & 0 \\ 0 & 0 & 11\,500 \end{bmatrix} \times 0.09$$

$$\begin{bmatrix} A_{xx} & A_{xy} & 0 \\ A_{yx} & A_{yy} & 0 \\ 0 & 0 & A_{ss} \end{bmatrix} = \begin{bmatrix} 113380 & 34110 & 0 \\ 34110 & 236380 & 0 \\ 0 & 0 & 29975 \end{bmatrix} \text{ kg/cm}$$

The computation of the laminate stiffness matrix $[A]$ from specified ply thicknesses is helpful in designing non-existing equipment, when the designer is free to choose the ply thicknesses.

Often, however, we need to know the stiffness matrix of existing laminates of unknown ply thicknesses. In such cases, we make use of the laminate composition determined from burnout tests. The stiffness matrix computed from the laminate composition is

$$[A] = \frac{3.12 \times t}{1.20 + 1.40(res)} \times [2.33(chop)[Q]_{chop} + 0.74(UD)[Q]_{UD} + 0.60(sand)[Q]_{sand} + 0.83K[Q]_r]$$

where

t is the known thickness of the existing laminate

(res) is the weight fraction of resin

$(chop)$ is the weight fraction of chopped fibers

(UD) is the weight fraction of UD fibers

$(sand)$ is the weight fraction of sand

K is the laminate dryness, computed as

$$K = (res) - \left[\frac{3}{7}(UD) + \frac{7}{3}(chop) + \frac{2}{8}(sand) \right]$$

$K > 1$ implies in “wet” laminates, with more resin than required by the standard plies. Conversely, $K < 1$ implies in “dry” laminates.

Example 4. Compute the stiffness matrix $[A]$ of an existing angle-ply laminate with the following composition.

$$(res) = 0.30, (chop) = 0.05, (UD55) = 0.65$$

The measured laminate thickness is 8.0 mm.

First, we compute the laminate dryness K

$$K = (res) - \left[\frac{3}{7}(UD) + \frac{7}{3}(chop) \right]$$

$$K = (0,30) - \left[\frac{3}{7}(0,65) + \frac{7}{3}(0,05) \right] = -0,10$$

The negative dryness $K = -0.10$ implies in a “dry” laminate, with less resin than required by the standard. The laminate dryness has nothing to do with quality.

Entering the data in the appropriate equation, we have

$$[A] = \frac{3.12 \times 0.8}{1.20 + 1.40 \times 0.30} \times \left[2.33 \times 0.05 \begin{bmatrix} 77000 & 23000 & 0 \\ 23000 & 77000 & 0 \\ 0 & 0 & 27000 \end{bmatrix} + 0.74 \times 0.65 \begin{bmatrix} 132470 & 92220 & 0 \\ 92220 & 235070 & 0 \\ 0 & 0 & 101220 \end{bmatrix} - 0.83 \times 0.10 \begin{bmatrix} 33\,000 & 10\,000 & 0 \\ 10\,000 & 33\,000 & 0 \\ 0 & 0 & 11\,500 \end{bmatrix} \right]$$

$$[A] = \begin{bmatrix} 107773 & 71192 & 0 \\ 71192 & 183810 & 0 \\ 0 & 0 & 78389 \end{bmatrix} \text{ kg/cm}$$

As we see, there are two formulas to compute the stiffness matrices $[A]$ of laminates. We can do it using the ply thicknesses or the laminate composition. The ply thickness approach is useful in designing non-existing laminates, when the engineer is free to choose the plies. The composition approach is useful to determine the properties of existing laminates. The composition of existing laminates is easily determined from simple burnout tests. The two approaches are identical and give exactly the same result.

3.2 – The thermal and hydric force resultants. The thermal and hydric force resultants $[N^T]$ and $[N^H]$ of laminates is the sum of the contributions from all plies.

$$[N^H] = [N^H]_{chop} \times t_{chop} + [N^H]_{UD} \times t_{UD} + [N^H]_{sand} \times t_{sand} + [N^H]_r \times t_r$$

$$[N^T] = [N^T]_{chop} \times t_{chop} + [N^T]_{UD} \times t_{UD} + [N^T]_{sand} \times t_{sand} + [N^T]_r \times t_r$$

The thermal and hydric force resultants of all plies, expressed in the global system $x - y$, are listed in chapter 1.

As in the case of the stiffness matrix $[A]$, the force resultants $[N^T]$ and $[N^H]$ can be computed from the laminate composition, instead of the ply thicknesses.

$$[N^T] = \frac{3.12 \times t}{1.20 + 1.40(res)} \times [2.33(chop)[N^T]_{chop} + 0.74(UD)[N^T]_{UD} + 0.60(sand)[N^T]_{sand} + 0.83K[N^T]_r]$$

$$[N^H] = \frac{3.12 \times t}{1.20 + 1.40(res)} \times [2.33(chop)[N^H]_{chop} + 0.74(UD)[N^H]_{UD} + 0.60(sand)[N^H]_{sand} + 0.83K[N^H]_r]$$

These statements will become clear as we move along.

4 – Laminate engineering constants: The engineering properties of laminates – elastic moduli, Poisson ratios and coefficients of expansion – derive from those of the constituent plies. In this section, we develop the expressions to compute the engineering constants of balanced circular cylindrical laminates.

4.1 – Laminate elastic properties. The average laminate moduli derive from the stiffness matrix $[A]$. The protocol is general and applicable to the computation of all elastic moduli of any circular cylindrical laminate. We start with the constitutive equation of balanced circular cylindrical laminates under the general loads N_x , N_y and N_{xy} .

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} & 0 \\ A_{yx} & A_{yy} & 0 \\ 0 & 0 & A_{ss} \end{bmatrix} \times \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix}$$

To illustrate the procedure, we derive the expression to compute the elastic modulus E_x in the longitudinal laminate direction. To do so, we simulate the tensile test to determine this modulus. This is done by retaining the force N_x and dropping the forces N_y and N_{xy} in the laminate constitutive equation.

$$\begin{bmatrix} N_x \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} & 0 \\ A_{yx} & A_{yy} & 0 \\ 0 & 0 & A_{ss} \end{bmatrix} \times \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix}$$

Expanding the above, we have

$$N_x = A_{xx}\epsilon_x + A_{xy}\epsilon_y$$

$$0 = A_{yx}\epsilon_x + A_{yy}\epsilon_y$$

By eliminating ϵ_y we have

$$N_x = \left(A_{xx} - \frac{A_{xy}^2}{A_{yy}} \right) \epsilon_x$$

Dividing through by the laminate thickness “t”

$$\sigma_x = \frac{N_x}{t} = \frac{1}{t} \left(A_{xx} - \frac{A_{xy}^2}{A_{yy}} \right) \epsilon_x$$

By definition of elastic modulus

$$\frac{\sigma_x}{\epsilon_x} = E_x = \frac{1}{t} \left(A_{xx} - \frac{A_{xy}^2}{A_{yy}} \right)$$

In a like manner, we have

$$\begin{bmatrix} 0 \\ N_y \\ 0 \end{bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} & 0 \\ A_{yx} & A_{yy} & 0 \\ 0 & 0 & A_{ss} \end{bmatrix} \times \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix}$$

$$E_y = \frac{1}{t} \left(A_{yy} - \frac{A_{xy}^2}{A_{xx}} \right)$$

The shear modulus is

$$\begin{bmatrix} 0 \\ 0 \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} & 0 \\ A_{yx} & A_{yy} & 0 \\ 0 & 0 & A_{ss} \end{bmatrix} \times \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix}$$

$$G_{xy} = \frac{A_{ss}}{t}$$

The laminate Poisson ratios are, by definition

$$\nu_{xy} = \frac{A_{xy}}{A_{yy}}$$

$$\nu_{yx} = \frac{A_{xy}}{A_{xx}}$$

As we see, knowledge of the stiffness matrix $[A]$ allows the computation of all engineering constants of circular cylindrical laminates.

Example 6: Compute the engineering constants of the laminate described in example 3.

From example 3, the laminate stiffness matrix is

$$\begin{bmatrix} A_{xx} & A_{xy} & 0 \\ A_{yx} & A_{yy} & 0 \\ 0 & 0 & A_{ss} \end{bmatrix} = \begin{bmatrix} 113380 & 34110 & 0 \\ 34110 & 236380 & 0 \\ 0 & 0 & 29975 \end{bmatrix} \text{ kg/cm}$$

Also from example 3, the laminate thickness is $t = 3.7 + 4.1 + 6.2 + 0.9 = 14.90 \text{ mm}$.

The laminate engineering constants are

$$E_y = \frac{1}{t} \left(A_{yy} - \frac{A_{xy}^2}{A_{xx}} \right)$$

$$E_y = \frac{1}{1.49} \left(236380 - \frac{34110^2}{113380} \right) = 226118 \text{ kg/cm}^2$$

$$E_x = \frac{1}{t} \left(A_{xx} - \frac{A_{xy}^2}{A_{yy}} \right)$$

$$E_x = \frac{1}{1.49} \left(113380 - \frac{34110^2}{236380} \right) = 108457 \text{ kg/cm}^2$$

$$G_{xy} = \frac{A_{ss}}{t} = \frac{29975}{1.49} = 20117 \text{ kg/cm}^2$$

$$v_{xy} = \frac{A_{xy}}{A_{yy}} = \frac{34110}{236380} = 0.14$$

$$v_{yx} = \frac{A_{xy}}{A_{xx}} = \frac{34110}{113380} = 0.30$$

4.2 – Laminate coefficients of expansion. In this section, we derive the expressions to compute the coefficients of thermal and hydric expansion of circular cylindrical laminates.

The laminate thermal resultants are

$$[N^T] = [N^T]_{chop} + [N^T]_{UD} + [N^T]_{sand} + [N^T]_r$$

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \left\{ \begin{bmatrix} 2,50 \\ 2,50 \\ 0 \end{bmatrix} \times t_{chop} + \begin{bmatrix} N_x^T \\ N_y^T \\ 0 \end{bmatrix} \times t_{UD} + \begin{bmatrix} 1,12 \\ 1,12 \\ 0 \end{bmatrix} \times t_{sand} + \begin{bmatrix} 2,58 \\ 2,58 \\ 0 \end{bmatrix} \times t_r \right\} \times (OT - PT)$$

The thermal expansion coefficients of the laminate comes from Hooke's equation with the thermal resultants $[N^T]$ in place of the external forces $[N]$.

$$\begin{bmatrix} N_x^T \\ N_y^T \\ 0 \end{bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} & 0 \\ A_{xy} & A_{yy} & 0 \\ 0 & 0 & A_{ss} \end{bmatrix} \times \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix} \times (OT - PT)$$

Entering the thermal resultants in this equation, the temperature factor cancels out leaving

$$\begin{bmatrix} A_{xx} & A_{xy} & 0 \\ A_{xy} & A_{yy} & 0 \\ 0 & 0 & A_{ss} \end{bmatrix} \times \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix} = \begin{bmatrix} 2,50 \\ 2,50 \\ 0 \end{bmatrix} \times t_{chop} + \begin{bmatrix} N_x^T \\ N_y^T \\ 0 \end{bmatrix} \times t_{UD} + \begin{bmatrix} 1,12 \\ 1,12 \\ 0 \end{bmatrix} \times t_{sand} + \begin{bmatrix} 2,58 \\ 2,58 \\ 0 \end{bmatrix} \times t_r$$

This is the general equation to compute the coefficients of thermal expansion of laminates.

The computation of the coefficients of hydric expansion follows the same protocol.

$$\begin{bmatrix} A_{xx} & A_{xy} & 0 \\ A_{xy} & A_{yy} & 0 \\ 0 & 0 & A_{ss} \end{bmatrix} \times \begin{bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{bmatrix} = \begin{bmatrix} 19000 \\ 19000 \\ 0 \end{bmatrix} \times t_{chop} + \begin{bmatrix} N_x^H \\ N_y^H \\ 0 \end{bmatrix} \times t_{UD} + \begin{bmatrix} 12800 \\ 12800 \\ 0 \end{bmatrix} \times t_{sand} + \begin{bmatrix} 17200 \\ 17200 \\ 0 \end{bmatrix} \times t_r$$

Example 7 – Compute the thermal and hydric coefficients of expansion of a UD70 laminate with the following plies.

$$t_{UD} = 3.5 \text{ mm}, \quad t_{chop} = 2.0 \text{ mm}, \quad t_r = 0.9 \text{ mm}$$

The laminate stiffness matrix $[A]$ is

$$[A] = \begin{bmatrix} 104000 & 61000 & 0 \\ 61000 & 334000 & 0 \\ 0 & 0 & 66000 \end{bmatrix} \times 0.35 + \begin{bmatrix} 77000 & 23000 & 0 \\ 23000 & 77000 & 0 \\ 0 & 0 & 27000 \end{bmatrix} \times 0.20$$

$$+ \begin{bmatrix} 33000 & 10000 & 0 \\ 10000 & 33000 & 0 \\ 0 & 0 & 11500 \end{bmatrix} \times 0.09$$

$$[A] = \begin{bmatrix} 54770 & 26900 & 0 \\ 26900 & 134970 & 0 \\ 0 & 0 & 29530 \end{bmatrix} \text{ kg/cm}$$

The coefficients of thermal expansion are

$$\begin{bmatrix} A_{xx} & A_{xy} & 0 \\ A_{xy} & A_{yy} & 0 \\ 0 & 0 & A_{ss} \end{bmatrix} \times \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix} = \begin{bmatrix} 2,50 \\ 2,50 \\ 0 \end{bmatrix} \times t_{chop} + \begin{bmatrix} N_x^T \\ N_y^T \\ 0 \end{bmatrix} \times t_{UD} + \begin{bmatrix} 2,58 \\ 2,58 \\ 0 \end{bmatrix} \times t_r$$

$$\begin{bmatrix} 54770 & 26900 & 0 \\ 26900 & 134970 & 0 \\ 0 & 0 & 29530 \end{bmatrix} \times \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix} = \begin{bmatrix} 2,50 \\ 2,50 \\ 0 \end{bmatrix} \times 0.20 + \begin{bmatrix} 5,22 \\ 8,50 \\ 0 \end{bmatrix} \times 0.35 + \begin{bmatrix} 2,58 \\ 2,58 \\ 0 \end{bmatrix} \times 0.09$$

$$\begin{bmatrix} 54770 & 26900 & 0 \\ 26900 & 134970 & 0 \\ 0 & 0 & 29530 \end{bmatrix} \times \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix} = \begin{bmatrix} 2,56 \\ 3,71 \\ 0 \end{bmatrix}$$

Solving this equation, we have

$$\begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix} = \begin{bmatrix} 36,8 \\ 20,1 \\ 0 \end{bmatrix} \times 10^{-6}/C$$

In the above computations, the ply thermal forces are those listed in chapter 1

The coefficients of hydric expansion are

$$\begin{bmatrix} A_{xx} & A_{xy} & 0 \\ A_{xy} & A_{yy} & 0 \\ 0 & 0 & A_{ss} \end{bmatrix} \times \begin{bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{bmatrix} = \begin{bmatrix} 19000 \\ 19000 \\ 0 \end{bmatrix} \times t_{chop} + \begin{bmatrix} N_x^H \\ N_y^H \\ 0 \end{bmatrix} \times t_{UD} + \begin{bmatrix} 17200 \\ 17200 \\ 0 \end{bmatrix} \times t_r$$

$$\begin{bmatrix} 54770 & 26900 & 0 \\ 26900 & 134970 & 0 \\ 0 & 0 & 29530 \end{bmatrix} \times \begin{bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{bmatrix} = \begin{bmatrix} 19000 \\ 19000 \\ 0 \end{bmatrix} \times 0.20 + \begin{bmatrix} 20640 \\ 32350 \\ 0 \end{bmatrix} \times 0.35 + \begin{bmatrix} 17200 \\ 17200 \\ 0 \end{bmatrix} \times 0.09$$

Where the ply hydric resultants come from chapter 1

$$\begin{bmatrix} 54770 & 26900 & 0 \\ 26900 & 134970 & 0 \\ 0 & 0 & 29530 \end{bmatrix} \times \begin{bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{bmatrix} = \begin{bmatrix} 12570 \\ 16670 \\ 0 \end{bmatrix}$$

The coefficients of hydric expansion are

$$\begin{bmatrix} \beta_x \\ \beta_y \\ \beta_{xy} \end{bmatrix} = \begin{bmatrix} 0.19 \\ 0.09 \\ 0 \end{bmatrix}$$

The protocol just described computes the expansion coefficients of any circular cylindrical laminate in a simple and straightforward manner.

Example 8. Compute the coefficients of thermal expansion of an existing laminate with the following data:

$t = 1.2$ cm is the laminate measured thickness

$(res) = 0.35$

$(UD90) = 0.25$

$(sand) = 0.30$

$(chop) = 0.10$

The stiffness matrix $[A]$ is

$$[A] = \frac{3.12 \times t}{1.20 + 1.40(res)} \times [2.33(chop)[Q]_{chop} + 0.74(UD)[Q]_{UD} + 0.60(sand)[Q]_{sand} + 0.83K[Q]_r]$$

Where the ply stiffness matrices are available in chapter 1

The laminate dryness is

$$K = (res) - \left[\frac{3}{7}(UD) + \frac{7}{3}(chop) + \frac{2}{8}(sand) \right]$$

$$K = (0.35) - \left[\frac{3}{7}(0.25) + \frac{7}{3}(0.10) + \frac{2}{8}(0.30) \right] = -0.07$$

$$[A] = \frac{3.12 \times 1.2}{1.20 + 1.40 \times 0.35} \times \left[2.33 \times 0.10 \times \begin{bmatrix} 77000 & 23000 & 0 \\ 23000 & 77000 & 0 \\ 0 & 0 & 27000 \end{bmatrix} + 0.74 \times 0.25 \times \begin{bmatrix} 100000 & 30000 & 0 \\ 30000 & 400000 & 0 \\ 0 & 0 & 35000 \end{bmatrix} + 0.60 \times 0.30 \times \begin{bmatrix} 66000 & 20000 & 0 \\ 20000 & 66000 & 0 \\ 0 & 0 & 23000 \end{bmatrix} - 0.83 \times 0.07 \times \begin{bmatrix} 33000 & 10000 & 0 \\ 10000 & 33000 & 0 \\ 0 & 0 & 11500 \end{bmatrix} \right]$$

$$[A] = \begin{bmatrix} 102800 & 30850 & 0 \\ 30850 & 225750 & 0 \\ 0 & 0 & 36000 \end{bmatrix} \text{ kg/cm}$$

The thermal resultant $[N^T]$ is

$$[N^T] = \frac{3.12 \times t}{1.20 + 1.40(res)} \times [2.33(chop)[N^T]_{chop} + 0.74(UD)[N^T]_{UD} + 0.60(sand)[N^T]_{sand} + 0.83K[N^T]_r]$$

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \frac{3.12 \times 1.2}{1.20 + 1.40 \times 0.35} \times \begin{bmatrix} 2.33 \times 0.10 \times \begin{bmatrix} 2,50 \\ 2,50 \\ 0 \end{bmatrix} + 0.74 \times 0.25 \times \begin{bmatrix} 4.73 \\ 4.35 \\ 0 \end{bmatrix} + 0.60 \times 0.30 \times \begin{bmatrix} 1,12 \\ 1,12 \\ 0 \end{bmatrix} \\ -0.83 \times 0.07 \times \begin{bmatrix} 2,58 \\ 2,58 \\ 0 \end{bmatrix} \end{bmatrix} \times (OT - PT)$$

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} 3.34 \\ 3.19 \\ 0 \end{bmatrix} \times (OT - PT)$$

The coefficients of thermal expansion are

$$\begin{bmatrix} 102800 & 30850 & 0 \\ 30850 & 225750 & 0 \\ 0 & 0 & 36000 \end{bmatrix} \times \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix} = \begin{bmatrix} 3.34 \\ 3.19 \\ 0 \end{bmatrix}$$

Solving this equation, we have

$$\begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{bmatrix} = \begin{bmatrix} 29.5 \\ 10.1 \\ 0 \end{bmatrix} \times 10^{-6}/C$$

In the above computations, the ply thermal forces are those listed in chapter 1

The computation of the coefficients of hydric expansion follows the same protocol.

Example 9: This numerical example shows how to obtain the standard ply stiffness matrices $[Q]_{12}$ from lab tests conducted on non-standard plies.

The properties of non-standard plies obtained in lab tests are never the same as those of the standard plies. The standard plies are mathematical fictions, impossible to replicate in the real world. The lab tests provide the actual engineering properties, the actual thickness and the glass weight in kg/m^2 of the tested ply. Of the preceding quantities, only the glass weight is the same in both the actual and the standard plies. The following equation shows how the standard ply matrix $[Q]_{12}$ can be obtained from actual lab tests.

$$[Q]_{12} = \begin{bmatrix} \frac{E_1}{1 - \nu_{12}\nu_{21}} & \frac{\nu_{21}E_1}{1 - \nu_{12}\nu_{21}} & 0 \\ \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}} & \frac{E_2}{1 - \nu_{12}\nu_{21}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \times \frac{t_{\text{actual}}}{t} - \begin{bmatrix} 33000 & 10000 & 0 \\ 10000 & 33000 & 0 \\ 0 & 0 & 11500 \end{bmatrix} \times \frac{t_{\text{actual}} - t}{t}$$

The only unknown in the above equation is the standard ply thickness “t”, which is readily computed from the measured glass weight and the formulas listed in table 3 of chapter 1. The quantities in the above equation are all known, allowing the direct computation of the standard ply matrix $[Q]_{12}$. Let us consider a numerical example.

A lab characterization of a UD ply yielded the following data.

Modulus in the fiber direction	$E_1 = 331\ 000\text{ kg/cm}^2$
Transverse modulus	$E_2 = 85\ 000\text{ kg/cm}^2$
Major Poisson ratio	$\nu_{12} = 0.31$
Minor Poisson ratio	$\nu_{21} = 0.08$
Shear modulus	$G_{12} = 31\ 000\text{ kg/cm}^2$
Actual ply thickness	3.0 mm
Glass weight	3400 g/m ²

We use this data to compute the ply stiffness $[Q]_{12}$ of the standard ply. First, we compute the standard ply thickness from the measured glass weight. For that, we use the appropriate equation listed in chapter 1.

$$\left[\frac{\text{kg}}{\text{m}^2}\right]_{UD} = 1.35 \times t_{UD}$$

$$3.4 = 1.35 \times t_{UD}$$

$$t_{UD} = 2.5\text{ mm}$$

This is the standard ply thickness, corresponding to the measured 3400 g/m² of glass. The actual ply thickness is 3.0 mm, meaning an excess resin of $3.0 - 2.5 = 0.5\text{ mm}$. Next, we enter the measured lab data in the equation for $[Q]_{12}$.

$$[Q]_{12} = \begin{bmatrix} \frac{331000}{1 - 0.08 \times 0.31} & \frac{0.08 \times 331000}{1 - 0.08 \times 0.31} & 0 \\ \frac{0.31 \times 85000}{1 - 0.08 \times 0.31} & \frac{85000}{1 - 0.08 \times 0.31} & 0 \\ 0 & 0 & 31000 \end{bmatrix} \times \frac{3.0}{2.5} - \begin{bmatrix} 33000 & 10000 & 0 \\ 10000 & 33000 & 0 \\ 0 & 0 & 11500 \end{bmatrix} \times \frac{3.0 - 2.5}{2.5}$$

$$[Q]_{12} = \begin{bmatrix} 400700 & 30500 & 0 \\ 30500 & 98000 & 0 \\ 0 & 0 & 34900 \end{bmatrix}$$

The stiffness matrix is nearly equal to the proposed standard listed in table 6 of chapter 1.

It is quite possible to correct the lab results using the ply composition. The procedure to do this is straightforward.

$$\begin{bmatrix} \frac{331000}{1 - 0.08 \times 0.31} & \frac{0.08 \times 331000}{1 - 0.08 \times 0.31} & 0 \\ \frac{0.31 \times 85000}{1 - 0.08 \times 0.31} & \frac{85000}{1 - 0.08 \times 0.31} & 0 \\ 0 & 0 & 31000 \end{bmatrix} = \frac{3,12 \times t}{1,20 + 1,40(\text{res})} \times \left[0,74(UD)[Q]_{UD} + 0,83K \begin{bmatrix} 33000 & 10000 & 0 \\ 10000 & 33000 & 0 \\ 0 & 0 & 11500 \end{bmatrix} \right]$$

In the above, the ply standard matrix $[Q]_{UD}$ is the only unknown

This numerical example brings to mind a comment by Ever Barbero in his book “Finite Element Analysis of Composite Materials”. See reference 30.

The elastic properties of unidirectional plies derive from micromechanics or experimental data. In the analysis of most composite structures, it is usual to avoid the micromechanics approach and to obtain experimentally the properties of the UD plies. However, the experimental approach is not ideal, because a change of constituents or volume fraction of reinforcement invalidates the material data and

requires a new experimental program for the new material. It is better to calculate the elastic properties of the ply using micromechanics formulas. Unfortunately, the micromechanics formulas are not accurate to predict strength, so experimental work cannot be ruled out completely.

This comment ignores the concept of standard plies and the mathematical corrections of experimental data. As shown in the preceding numerical examples, the experimental lab data serve very well to characterize standard plies.

Chapter 4

Computing ply strains

1 – Introduction. The first step in assessing the load-dependent durability of equipment is the accurate computation of the total strains on the critical ply, i.e., the ply actually governing the failure. We will have more to say about the critical plies and the importance of their strains in the part 2 of this book. For now, let us tackle the strain issue.

In this chapter, we develop a simple and accurate protocol to compute the total strains of plies embedded in circular cylindrical laminates subjected to thermal, hydric and mechanical loadings. The published literature is highly competent in computing the ply strains from mechanical loadings, but less so for the thermal and hydric residual strains. In fact, the published literature is unsatisfactory in the computation of the hydric and thermal residual strains. The simple and accurate protocol developed in this chapter fills this existing gap.

The total ply strains of embedded plies have three independent components.

$$\begin{bmatrix} total \\ strain \end{bmatrix} = \begin{bmatrix} mechanical \\ strain \end{bmatrix} + \begin{bmatrix} residual\ thermal \\ strain \end{bmatrix} + \begin{bmatrix} residual\ hydric \\ strain \end{bmatrix}$$

To facilitate the discussion and clarify the issue, we will develop separate computation protocols for each component. The reader will realize the similarities in each case and see the small differences as obvious and intuitive. Each of the three strain components are first computed in the global system $x - y$ and subsequently rotated to the local ply frame 1 – 2.

The computation protocol developed in this book is applicable to circular cylindrical laminates subjected to membrane strains. As explained in chapter two, the strains in such laminates are independent of the coupling matrix [B]. This is a tremendous simplification. However, despite this simplification, the computation of all strains, for all plies, on the local and global frames, requires patience and attention. The prudent designer always rechecks his computation to avoid possible mistakes.

We start with the residual thermal strains.

2 – Computing the thermal strains – The commercial laminates are laid up at room temperature, combining several plies of glass fibers impregnated with liquid resin. The liquid resin gradually undergoes a curing process, becoming a gel that subsequently turns solid. The cure process releases chemical energy that increases the laminate temperature. In addition to this internal released heat, the manufacturer may also provide outside heating to expedite the process. The laminate heats up from the room temperature (RT) to a peak cure temperature (PT) before cooling down. On reaching the peak temperature (PT), the resin in the individual plies is still in a stress-free condition. The plies and the laminate remain at the peak temperature (PT) for a while, as the resin changes from liquid into a hard solid. The important thing to consider at this point is the stress-free condition of the laminate at the temperature PT. After a while, when the resin has set into a solid, the temperature starts to drop, shrinking the laminate as a single body. This shrinkage induces thermal residual strains on the plies.

In the early phase of the cure process, when the temperature first reaches the peak process temperature PT, the crosslinking has not advanced much, and the still liquid resin can flow to keep the plies in a stress-free condition. Thereupon, the cure proceeds quickly to set the resin into a rigid solid that congeals and immobilizes the plies in this stress-free condition. The thermal stresses develop as the temperature drops from PT. All plies cool off and shrink together, as a laminate, to take the same equilibrium dimensions. At the equilibrium temperature OT, the plies that shrink more come to rest in a stretched condition, while those that shrink less are compressed. This is because all plies shrink together as a laminate.

The stress-free temperature PT is determined by taking into consideration four temperatures.

- **Heat distortion temperature HDT.** *This is the temperature at which the cured resin gains molecular mobility that dissipates all residual stresses. The HDT sets the maximum possible value for the stress-free temperature. Therefore, even if $PT > HDT$, the maximum stress-free temperature is $PT = HDT$.*
- **Peak process temperature PPT.** *This is the highest temperature achieved in unassisted room temperature cure. In laminates that are not post-cured, the stress-free temperature is $PT = PPT$, subject to $PT < HDT$.*
- **Post-cure temperature PCT.** *This is the highest temperature reached in laminates that are post-cured. When $PCT < HDT$, we have $PT = PCT$. If $PCT > HDT$, then $PT = HDT$.*

- **Operating temperature OT.** Also known as working temperature. At OT, the laminate takes its equilibrium position at which the plies develop their residual thermal strains.

The stress-free temperature falls between a minimum at $PT = PPT$ in laminates that are not post-cured and a maximum at $PT = HDT$ in laminates that are post-cured above the HDT. In laminates that are post-cured at temperatures less than the HDT, $PT = PCT$. Figure 1 illustrates this.

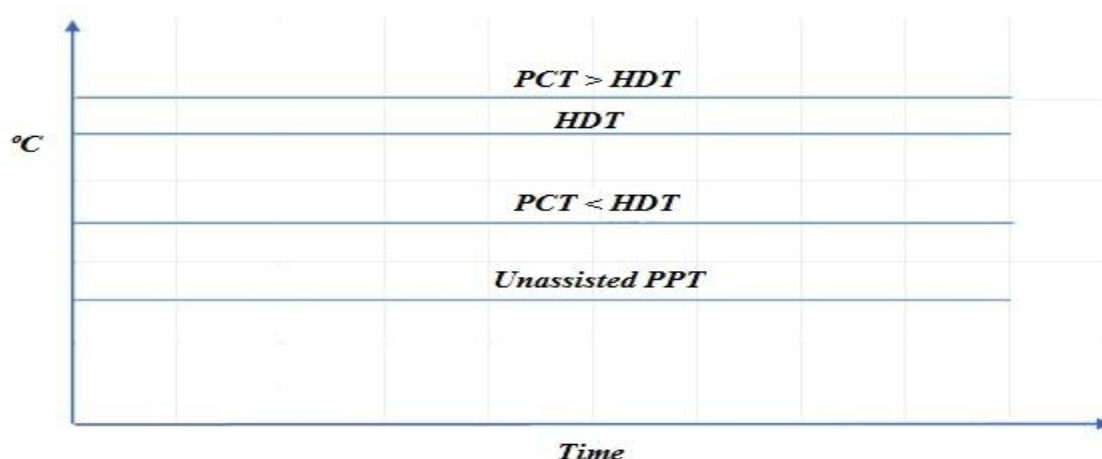


Figure 1

The stress-free temperature PT is the maximum temperature achieved in the cure process. It ranges from a minimum unassisted peak process temperature PPT to a maximum limited by the resin HDT .

Example 1: This numerical example clarifies the preceding discussion. Let us assume a laminate of vinyl ester Derakane DK 411 of $HDT = 105C$. The unassisted peak process temperature is $PPT = 70C$.

- Suppose the laminate is not post-cured. The stress-free temperature in this case is $PT = PPT = 70C$.
- Suppose the laminate is post-cured for 1 hour at $PCT = 90C$. The stress-free temperature in this case is $PT = PCT = 90C$.
- Suppose the laminate is post-cured for 1 hour at $PCT = 120C$. The stress-free temperature in this case is $PT = HDT = 105C$.

The computation of the equilibrium thermal strains in circular cylindrical laminates requires knowledge of the laminate stiffness matrix $[A]$ and the thermal force resultants $[N^T]$, all referred to the global system $x - y$. Before starting the computations, the ply matrices $[Q]$ and $[\alpha]$ must be rotated from their local frames $1 - 2$ to the global frame $x - y$.

The complete protocol to compute the residual thermal strains is

1. Rotation of the ply elastic matrices $[Q]$ from the local to the global frame.
2. Computation of the laminate matrix $[A]$
3. Rotation of the ply coefficients of thermal expansion to the global frame.
4. Computation of the laminate thermal resultants
5. Computation of the laminate thermal equilibrium strains
6. Rotation of the equilibrium strains from the global to the local ply frame.
7. Computation of the residual ply strains in the local system 1 – 2

The seven steps outlined above compute the residual thermal strains of plies embedded in circular cylindrical laminates. The rest of this section is dedicated to a full and detailed description of the entire procedure, often repeating things and concepts for the sake of clarity. The experienced reader will bear with this repetition.

1. Rotation of the ply elastic matrices $[Q]$ from the local to the global frame. This rotation is required only for UD plies. The ply matrices $[Q]$ in the local 1 – 2 and in the global x-y systems are.

$$[Q]_{12} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}_{LOCAL} \quad [Q]_{xy} = \begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xs} \\ Q_{yx} & Q_{yy} & Q_{ys} \\ Q_{sx} & Q_{sy} & Q_{ss} \end{bmatrix}_{GLOBAL}$$

The elastic matrices $[Q]_{12}$ of all plies in the local frame are tabulated in chapter 1. The elastic matrices of UD55, UD70 and UD90 plies in the global frame are also available in chapter 1. For any other ply, the elements of the rotated matrices $[Q]_{xy}$ are

$$\begin{aligned} Q_{xx} &= m^4 Q_{11} + n^4 Q_{22} + 2m^2 n^2 Q_{12} + 4m^2 n^2 Q_{66} \\ Q_{xy} &= Q_{yx} = m^2 n^2 Q_{11} + m^2 n^2 Q_{22} + (m^4 + n^4) Q_{12} - 4m^2 n^2 Q_{66} \\ Q_{xs} &= Q_{sx} = m^3 n Q_{11} - mn^3 Q_{22} + (mn^3 - m^3 n) Q_{12} + 2(mn^3 - m^3 n) Q_{66} \\ Q_{yy} &= n^4 Q_{11} + m^4 Q_{22} + 2m^2 n^2 Q_{12} + 4m^2 n^2 Q_{66} \\ Q_{ys} &= Q_{sy} = mn^3 Q_{11} - m^3 n Q_{22} + (m^3 n - mn^3) Q_{12} + 2(m^3 n - mn^3) Q_{66} \\ Q_{ss} &= m^2 n^2 Q_{11} + m^2 n^2 Q_{22} - 2m^2 n^2 Q_{12} + (m^2 - n^2)^2 Q_{66} \end{aligned}$$

Where “m” is the cosine and “n” is the sine of the angle formed by the global laminate axis “x” and the local axis 1 of the UD ply. The above computations include all UD plies. They are rather complex and full of opportunities for errors. The prudent designer will repeat the computations to detect possible mistakes. The orthotropic UD plies are the only ones requiring rotation. The isotropic plies require no rotation.

2. Computation of the laminate stiffness matrix $[A]$. The elements of the laminate stiffness matrix $[A]$ include contributions from all plies. The expression below is self-explanatory.

$$[A] = \begin{bmatrix} \sum Q_{xx}t & \sum Q_{xy}t & \sum Q_{xs}t \\ \sum Q_{yx}t & \sum Q_{yy}t & \sum Q_{ys}t \\ \sum Q_{sx}t & \sum Q_{sy}t & \sum Q_{ss}t \end{bmatrix}$$

In the above equation, “t” are the individual ply thicknesses and $[Q]_{xy}$ are the elastic matrices of each ply in the global system.

3. Rotation of the ply coefficients of thermal expansion into the global frame. This rotation is required only for UD plies

$$\alpha_x = m^2\alpha_1 + n^2\alpha_2$$

$$\alpha_y = n^2\alpha_1 + m^2\alpha_2$$

$$\alpha_{xy} = 2mn(\alpha_2 - \alpha_1)$$

Where again $m = \cos(\alpha)$ and $n = \sin(\alpha)$, α being the angle between the fiber direction 1 and the longitudinal global axis x. The rotation applies to all UD plies.

4. Computation of the thermal resultants. The general equations to compute the thermal resultants are self-explanatory.

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} \sum (Q_{xx}t\alpha_x + Q_{xy}t\alpha_y + Q_{xs}t\alpha_{xy}) \\ \sum (Q_{yx}t\alpha_x + Q_{yy}t\alpha_y + Q_{ys}t\alpha_{xy}) \\ \sum (Q_{sx}t\alpha_x + Q_{sy}t\alpha_y + Q_{ss}t\alpha_{xy}) \end{bmatrix} \times (OT - PT)$$

Since all commercial laminates are balanced, the above equation reduces to

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} \sum (Q_{xx}t\alpha_x + Q_{xy}t\alpha_y + Q_{xs}t\alpha_{xy}) \\ \sum (Q_{yx}t\alpha_x + Q_{yy}t\alpha_y + Q_{ys}t\alpha_{xy}) \\ 0 \end{bmatrix} \times (OT - PT)$$

Balanced laminates have no shear thermal resultant in the global system.

5. Computation of the equilibrium thermal strains. The equilibrium takes place when the laminate cools down from the peak temperature PT to the operating temperature OT. The circularity condition guarantees the same equilibrium strains for all plies. The equilibrium strains depend on the laminate stiffness matrix $[A]$ and the thermal resultants $[N^T]$.

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} & A_{xs} \\ A_{yx} & A_{yy} & A_{ys} \\ A_{sx} & A_{sy} & A_{ss} \end{bmatrix} \times \begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix}$$

Since all commercial laminates are balanced, the above equation reduces to

$$\begin{bmatrix} N_x^T \\ N_y^T \\ 0 \end{bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} & 0 \\ A_{yx} & A_{yy} & 0 \\ 0 & 0 & A_{ss} \end{bmatrix} \times \begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix}$$

This equation immediately leads to

$$\gamma_{xy}^e = 0$$

The equilibrium thermal strains are

$$\begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ 0 \end{bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} & 0 \\ A_{yx} & A_{yy} & 0 \\ 0 & 0 & A_{ss} \end{bmatrix}^{-1} \times \begin{bmatrix} N_x^T \\ N_y^T \\ 0 \end{bmatrix}$$

In this equation, the superscript “e” designates equilibrium strains. The equilibrium strains take as reference the laminate dimensions at the stress-free peak temperature PT, not at the room temperature RT or the operating temperature OT.

Figure 2 shows a laminate with three plies of equal dimensions at the stress-free temperature PT. Also shown are the ply dimensions if allowed to shrink freely from PT to OT. As already discussed, the embedded plies develop residual strains when forced to take the same equilibrium dimensions at the operating temperature OT. The equilibrium strains are negative, since the laminate shrinks when cooling from PT to the operating temperature OT. The residual strains, however, can be positive or negative, as shown in figure 2.

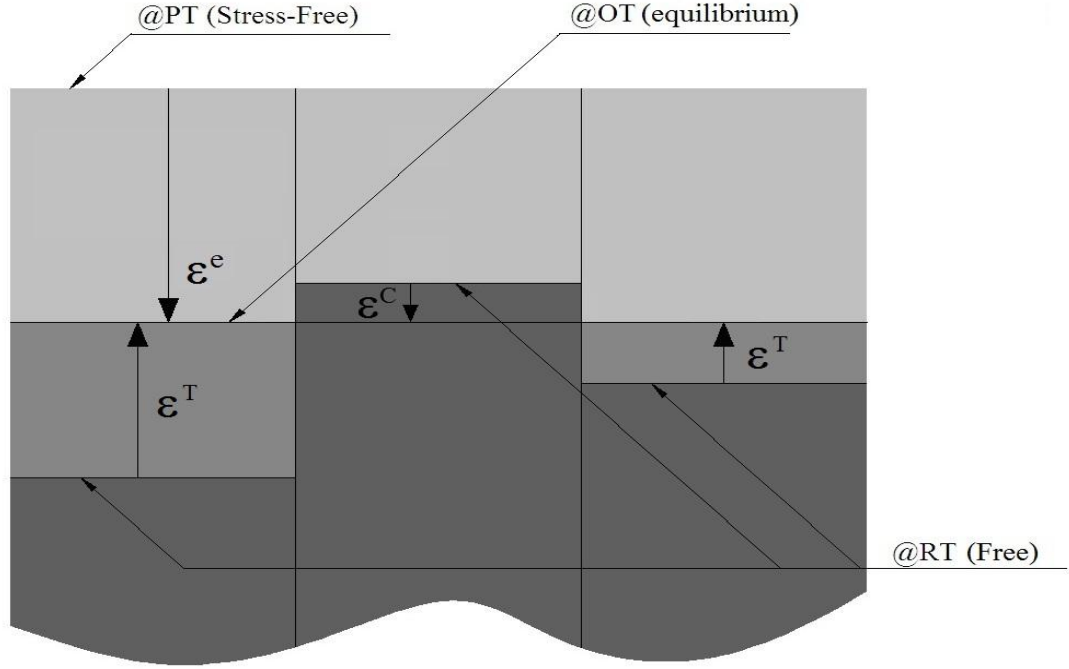


Figure 2

The thermal strains are zero (stress-free condition) for all plies at the temperature PT. The equilibrium strains are equal for all plies at the operating temperature OT. The heavy shaded areas show the rest position of each ply if allowed to contract freely.

The preservation of circularity assures that all plies have the same equilibrium strains in the global system $x-y$. These strains need rotation to the local systems 1-2 of each ply.

6. Rotation of the equilibrium strains. This rotation is required only for UD plies. The rotation involves the multiplication of the equilibrium strains by the matrix $[T]$.

$$[T] = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix}$$

Where “ m ” is the cosine and “ n ” is the sine of the angle defining the ply orientation. The equilibrium thermal strains referred to the local system 1-2 are:

$$\begin{bmatrix} \epsilon_1^e \\ \epsilon_2^e \\ \frac{1}{2}\gamma_{12}^e \end{bmatrix} = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix} \times \begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ 0 \end{bmatrix}$$

Multiplication by the matrix $[T]$ rotates the strains from the global $x - y$ to the local $1 - 2$ system of the UD plies. The equilibrium shear strain is not zero in the local frame. In fact, the equilibrium shear strain in the local system is

$$\gamma_{12}^e = 2 \times mn \times (\epsilon_y^e - \epsilon_x^e)$$

7. Computing the residual thermal strains. The residual thermal strains come from subtracting the free ply expansion from the equilibrium strains.

$$\begin{bmatrix} \epsilon_1^r \\ \epsilon_2^r \\ \gamma_{12}^r \end{bmatrix} = \begin{bmatrix} \epsilon_1^e - \alpha_1(OT - PT) \\ \epsilon_2^e - \alpha_2(OT - PT) \\ \gamma_{12}^e \end{bmatrix}$$

The residual thermal strains (superscript “r”) exist because the embedded plies take equilibrium dimensions different from those they would take if allowed to shrink freely. They can be negative or positive, depending on the thermal contraction of each ply.

Note: The residual thermal stresses are not required to predict the load-dependent durability. Their computation is not necessary. As a curiosity, the residual stresses derive from the ply constitutive equation and the residual strains.

$$\begin{bmatrix} \sigma_1^r \\ \sigma_2^r \\ \tau_{12}^r \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{ss} \end{bmatrix} \times \begin{bmatrix} \epsilon_1^r \\ \epsilon_2^r \\ \gamma_{12}^r \end{bmatrix}$$

3.3 – Computing the hydric residual strains – The hydric residual strains are computed in the same manner as their thermal counterparts. The protocol is essentially identical, except for the hydric expansion coefficients replacing their thermal counterparts. Furthermore, the resin water pickup Δm replaces the temperature drop $PT - OT$. The complete protocol to compute the hydric residual strains is

1. Rotation of the ply matrices $[Q]$ to the global frame.
2. Computation of the matrix $[A]$
3. Rotation of coefficients of hydric expansion to the global frame.
4. Computation of the hydric resultants
5. Computation of the equilibrium strains in the global system $x - y$
6. Rotation of the equilibrium strains to the local system $1 - 2$ of each UD ply
7. Computation of the residual hydric strains in the local system $1 - 2$.

The computations are similar to those for the thermal case. What follows is a highlight of the differences. The hydric coefficients of expansion in the global system are.

$$\begin{aligned}\beta_x &= m^2 \beta_1 + n^2 \beta_2 \\ \beta_y &= n^2 \beta_1 + m^2 \beta_2 \\ \beta_{xy} &= 2mn(\beta_2 - \beta_1)\end{aligned}$$

The hydric force resultants are

$$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = \begin{bmatrix} \sum (Q_{xx} t \beta_x + Q_{xy} t \beta_y + Q_{xs} t \beta_{xy}) \\ \sum (Q_{yx} t \beta_x + Q_{yy} t \beta_y + Q_{ys} t \beta_{xy}) \\ \sum (Q_{sx} t \beta_x + Q_{sy} t \beta_y + Q_{ss} t \beta_{xy}) \end{bmatrix} \times (\Delta m)$$

In the above, “t” is the thickness and “β” is the coefficient of hydric expansion of each ply. “Δm” is the amount of water picked up by the resin.

The hydric equilibrium strains are

$$\begin{bmatrix} \varepsilon_x^e \\ \varepsilon_y^e \\ 0 \end{bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} & 0 \\ A_{yx} & A_{yy} & 0 \\ 0 & 0 & A_{ss} \end{bmatrix}^{-1} \times \begin{bmatrix} N_x^H \\ N_y^H \\ 0 \end{bmatrix}$$

The superscript “e” denotes the equilibrium strain, reached when the resin saturates with water. These strains are measured in the global system x – y, taking as reference the stress-free dry dimensions of the laminate. The equilibrium strains are positive, since all laminates expand on picking up water. The hydric equilibrium strains expressed in the global system are directly proportional to the amount of water picked up by the resin. The global equilibrium strains rotated to the local 1 – 2 system in the usual way.

$$\begin{bmatrix} \varepsilon_1^e \\ \varepsilon_2^e \\ 1/2 \gamma_{12}^e \end{bmatrix} = [T] \times \begin{bmatrix} \varepsilon_x^e \\ \varepsilon_y^e \\ 0 \end{bmatrix}$$

Where [T] is the previously defined rotation matrix

The residual hydric strains (indicated by the superscript “r”) occur when the laminate constrains the embedded plies and prevent their free expansion as they soak up water. The residual strains result from the subtraction of the free expansion of each ply from the equilibrium strain. They can be negative or positive, depending on the hydric coefficient of expansion of each ply.

$$\begin{bmatrix} \varepsilon_1^r \\ \varepsilon_2^r \\ \gamma_{12}^r \end{bmatrix} = \begin{bmatrix} \varepsilon_1^e - \beta_1 \Delta m \\ \varepsilon_2^e - \beta_2 \Delta m \\ \gamma_{12}^e \end{bmatrix}$$

The residual hydric and thermal strains are part of the total ply strains. They enter the constitutive equations to compute the residual stresses of each ply.

The residual stresses are not required to compute the load-dependent durability. Their computation, shown below, is not required.

$$\begin{bmatrix} \sigma_1^r \\ \sigma_2^r \\ \tau_{12}^r \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{ss} \end{bmatrix} \times \begin{bmatrix} \varepsilon_1^r \\ \varepsilon_2^r \\ \gamma_{12}^r \end{bmatrix}$$

4 – Computing the mechanical strains – The protocol to compute the mechanical strains is like follows:

1. *Rotation of the ply elastic matrices [Q] from the local to the global frame*
2. *Computation of the laminate matrix [A]*
3. *There are no coefficients of mechanical expansion*
4. *The mechanical resultants are given.*
5. *Computation of the global mechanical strains*
6. *Rotation of the global strains to the local system 1 – 2.*

The computation details are essentially identical to those discussed before and are not repeated here. The laminate stiffness matrix [A] is the same. There are no mechanical coefficients of expansion. The mechanical force resultants are the given external forces. The mechanical equilibrium strains in the global system x – y are computed in the same way as the equilibrium thermal and hydric strains.

Since all commercial laminates are balanced, we have

$$\begin{bmatrix} \varepsilon_x^e \\ \varepsilon_y^e \\ \gamma_{xy}^e \end{bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} & 0 \\ A_{yx} & A_{yy} & 0 \\ 0 & 0 & A_{ss} \end{bmatrix}^{-1} \times \begin{bmatrix} N_x^M \\ N_y^M \\ N_{xy}^M \end{bmatrix}$$

Where $[N^M]$ is the matrix of external forces.

In the absence of external torque, the mechanical shear resultant in the global frame is zero. The preservation of circularity assures the same equilibrium mechanical strains for all plies. The equilibrium strains rotate from the global to the local system in the usual way.

$$\begin{bmatrix} \varepsilon_1^e \\ \varepsilon_2^e \\ 1/2 \gamma_{12}^e \end{bmatrix} = [T] \times \begin{bmatrix} \varepsilon_x^e \\ \varepsilon_y^e \\ 1/2 \gamma_{xy}^e \end{bmatrix}$$

The equilibrium strains are the mechanical strains we are looking for.

Example 2: Assuming preservation of circularity, explain the difference between the equilibrium and the residual strains.

The preservation of circularity assures that all plies come to rest equally strained at the laminate equilibrium strain. The equilibrium strains occur when the mechanical loading, the operating temperature and the water pick up stabilize. The following equations give the equilibrium thermal, hydric and mechanical strains.

$$\begin{bmatrix} N_x^T \\ N_y^T \\ 0 \end{bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} & 0 \\ A_{yx} & A_{yy} & 0 \\ 0 & 0 & A_{ss} \end{bmatrix} \times \begin{bmatrix} \varepsilon_x^e \\ \varepsilon_y^e \\ 0 \end{bmatrix}_{Thermal}$$

$$\begin{bmatrix} N_x^H \\ N_y^H \\ 0 \end{bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} & 0 \\ A_{yx} & A_{yy} & 0 \\ 0 & 0 & A_{ss} \end{bmatrix} \times \begin{bmatrix} \varepsilon_x^e \\ \varepsilon_y^e \\ 0 \end{bmatrix}_{Hydric}$$

$$\begin{bmatrix} N_x^M \\ N_y^M \\ N_{xy}^M \end{bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} & 0 \\ A_{yx} & A_{yy} & 0 \\ 0 & 0 & A_{ss} \end{bmatrix} \times \begin{bmatrix} \varepsilon_x^e \\ \varepsilon_y^e \\ \gamma_{xy}^e \end{bmatrix}_{Mechanical}$$

The above are the laminate equilibrium strains. From the preservation of circularity, they are the same for all plies. Two things we should know about the equilibrium strains:

- They refer to the global laminate system, not to the local ply system.
- They are not the strains we are looking for. We are looking for the residual, not the equilibrium, strains.

The residual strains occur on those plies that expand or contract at a different rate than the laminate. The concept of residual strain is applicable to plies, not to laminates. The residual strains depend on the expansion coefficient of each ply and have the same nature as the mechanical strains. They accu-

mulate elastic energy and promote crack growth in the same way as the strains from external mechanical loads.

5 – Total strains – To obtain the total ply strains, we add the mechanical, thermal and hydric components. This addition is possible, since the three components act in the same ply directions.

The total ply strains are

$$\begin{bmatrix} Total \\ strain \end{bmatrix} = \begin{bmatrix} mechanical \\ strain \end{bmatrix} + \begin{bmatrix} residual \\ thermal strain \end{bmatrix} + \begin{bmatrix} residual \\ hydric strain \end{bmatrix}$$

From the protocol we have just described

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix}_{TOTAL} = \begin{bmatrix} \varepsilon_1^e \\ \varepsilon_2^e \\ \gamma_{12}^e \end{bmatrix}_{MEC} + \begin{bmatrix} \varepsilon_1^e - \alpha_1(OT - PT) \\ \varepsilon_2^e - \alpha_2(OT - PT) \\ \gamma_{12}^e \end{bmatrix}_{TER} + \begin{bmatrix} \varepsilon_1^e - \beta_1 \Delta m \\ \varepsilon_2^e - \beta_2 \Delta m \\ \gamma_{12}^e \end{bmatrix}_{HID}$$

The commercial product standards usually ignore the residual thermal and hydric strains. This oversight may have great impact on the analysis of the load-dependent durability.

In conclusion, we have described a simple protocol to compute the total strains in circular cylindrical laminates. The computed strains referred to the local ply frame are ready to enter the load-dependent durability equations described in the part 2 of this book.

The total strain in the fiber direction of UD plies governs the long-term rupture failure of pressure pipes or wind blades. In fact, the strains in the fiber direction of UD plies govern the long-term rupture of any structural laminate.

The total transverse strain in UD fibers governs the infiltration, stiffness and weep failures.

The same comments hold for other plies, like those of chopped fibers. We will have more to say about this in the part 2 of this book.

Appendix I

Stress-free temperature

In the first draft of this chapter, I wrote that the resin molecules congeal in a stress-free condition at the peak process temperature, PPT. Any later temperature excursion above the PPT – either in operation or in post-cure – would increase the resin cross-linking with no effect on the stress-free state. Assuming this statement as true, we conclude that the post-cure process is just a brief upset, a short-term excursion, into a temperature that happens to be higher than the stress-free temperature PPT. This argument seems plausible. The post-cure temperature may be irrelevant in the computation of the residual thermal strains.

I am reluctant to discard this powerful argument. I still feel that I was right in my first approach, and the cross-linking attained at the peak process temperature, PPT, is sufficient to congeal the resin in a stress-free condition. Should this be correct, the stress-free temperature PT to compute the thermal strains is

$$PT = PPT \text{ if } PPT < HDT$$

$$PT = HDT \text{ if } PPT > HDT$$

From this approach, the post-cure treatment has no effect on the stress-free temperature PT.

Example 7: Let us compute the stress-free temperature PT of the laminate processed as described in the numerical example 1. The HDT of the vinyl ester resin is $HDT = 105C$. The unassisted peak process temperature is $PPT = 70C$.

- Suppose the laminate is not post-cured. The stress-free temperature is $PT = PPT = 70C$.
- Suppose the laminate is post-cured for 1 hour at $PCT = 90C$. The stress-free temperature is still $PT = PPT = 70C$.

- Suppose the laminate is post-cured for 1 hour at $PCT = 120C$. The stress-free temperature is still $PT = PPT = 70C$.

Example 8: Compute the stress-free temperature PT of the laminate in example 7, assuming it has been overheated in the cure process and attained a peak process temperature $PPT = 120C$.

The resin HDT limits the stress-free temperature PT , which is therefore

$$PT = HDT = 105C.$$

Example 9: The accurate computation of the residual thermal strains at the ply level requires a good knowledge of the stress-free temperature PT . We have described, without committing ourselves, two equally reasonable approaches to estimate the stress-free temperature. Describe a simple experiment to determine the correct approach.

We start from the fact that all flat warped unbalanced laminates straighten out and regain flatness at the stress-free temperature. The experiment consists in the following steps:

First, we lay up several thin cross-ply flat laminates cured at various unassisted PPT . Second, we post-cure some of these laminates. The thin flat cross-ply laminates will show substantial warpage at room temperature. The experiment proceeds by heating up the warped laminates until they flatten out. The temperatures that eliminate the warping and flatten out the laminates are the stress-free temperatures.

The above experiment can decide which of the two models presented in this chapter best describes the stress-free temperature.

Appendix II

Cure shrinkage and residual strains

The polyester and vinyl ester resins display a high volumetric contraction of 6.0% to 8.0% in the cure process. This appendix explains why the resin cure shrinkage, although very high, has not been included in the protocol developed in this chapter.

Cure shrinkage – The resin cure shrinkage does not generate residual strains at the ply level. The residual strains from cure shrinkage are relevant only at the fiber level, not at the ply level, as discussed in chapter 10. As explained there, the process of ply characterization accounts for these residual cure strains that are, therefore, not relevant to the analyst.

The heat released in the cure process expands the still liquid and plastic resin. This thermal expansion is constrained by the mold, forcing the liquid and plastic resin to flow past the fibers to increase the laminate thickness. In the specific case of circular laminates, the resin-mold friction prevents any circumferential or axial expansion. The resin expands in the thickness direction and cures at the stress-free peak temperature PT . At the stress-free temperature PT , the hot laminate has full contact with the mold and there are no residual stresses in the plies.

Most of the cure shrinkage (6.0% to 8.0% in volume) occurs while the resin is still in the liquid and plastic state and therefore capable of free flow. The mold presence prevents the circumferential and axial shrinkage, which occurs in the thickness direction. At the stress-free peak temperature PT , the cured and already solid laminate is in the following condition:

- All plies are stress-free.
- The cured and solid laminate has full contact with the mold surface
- Most of the cure shrinkage has been absorbed as thickness reduction.
- The cure shrinkage originates resin-fiber micro-strains, not ply stresses.

Thermal shrinkage – The cured and solid laminates shrink and cool down from the peak temperature PT to the operating temperature OT . The residual thermal strains arise from this shrinkage.

Example 7 – Compute the final diameter of a composite cylinder molded in the following conditions:

Mold diameter $D = 3000$ mm.

Peak cure temperature $PT = PPT = 85C$

Operating temperature $OT = 25C$

Coefficient of thermal expansion $\alpha_y = 20 \times 10^{-6}/C$

As discussed above, the resin cure shrinkage is not relevant. The thermal shrinkage is

$$\text{thermal shrinkage} = \alpha_y(PT - OT) \times D$$

$$\text{thermal shrinkage} = 20 \times 10^{-6}(85 - 25) \times 3000 = 3.6 \text{ mm}$$

The final diameter is $3000 - 3.6 = 2996.4$ mm, a decrease of 1.2%.

Chapter 5

Total strains in UD55 pipes

1 – Introduction: This chapter illustrates the numerical computation of the total strains in plies of oil pipes, using the protocol developed in chapter 4 and the numerical matrices derived in chapter 3. The strains computed in this manner are essential inputs for the analysis of load-dependent durability, discussed in the part 2 of this book.

The wall construction of oil pipes consists of a thin liner of neat resin covered by several structural UD55 plies. The failure mode of these pipes is weeping of the UD55 plies, since they have no sand core or corrosion barrier, and never see highly corrosive service. The design for weep failure means the allowable strain should be less than the weep threshold of the critical UD55 plies. We assume plies of vinyl ester resin with a transverse weep threshold $T_w = 0.55\%$. We also assume a weeping short-term safety factor $SF = 2.0$. Therefore, the allowable transverse strain is

$$\text{allowable transverse strain} = \frac{\text{weep threshold}}{\text{short-term safety factor}}$$

$$\epsilon_2 = \frac{0.0055}{2} = 0.28\%$$

The total thickness of the UD55 plies is t_{UD} . The thickness of the thin liner of neat resin is $t_r = 0.3 \text{ mm}$. The analysis assumes above ground and freely supported pipes (no anchors). Our task is to compute the thickness t_{UD} taking into consideration the residual thermal and hydric strains. The transverse strain cannot exceed the allowable limit $\epsilon_2 = 0.28\%$

2 – Residual thermal strains. The laminate stiffness matrix is

$$[A] = \begin{bmatrix} 132470 & 96220 & 0 \\ 96220 & 235070 & 0 \\ 0 & 0 & 101220 \end{bmatrix} \times t_{UD} + \begin{bmatrix} 33000 & 10000 & 0 \\ 10000 & 33000 & 0 \\ 0 & 0 & 11500 \end{bmatrix} \times t_r$$
$$[A] = \begin{bmatrix} 132470 & 96220 & 0 \\ 96220 & 235070 & 0 \\ 0 & 0 & 101220 \end{bmatrix} \times t_{UD} + \begin{bmatrix} 33000 & 10000 & 0 \\ 10000 & 33000 & 0 \\ 0 & 0 & 11500 \end{bmatrix} \times 0.03$$

Ignoring the small contribution from the neat resin, the laminate stiffness matrix reduces to

$$[A] = \begin{bmatrix} 132470 & 96220 & 0 \\ 96220 & 235070 & 0 \\ 0 & 0 & 101220 \end{bmatrix} \times t_{UD}$$

The laminate thermal resultants are

$$[N^T] = [N^T]_{chop} \times t_{chop} + [N^T]_{UD} \times t_{UD} + [N^T]_{sand} \times t_{sand} + [N^T]_r \times t_r$$

$$[N^T] = [N^T]_{UD} \times t_{UD} + [N^T]_r \times t_r$$

Where the ply thermal matrices are in chapter 1.

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} 7.86 \\ 11.14 \\ 0 \end{bmatrix} \times t_{UD} \times (OT - PT) + \begin{bmatrix} 2.58 \\ 2.58 \\ 0 \end{bmatrix} \times t_r \times (OT - PT)$$

Again, ignoring the small contribution from the neat resin, the thermal resultants reduce to

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} 7.86 \\ 11.14 \\ 0 \end{bmatrix} \times t_{UD} \times (OT - PT)$$

The global equilibrium thermal strains are

$$\begin{bmatrix} 7.86 \\ 11.14 \\ 0 \end{bmatrix} \times (OT - PT) = \begin{bmatrix} 132470 & 96220 & 0 \\ 96220 & 235070 & 0 \\ 0 & 0 & 101220 \end{bmatrix} \times \begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ 0 \end{bmatrix} = \begin{bmatrix} 0.3545 \\ 0.3288 \\ 0 \end{bmatrix} \times 10^{-4} \times (OT - PT)$$

The equilibrium thermal strains do not depend on the laminate thickness or the pipe diameter. The embedded UD plies move together and take the above equilibrium strains as the laminate cools from PT to OT. The preservation of circularity ensures that all plies have the same global equilibrium strain. To obtain the strains in the local frame, we multiply the global strains by the rotation matrix [T].

$$[T] = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix}$$

Where $m = \cos 55 = 0.57$ and $n = \sin 55 = 0.82$

$$[T] = \begin{bmatrix} 0.33 & 0.67 & \pm 0.94 \\ 0.67 & 0.33 & \mp 0.94 \\ \mp 0.47 & \pm 0.47 & -0.34 \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_1^e \\ \epsilon_2^e \\ \frac{1}{2}\gamma_{12}^e \end{bmatrix} = \begin{bmatrix} 0.33 & 0.67 & \pm 0.94 \\ 0.67 & 0.33 & \mp 0.94 \\ \mp 0.47 & \pm 0.47 & -0.34 \end{bmatrix} \times \begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ 0 \end{bmatrix}$$

Finally, the thermal equilibrium strains in the local frame of the UD plies are

$$\begin{bmatrix} \epsilon_1^e \\ \epsilon_2^e \\ \gamma_{12}^e \end{bmatrix} = \begin{bmatrix} 0.3373 \\ 0.3460 \\ \pm 0.0241 \end{bmatrix} \times 10^{-4} \times (OT - PT)$$

The thermal equilibrium strains are negative, since OT is usually less than PT. The residual thermal strains are the differences between the laminate equilibrium strain and the free ply contractions.

$$\begin{bmatrix} \epsilon_1^r \\ \epsilon_2^r \\ \gamma_{12}^r \end{bmatrix} = \begin{bmatrix} \epsilon_1^e - \alpha_1(OT - PT) \\ \epsilon_2^e - \alpha_2(OT - PT) \\ \gamma_{12}^e \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_1^r \\ \epsilon_2^r \\ \gamma_{12}^r \end{bmatrix} = \begin{bmatrix} 0.3373 \times 10^{-4} (OT - PT) - 7.5 \times 10^{-6} (OT - PT) \\ 0.3460 \times 10^{-4} (OT - PT) - 45.0 \times 10^{-6} (OT - PT) \\ \pm 0.0241 \times 10^{-4} \times (OT - PT) \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_1^r \\ \epsilon_2^r \\ \gamma_{12}^r \end{bmatrix} = \begin{bmatrix} +0.2623 \\ -0.1040 \\ \pm 0.0241 \end{bmatrix} \times 10^{-4} \times (OT - PT)$$

This equation computes the residual thermal strains for all UD55 plies. In general $OT < PT$, which makes the residual thermal strains negative – compressive – in the fiber direction and positive – tensile – in the transverse direction. The compressive strains in the fiber direction increase the rupture durability, while the tensile strains in the transverse direction are detrimental to stiffness, infiltration and weep failures.

We call the reader's attention to an interesting situation. The above equation shows the transverse residual thermal strain ϵ_2 always tensile

$$\epsilon_2^r = 0.1040 \times 10^{-4} \times (PT - OT)$$

This tensile transverse strain is detrimental to weep. However, from the above equation, we see it decreasing with increases in the operating temperature OT. In other words, increments in the operating temperature OT decrease the transverse strain ϵ_2 , favoring the pipe's performance. This amazing conclusion contradicts the widely accepted wisdom that higher operating temperatures are detrimental to weep performance. We will have more to say about this in the part 2 of this book.

The residual thermal strains in UD55 plies depend exclusively on the temperature drop ($PT - OT$), regardless of the wall thickness and the cylinder diameter. This conclusion holds also for the residual hydric strains discussed in the next section.

3 – Residual hydric strains. The computation of the residual hydric strains in UD55 oil pipes follows the same protocol used in the thermal case. The computation is straightforward

and is, for the most part, as described in chapter 4. The residual hydric strains in the local frame of the UD55 plies are.

$$[N^H] = [N^H]_{chop} \times t_{chop} + [N^H]_{UD} \times t_{UD} + [N^H]_{sand} \times t_{sand} + [N^H]_r \times t_r$$

$$[N^H] = [N^H]_{UD} \times t_{UD} + [N^H]_r \times t_r$$

$$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = \begin{bmatrix} 28660 \\ 39340 \\ 0 \end{bmatrix} \times t_{UD} \times \Delta m + \begin{bmatrix} 17200 \\ 17200 \\ 0 \end{bmatrix} \times t_r \times \Delta m$$

Ignoring the resin ply, this equation simplifies to

$$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = \begin{bmatrix} 28660 \\ 39340 \\ 0 \end{bmatrix} \times t_{UD} \times \Delta m$$

The equilibrium strains in the global frame are

$$\begin{bmatrix} 28660 \\ 39340 \\ 0 \end{bmatrix} \times t_{UD} \times \Delta m = \begin{bmatrix} 132470 & 96220 & 0 \\ 96220 & 235070 & 0 \\ 0 & 0 & 101220 \end{bmatrix} \times t_{UD} \times \begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix} = \begin{bmatrix} 0.14 \\ 0.11 \\ 0 \end{bmatrix} \times \Delta m$$

Rotating the above into the local frame, we have

$$\begin{bmatrix} \epsilon_1^e \\ \epsilon_2^e \\ \frac{1}{2}\gamma_{12}^e \end{bmatrix} = \begin{bmatrix} 0.33 & 0.67 & \pm 0.94 \\ 0.67 & 0.33 & \mp 0.94 \\ \mp 0.47 & \pm 0.47 & -0.34 \end{bmatrix} \times \begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_1^e \\ \epsilon_2^e \\ \gamma_{12}^e \end{bmatrix} = \begin{bmatrix} 0.12 \\ 0.13 \\ \mp 0.03 \end{bmatrix} \times \Delta m$$

The residual hydric strains are

$$\begin{bmatrix} \epsilon_1^r \\ \epsilon_2^r \\ \gamma_{12}^r \end{bmatrix} = \begin{bmatrix} 0.12 - 0.04 \\ 0.13 - 0.18 \\ \mp 0.03 \end{bmatrix} \times \Delta m$$

Where the coefficients of hydric expansion for UD plies are $\beta_1 = 0.04$ and $\beta_2 = 0.18$.

$$\begin{bmatrix} \epsilon_1^r \\ \epsilon_2^r \\ \gamma_{12}^r \end{bmatrix} = \begin{bmatrix} 800 \\ -700 \\ \mp 300 \end{bmatrix} \times 10^{-4} \times \Delta m$$

The residual hydric strains are tensile in the fiber direction and compressive in the transverse direction. The transverse compression is desirable for weep performance, indicating that saturated pipes perform better than dry ones. This unexpected conclusion is a topic for chapter 9.

The reader may wonder why the residual thermal and hydric strains have opposite signs. The reason is simple and obvious. In the thermal case, the laminate shrinks from a high stress-free temperature T_T , whereas in the hydric case they expand from a stress-free dry condition.

4 – Mechanical strains. In pressurized non-anchored pipelines, the hoop force resultant is twice as large as the longitudinal force resultant. This loading condition, known in the industry as 2:1 loading, is present in pressure vessels and in non-anchored above ground pipelines. The mechanical force resultants on pressurized circular cylinders under 2:1 loadings are

$$\begin{bmatrix} N_x^M \\ N_y^M \\ N_{xy}^M \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.50 \\ 0 \end{bmatrix} \times (PD)$$

Where P is the internal pressure and D is the pipe diameter. The equilibrium mechanical strains for UD55 oil pipes are

$$\begin{bmatrix} 132470 & 96220 & 0 \\ 96220 & 235070 & 0 \\ 0 & 0 & 101220 \end{bmatrix} \times \begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0.50 \\ 0 \end{bmatrix} \times \left(\frac{PD}{t} \right)$$

$$\begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix} = \begin{bmatrix} 0.0049 \\ 0.0193 \\ 0 \end{bmatrix} \times 10^{-4} \times \left(\frac{PD}{t} \right)$$

The mechanical strains under 2:1 loadings are tensile and – unlike the residual thermal and hydric strains – depend on the pipe thickness and diameter. Following the protocol in chapter 4, we rotate the above equilibrium strains from the global to the local system.

$$\begin{bmatrix} \epsilon_1^e \\ \epsilon_2^e \\ \frac{1}{2}\gamma_{12}^e \end{bmatrix}_{\pm 55} = \begin{bmatrix} 0.33 & 0.67 & \pm 0.94 \\ 0.67 & 0.33 & \mp 0.94 \\ \mp 0.47 & \pm 0.47 & -0.34 \end{bmatrix} \times \begin{bmatrix} 0.0049 \\ 0.0193 \\ 0 \end{bmatrix} \times 10^{-4} \times \left(\frac{PD}{t} \right)$$

$$\begin{bmatrix} \epsilon_1^e \\ \epsilon_2^e \\ \gamma_{12}^e \end{bmatrix}_{\pm 55} = \begin{bmatrix} +0.015 \\ +0.010 \\ \pm 0.007 \end{bmatrix} \times 10^{-4} \times \left(\frac{PD}{t} \right)$$

These are the local mechanical strains on UD55 plies of oil pipes subjected to 2:1 pressure loadings. We proceed now to compute the total ply strains.

5 – Total strains: The total strains on the UD55 plies referred to their natural frames 1 – 2 are

$$\begin{bmatrix} Total \\ strain \end{bmatrix} = \begin{bmatrix} mechanical \\ strain \end{bmatrix} + \begin{bmatrix} residual \\ thermal strain \end{bmatrix} + \begin{bmatrix} residual \\ hydric strain \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 0.015 \\ 0.010 \\ \pm 0.007 \end{bmatrix} \times 10^{-4} \times \frac{PD}{t} + \begin{bmatrix} 0.262 \\ -0.104 \\ \pm 0.024 \end{bmatrix} \times 10^{-4} \times (OT - PT) + \begin{bmatrix} 800 \\ -700 \\ \mp 300 \end{bmatrix} \times 10^{-4} \times \Delta m$$

This equation computes the total strains on plies of standard UD55 oil pipes under 2:1 loading.

Example 1: Compute the mechanical stresses on UD55 oil pipes under a 2:1 loading.

The mechanical stresses come from the ply constitutive equation

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}_{\pm 55} = \begin{bmatrix} 400000 & 30000 & 0 \\ 30000 & 100000 & 0 \\ 0 & 0 & 35000 \end{bmatrix} \times \begin{bmatrix} 0.015 \\ 0.010 \\ \pm 0.007 \end{bmatrix} \times 10^{-4} \times \left(\frac{PD}{t} \right)$$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}_{\pm 55} = \begin{bmatrix} 0.630 \\ 0.145 \\ \pm 0.025 \end{bmatrix} \times \left(\frac{PD}{t} \right)$$

Assuming $P = 10$ bar, $D = 500$ mm and $t = 5.0$ mm, we have

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}_{\pm 55} = \begin{bmatrix} 0.630 \\ 0.145 \\ \pm 0.025 \end{bmatrix} \times \left(\frac{10 \times 500}{5} \right)$$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix}_{\pm 55} = \begin{bmatrix} 630 \\ 145 \\ \pm 25 \end{bmatrix} kg/cm^2$$

The protocols to compute the load-dependent durability of composites makes use of ply strains, not stresses. The above stresses are not required in the analysis of durability. The reader is asked to compare the relation $630/145 = 4.5$ between the fiber and transverse stresses with the same relation $0.015/0.010 = 1.5$ for strains. This enormous difference comes from the strong Poisson coupling.

Example 2: Compute the minimum thickness of a Derakane 411 oil pipe. The operating conditions are

- 2:1 loading

- The HDT of the vinyl ester resin matrix is $HDT = 105C$
- The peak post-cure temperature is $PCT = 100C$.
- The pipes carry water at room temperature $OT = RT = 25C$
- The operating pressure is $P = 10$ bar under a 2:1 loading.
- The pipe diameter is $D = 500$ mm
- The water pick-up of vinyl ester resins is $\Delta m = 1.0\% = 0.01$
- The allowable total strain in the direction transverse to the fibers is $\epsilon_2 = 0.28\%$.

The design criterion is weeping, and the pipe thickness is determined to keep the transverse strain less than $\epsilon_2 = 0.28\%$.

The total transverse strain in the UD55 plies is

$$0.010 \times 10^{-4} \times \frac{PD}{t} - 0.104 \times 10^{-4}(OT - PT) - 700 \times 10^{-4} \times \Delta m = \epsilon_2$$

Entering the problem data in the above, we have

$$0.010 \times 10^{-4} \times \frac{10 \times 500}{t} - 0.104 \times 10^{-4}(25 - 100) - 700 \times 10^{-4} \times 0.01 = 0.0028$$

$$\frac{0.005}{t} + 0.0008 - 0.0007 = 0.0028$$

Solving for the laminate thickness t , we have

$$t = 1.8 \text{ mm}$$

The computed thickness – 1.8 mm – meets the stated design criterion of 0.28% strain in the transverse ply direction. The question now is..... How long would this pipeline last before it weeps? The unified equation, discussed in chapter 16, gives the answer to this question.

Example 3: This numerical example compares the residual transverse strains on UD55 plies of standard oil pipes at different operating conditions. Assume 2:1 loading.

The total transverse strain comes from the general equation developed in section 5.5, repeated below to facilitate the presentation.

$$\epsilon_2 = 0.010 \times 10^{-4} \times \frac{PD}{t} + 0.104 \times 10^{-4}(PT - OT) - 700 \times 10^{-4} \times \Delta m$$

The hydric strain component varies from zero in dry air ($\Delta m = 0$) to a compressive – 0.07% in saturated conditions ($\Delta m = 1.0\%$). Assuming a typical allowable transverse strain of 0.28%, the water saturation alone improves the pipe rating by a whopping $0.07/0.28 = 25\%$. This is a surprising result. Water saturation actually improves the pressure rating. Most people would expect otherwise.

The thermal strain component decreases with increments in the operating temperature, meaning better pipe performance at higher OT. As an example, consider the pipes post-cured at $PCT = 100C$ and operating at two different temperatures, say $OT = 25C$ and $OT = 80C$. The stress-free temperature for pipes of vinyl ester resins would be $PT = 100C$. From the above equation, the residual thermal strains would be + 0.08% at 25C and + 0.02% at 80C, indicating a substantially improved performance at higher temperatures. This unexpected conclusion runs against the conventional wisdom, which degrades the pipe performance at higher operating temperatures.

In chapter 10, we present a detailed discussion of this subject. We strongly oppose the classical and false view of pipe degradation when water saturated or at high temperatures.

Example 4: Let us estimate the rupture durability of the pipe described in example 2. Assume loading 2:1.

The total strain in the fiber direction controls the long-term rupture failure.

$$\epsilon_1 = 0.015 \times 10^{-4} \left(\frac{PD}{t} \right) + 800 \times 10^{-4} \Delta m - 0.262 \times 10^{-4} (PT - OT)$$

$$\epsilon_1 = 0.015 \times 10^{-4} \left(\frac{10 \times 500}{2.1} \right) + 800 \times 10^{-4} \times 0.01 - 0.262 \times 10^{-4} (100 - 25)$$

$$\epsilon_1 = 0.0032 + 0.0008 - 0.0020$$

$$\epsilon_1 = 0.002$$

The compressive thermal strain (-0.20%) in the fiber direction improves the long-term rupture performance. The tensile hydric strain ($+0.08\%$) reduces the long-term performance. On balance, the inclusion of the residual strains reduces the mechanical strain in the fiber direction from 0.32% to 0.20% .

The last two numerical examples show the importance of the thermal and hydric residual strains, highlighting their role in the analysis of load-dependent durability. The question is.... How long will it take before the glass fibers fail, or the pipe weeps? As explained before, we discuss all questions involving durability in the part 2 of this book.

Example 5: It is required to compute the minimum thickness of the pipe described in example 2, assuming the operating temperature increased from $OT = 25C$ to $OT = 80C$. The loading is 2:1.

The mode of failure is weep, and the allowable transverse strain is $\epsilon_2 = 0.28\%$. The total strain in the transverse direction of the UD plies is as before

$$\epsilon_2 = 0.010 \times 10^{-4} \left(\frac{PD}{t} \right) - 700 \times 10^{-4} \Delta m + 0.104 \times 10^{-4} (PT - OT)$$

Entering the given operating data in the above, we have

$$\epsilon_2 = 0.010 \times 10^{-4} \left(\frac{10 \times 500}{t} \right) - 700 \times 10^{-4} \times 0.01 + 0.104 \times 10^{-4} (100 - 80)$$

$$0.0028 = \left(\frac{0.0050}{t} \right) - 0.0007 + 0.0002$$

From which we obtain the minimum pipe thickness.

$$t = 1.5 \text{ mm}$$

The required pipe thickness decreases from 1.8 mm at $OT = 25C$ to 1.5 mm at $OT = 80C$. The commercial pipe standards and specifications say otherwise and demand thicker pipes at higher operating temperatures.

There are two arguments supporting the idea that higher operating temperatures improve the weep performance of oil pipes. The first argument, developed in this chapter, is the drop in the transverse residual thermal strain with increments in the operating temperature. The second argument – detailed in chapter 9 – is the increased value of the weep threshold at higher temperatures. These arguments taken together form a strong platform suggesting improved weep performance of oil pipes operating at higher temperatures.

On the other hand, the strain in the fiber direction increases with the operating temperature. This, as discussed in the part 2 of the book, is detrimental to long-term fiber rupture. However, since long-term weep failure is far more important than long-term rupture, the argument in favor of improved performance of oil pipes at higher operating temperatures still stands.

We end this chapter with three numerical examples that provide useful results for use in the part 5 of chapter 16.

Example 6. Compute the total strains on UD55 oil pipes under 3:1 loading

The mechanical forces on the pipe are

$$\begin{bmatrix} N_x^M \\ N_y^M \\ N_{xy}^M \end{bmatrix} = \begin{bmatrix} 0.17 \\ 0.50 \\ 0 \end{bmatrix} \times PD$$

Where P is the internal pressure and D is the pipe diameter. The equilibrium mechanical strains are

$$\begin{bmatrix} 132470 & 96220 & 0 \\ 96220 & 235070 & 0 \\ 0 & 0 & 101220 \end{bmatrix} \times \begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix} = \begin{bmatrix} 0.17 \\ 0.50 \\ 0 \end{bmatrix} \times \left(\frac{PD}{t} \right)$$

$$\begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix} = \begin{bmatrix} -0.0037 \\ 0.0228 \\ 0 \end{bmatrix} \times 10^{-4} \times \left(\frac{PD}{t} \right)$$

The axial mechanical strain for 3:1 loading is negative, because of the Poisson effect. The equilibrium strains in the local system are

$$\begin{bmatrix} \epsilon_1^M \\ \epsilon_2^M \\ 1/2 \gamma_{12}^M \end{bmatrix} = \begin{bmatrix} 0.33 & 0.67 & \pm 0.94 \\ 0.67 & 0.33 & \mp 0.94 \\ \mp 0.47 & \pm 0.47 & -0.34 \end{bmatrix} \times \begin{bmatrix} -0.0037 \\ 0.0228 \\ 0 \end{bmatrix} \times 10^{-4} \times \left(\frac{PD}{t} \right)$$

$$\begin{bmatrix} \epsilon_1^M \\ \epsilon_2^M \\ \gamma_{12}^M \end{bmatrix} = \begin{bmatrix} 0.0141 \\ 0.0050 \\ \pm 0.0250 \end{bmatrix} \times 10^{-4} \times \left(\frac{PD}{t} \right)$$

These are the local mechanical strains on UD55 oil pipes under 3:1 pressure loading. The reader will notice the shear strain 5 times higher than the transverse tensile strain.

The total strains referred to the ply frame 1 – 2 are

$$\begin{bmatrix} Total \\ strain \end{bmatrix} = \begin{bmatrix} mechanical \\ strain \end{bmatrix} + \begin{bmatrix} residual \\ thermal strain \end{bmatrix} + \begin{bmatrix} residual \\ hydric strain \end{bmatrix}$$

The residual strains are the same as those computed before

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 0.0141 \\ 0.0050 \\ \pm 0.0250 \end{bmatrix} \times 10^{-4} \times \left(\frac{PD}{t} \right) + \begin{bmatrix} 0.262 \\ -0.104 \\ \pm 0.024 \end{bmatrix} \times 10^{-4} \times (OT - PT) + \begin{bmatrix} 800 \\ -700 \\ \mp 300 \end{bmatrix} \times 10^{-4} \times \Delta m$$

These are the total strains on UD55 pipes under 3:1 loading. We will make use of this equation in the part 5 of chapter 16.

Example 7. Compute the total strains on standard UD55 oil pipes under 1:0 loading

The mechanical forces on the pipe are

$$\begin{bmatrix} N_x^M \\ N_y^M \\ N_{xy}^M \end{bmatrix} = \begin{bmatrix} 0 \\ 0.50 \\ 0 \end{bmatrix} \times PD$$

The equilibrium mechanical strains are

$$\begin{bmatrix} 132470 & 96220 & 0 \\ 96220 & 235070 & 0 \\ 0 & 0 & 101220 \end{bmatrix} \times \begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix} = \begin{bmatrix} 0 \\ 0.50 \\ 0 \end{bmatrix} \times \left(\frac{PD}{t} \right)$$

$$\begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix} = \begin{bmatrix} -0.0220 \\ 0.0303 \\ 0 \end{bmatrix} \times 10^{-4} \times \left(\frac{PD}{t} \right)$$

The equilibrium strains in the local system are

$$\begin{bmatrix} \epsilon_1^M \\ \epsilon_2^M \\ 1/2 \gamma_{12}^M \end{bmatrix} = \begin{bmatrix} 0.33 & 0.67 & \pm 0.94 \\ 0.67 & 0.33 & \mp 0.94 \\ \mp 0.47 & \pm 0.47 & -0.34 \end{bmatrix} \times \begin{bmatrix} -0.0220 \\ 0.0303 \\ 0 \end{bmatrix} \times 10^{-4} \times \left(\frac{PD}{t} \right)$$

$$\begin{bmatrix} \epsilon_1^M \\ \epsilon_2^M \\ \gamma_{12}^M \end{bmatrix} = \begin{bmatrix} 0.0130 \\ -0.0047 \\ \pm 0.0492 \end{bmatrix} \times 10^{-4} \times \left(\frac{PD}{t} \right)$$

These are the local mechanical strains on UD55 oil pipes under 1:0 pressure loading. The reader will notice the high shear strain compared to the negative (compressive) transverse strain. This indicates weep failure from shear strain.

The total strains referred to the natural frames 1 – 2 are

$$\begin{bmatrix} \text{Total} \\ \text{strain} \end{bmatrix} = \begin{bmatrix} \text{mechanical} \\ \text{strain} \end{bmatrix} + \begin{bmatrix} \text{residual} \\ \text{thermal strain} \end{bmatrix} + \begin{bmatrix} \text{residual} \\ \text{hydric strain} \end{bmatrix}$$

The residual strains are the same as computed before

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 0.0130 \\ -0.0047 \\ \pm 0.0492 \end{bmatrix} \times 10^{-4} \times \left(\frac{PD}{t}\right) + \begin{bmatrix} 0.262 \\ -0.104 \\ \pm 0.024 \end{bmatrix} \times 10^{-4} \times (OT - PT) + \begin{bmatrix} 800 \\ -700 \\ \mp 300 \end{bmatrix} \times 10^{-4} \times \Delta m$$

These are the total strains on UD55 pipes under 1:0 loading. We will make use of this equation in the part 5 of chapter 16.

Example 8. Compute the total strains on standard UD55 oil pipes under 0:1 loading

The mechanical forces on the pipe are

$$\begin{bmatrix} N_x^M \\ N_y^M \\ N_{xy}^M \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0 \\ 0 \end{bmatrix} \times PD$$

The equilibrium mechanical strains are

$$\begin{bmatrix} 132470 & 96220 & 0 \\ 96220 & 235070 & 0 \\ 0 & 0 & 101220 \end{bmatrix} \times \begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix} = \begin{bmatrix} 0.25 \\ 0 \\ 0 \end{bmatrix} \times \left(\frac{PD}{t}\right)$$

$$\begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix} = \begin{bmatrix} 0.0269 \\ -0.0110 \\ 0 \end{bmatrix} \times 10^{-4} \times \left(\frac{PD}{t}\right)$$

The hoop mechanical strain for 0:1 loading is negative, due to the Poisson effect. The equilibrium strains rotated to the local system are.

$$\begin{bmatrix} \epsilon_1^M \\ \epsilon_2^M \\ 1/2 \gamma_{12}^M \end{bmatrix} = \begin{bmatrix} 0.33 & 0.67 & \pm 0.94 \\ 0.67 & 0.33 & \mp 0.94 \\ \mp 0.47 & \pm 0.47 & -0.34 \end{bmatrix} \times \begin{bmatrix} 0.0269 \\ -0.0110 \\ 0 \end{bmatrix} \times 10^{-4} \times \left(\frac{PD}{t}\right)$$

$$\begin{bmatrix} \epsilon_1^M \\ \epsilon_2^M \\ \gamma_{12}^M \end{bmatrix} = \begin{bmatrix} 0.0015 \\ 0.0144 \\ \mp 0.0356 \end{bmatrix} \times 10^{-4} \times \left(\frac{PD}{t}\right)$$

These are the local mechanical strains on UD55 oil pipes under 0:1 loading. The reader will notice the high shear strain compared to the transverse tensile strain.

The total strains referred to the natural frames 1 – 2 are

$$\begin{bmatrix} \text{Total} \\ \text{strain} \end{bmatrix} = \begin{bmatrix} \text{mechanical} \\ \text{strain} \end{bmatrix} + \begin{bmatrix} \text{residual} \\ \text{thermal strain} \end{bmatrix} + \begin{bmatrix} \text{residual} \\ \text{hydric strain} \end{bmatrix}$$

The residual strains are the same as computed before

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 0.0015 \\ 0.0144 \\ \mp 0.0356 \end{bmatrix} \times 10^{-4} \times \left(\frac{PD}{t}\right) + \begin{bmatrix} 0.262 \\ -0.104 \\ \pm 0.024 \end{bmatrix} \times 10^{-4} \times (OT - PT) + \begin{bmatrix} 800 \\ -700 \\ \mp 300 \end{bmatrix} \times 10^{-4} \times \Delta m$$

These are the total strains on UD55 pipes under 0:1 loading. We will make use of this equation in the part 5 of chapter 16.

Chapter 6

Total strains in UD70 laminates

1 – Introduction: The corrosion barriers of UD70 laminates are not included in the structural design. However, they retain their role as critical plies governing the laminate design in the transportation or storage of liquids.

- *In mild service, where chemical attack is not a problem, the allowable strain derives from the pristine weep threshold of the corrosion barrier.*
- *In aggressive service, where chemical attack is a problem, the allowable strain derives from the aged infiltration threshold of the corrosion barrier.*

As we see, even if excluded as a structural contributor, the corrosion barrier still determines the infiltration threshold to compute the equipment durability

The following computations combine the protocol developed in chapter 4 with the numerical matrices derived in chapter 3. The computation of the total strains in UD70 laminates follows essentially the same protocol developed in the previous chapter for UD55 oil pipes. Our discussions assume circular cylindrical laminates operating in the following conditions:

- *The corrosion barrier is ignored as a structural contributor*
- *The internal pressure is P*
- *The structural laminate thickness is t_{UD}*
- *The cylinder diameter is D .*
- *The operating temperature is OT .*
- *The water absorbed by resin matrices is $\Delta m = 1.0\%$.*

2 – Residual thermal strains: The thermal resultants $[N^T]$ and the stiffness matrix $[A]$ of standard UD70 laminates are

$$[A]_{\pm 70} = \begin{bmatrix} 104000 & 61000 & 0 \\ 61000 & 334000 & 0 \\ 0 & 0 & 66000 \end{bmatrix} \times (t_{UD})$$

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} 5.22 \\ 8.50 \\ 0 \end{bmatrix} \times t_{UD} \times (OT - PT)$$

From chapter 4, the equilibrium thermal strains are

$$\begin{bmatrix} N_x^T \\ N_y^T \\ 0 \end{bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} & 0 \\ A_{yx} & A_{yy} & 0 \\ 0 & 0 & A_{ss} \end{bmatrix} \times \begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix}$$

$$\begin{bmatrix} 5.22 \\ 8.50 \\ 0 \end{bmatrix} \times (OT - PT) = \begin{bmatrix} 104000 & 61000 & 0 \\ 61000 & 334000 & 0 \\ 0 & 0 & 66000 \end{bmatrix} \times \begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix}$$

The global equilibrium thermal strains of UD70 laminates are

$$\begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix} = \begin{bmatrix} 0.395 \\ 0.182 \\ 0 \end{bmatrix} \times 10^{-4} (OT - PT)$$

As explained earlier, the embedded plies shrink as a laminate, taking the same equilibrium position. The preservation of circularity ensures the same global equilibrium strain for all plies. Now, at this point we depart from the analysis of UD55 oil pipes discussed in the previous chapter.

This is a necessary departure, since the critical plies in UD70 laminates are the isotropic plies of chopped fibers in the corrosion barrier. Therefore, the residual thermal strains of the ignored corrosion barrier controls the laminate design. To compute the residual thermal strains of the corrosion barrier we subtract the free ply contractions from the equilibrium strains, all taken in the global system.

$$\begin{bmatrix} \epsilon_1^r \\ \epsilon_2^r \\ \gamma_{12}^r \end{bmatrix}_{chop} = \begin{bmatrix} 0.395 \times 10^{-4} (OT - PT) - 25 \times 10^{-6} (OT - PT) \\ 0.182 \times 10^{-4} (OT - PT) - 25 \times 10^{-6} (OT - PT) \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_x^r \\ \epsilon_y^r \\ \gamma_{xy}^r \end{bmatrix}_{chop} = \begin{bmatrix} \epsilon_1^r \\ \epsilon_2^r \\ \gamma_{12}^r \end{bmatrix}_{chop} = \begin{bmatrix} -0.145 \\ 0.068 \\ 0 \end{bmatrix} \times 10^{-4} \times (PT - OT)$$

The residual thermal strains of the chopped plies depend only on the temperature drop (PT – OT), regardless of the cylinder thickness and diameter. This conclusion holds for the residual hydric strains as well.

3 – Residual hydric strains: The computation of the residual hydric strains follows the same protocol. The computation is straightforward, as shown in chapter 4. Following the protocol, we have

$$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = \begin{bmatrix} 20640 \\ 32350 \\ 0 \end{bmatrix} \times \Delta m$$

From chapter 4, the equilibrium hydric strains in the global system are

$$\begin{bmatrix} 20640 \\ 32350 \\ 0 \end{bmatrix} \times \Delta m = \begin{bmatrix} 104000 & 61000 & 0 \\ 61000 & 334000 & 0 \\ 0 & 0 & 66000 \end{bmatrix} \times \begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix} = \begin{bmatrix} 0.159 \\ 0.068 \\ 0 \end{bmatrix} \times \Delta m$$

The residual hydric strains of the corrosion barrier are

$$\begin{bmatrix} \epsilon_x^r \\ \epsilon_y^r \\ \gamma_{xy}^r \end{bmatrix}_{chop} = \begin{bmatrix} \epsilon_1^r \\ \epsilon_2^r \\ \gamma_{12}^r \end{bmatrix}_{chop} = \begin{bmatrix} 0.159 - 0.19 \\ 0.068 - 0.19 \\ 0 \end{bmatrix} \times \Delta m = \begin{bmatrix} -0.031 \\ -0.122 \\ 0 \end{bmatrix} \times \Delta m$$

$$\begin{bmatrix} \epsilon_x^r \\ \epsilon_y^r \\ \gamma_{xy}^r \end{bmatrix}_{chop} = \begin{bmatrix} \epsilon_1^r \\ \epsilon_2^r \\ \gamma_{12}^r \end{bmatrix}_{chop} = \begin{bmatrix} -310 \\ -1220 \\ 0 \end{bmatrix} \times 10^{-4} \times \Delta m$$

The residual hydric strains of the corrosion barrier are both compressive. This is highly desirable, since compressive strains favor the infiltration and weep performance. The full importance of this statement will become apparent in the part 2 of this book.

4 – Mechanical strains on vertical storage tanks: The hoop tensile force in vertical storage tanks is $N_y = PD/2$, where P is the hydrostatic pressure and D is the tank diameter. The longitudinal force in atmospheric vertical tanks is $N_x = 0$. The mechanical force resultants on the cylindrical wall of vertical storage tanks are

$$\begin{bmatrix} N_x^M \\ N_y^M \\ N_{xy}^M \end{bmatrix}_{\pm 70} = \begin{bmatrix} 0 \\ 0.50 \\ 0 \end{bmatrix} \times (PD)$$

To compute the equilibrium mechanical strains, we use the force resultants and the laminate matrix [A], as shown in the protocol developed in chapter 4. For standard UD70 angle-ply laminates, we have

$$\begin{bmatrix} 104000 & 61000 & 0 \\ 61000 & 334000 & 0 \\ 0 & 0 & 66000 \end{bmatrix} \times \begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix} = \begin{bmatrix} 0 \\ 0.50 \\ 0 \end{bmatrix} \times \left(\frac{PD}{t_{UD}} \right)$$

$$\begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix} = \begin{bmatrix} -0.0098 \\ 0.0168 \\ 0 \end{bmatrix} \times 10^{-4} \times \left(\frac{PD}{t_{UD}} \right)$$

Since the corrosion barrier is isotropic and requires no strain rotation, these are the principal mechanical strains on the corrosion barrier.

5 – Total strains in vertical UD70 storage tanks. The total strains on the corrosion barrier of UD70 storage tanks are.

$$[Total\ strain] = [mechanical\ strain] + [thermal\ strain] + [residual\ hydric\ strain]$$

$$\begin{bmatrix} \epsilon_x^{total} \\ \epsilon_y^{total} \\ \gamma_{xy}^{total} \end{bmatrix} = \begin{bmatrix} -0.0098 \\ 0.0168 \\ 0 \end{bmatrix} \times 10^{-4} \times \left(\frac{PD}{t_{UD}} \right) + \begin{bmatrix} -310 \\ -1220 \\ 0 \end{bmatrix} \times 10^{-4} \times \Delta m + \begin{bmatrix} -0.145 \\ 0.068 \\ 0 \end{bmatrix} \times 10^{-4} \times (PT - OT)$$

This equation computes the total strains in the corrosion barrier of UD70 cylindrical vertical tanks under hydrostatic loading. The total hoop strain ϵ_y controls the tank design.

$$\epsilon_y = \left(0.017 \times \frac{PD}{t_{UD}} + 0.068 \times (PT - OT) - 1220 \times \Delta m \right) \times 10^{-4}$$

The above equation indicates that higher operating temperatures OT reduce the hoop strain on the corrosion barrier and favor the tank's performance. We discuss this situation in numerical examples.

Example 1: Compute the minimum structural thickness of a standard UD70 vertical cylindrical storage tank under hydrostatic loads. The following information applies.

- The HDT of the vinyl ester resin is $HDT = 105C$
- The peak post-cure temperature is $PCT = 100C$.
- The tank stores 20% solution of HCl at room temperature $OT = RT = 25C$
- The operating hydrostatic pressure is $P = 1.0$ bar.
- The tank diameter is $D = 5000$ mm
- The water pickup by resin matrices is $\Delta m = 1.0\%$
- The allowable total strain in the corrosion barrier $\epsilon_y = T_i/2 = 0.50/2 = 0.25\%$

The corrosion barrier of chopped fibers is the critical ply. The failure criterion in such cases is infiltration. See chapter 8. From the equation in section 5, the maximum total strain on the corrosion barrier is in the hoop direction y . The design criterion is infiltration, and the wall thickness is determined to keep the strain in the hoop direction of the corrosion barrier less than 0.25%.

$$\epsilon_y = 0.017 \times 10^{-4} \times \frac{PD}{t_{UD}} + 0.068 \times 10^{-4} \times (PT - OT) - 1220 \times 10^{-4} \times \Delta m$$

Entering the problem data in the above

$$0.017 \times 10^{-4} \times \frac{1.0 \times 5000}{t_{UD}} + 0.068 \times 10^{-4} \times (100 - 25) - 1220 \times 10^{-4} \times 0.01 = 0.0025$$

$$\frac{0.009}{t_{UD}} + 0.0005 - 0.0012 = 0.0025$$

Solving for the laminate thickness t_{UD} , we have

$$t_{UD} = 2.8 \text{ mm}$$

The minimum required thickness to meet the stated design criterion is $t_{UD} = 2.6$ mm. The total laminate thickness includes the corrosion barrier, the liner and the topcoat

$$\text{total thickness} = 2.8 + 2.1 + 0.3 + 0.3 = 5.50 \text{ mm}$$

The question now is..... How thick should the corrosion barrier be for a desired service life of, say, 25 years? The answer to this question is in chapter 10.

Example 2: This numerical example compares the residual strains on the corrosion barrier of standard UD70 vertical storage tanks at different operating conditions.

To facilitate the presentation, we repeat the equation to compute the total hoop strain in the corrosion barrier.

$$\epsilon_y = 0.017 \times 10^{-4} \times \frac{PD}{t_{UD}} + 0.068 \times 10^{-4} \times (PT - OT) - 1220 \times 10^{-4} \times \Delta m$$

The residual hoop hydric strain component varies from zero in air ($\Delta m = 0$) to a compressive -0.12% in saturated conditions ($\Delta m = 1.0\%$). Assuming a typical allowable infiltration strain of 0.25% , the water saturation improves the tank rating by a whopping $0.12/0.25 = 48\%$. This is a surprising result.

The residual hoop thermal strain component decreases with increments in the operating temperature, meaning better infiltration performance at higher OT. This result is also unexpected.

The preceding analysis indicates improved chemical performance of tanks operating at higher temperature and saturated in water.

Example 3. Consider the vertical storage tank discussed in the numerical example 1. What would the required structural thickness be if the operating temperature increased from $OT = 25C$ to $OT = 80C$? The mode of failure is infiltration and the allowable hoop strain in the corrosion barrier is 0.25% .

The total strain on the corrosion barrier in the hoop direction is

$$\epsilon_y = 0.017 \times 10^{-4} \times \frac{PD}{t_{UD}} + 0.068 \times 10^{-4} \times (PT - OT) - 1220 \times 10^{-4} \times \Delta m$$

Entering the given operating data in the above, we have

$$0.0025 = 0.017 \times 10^{-4} \times \frac{1 \times 5000}{t_{UD}} + 0.068 \times 10^{-4} \times (100 - 80) - 1220 \times 10^{-4} \times 0.01$$

From which we have

$$t_{UD} = 2.4 \text{ mm}$$

The required structural wall thickness decreases from 2.8 mm at $OT = 25C$ to 2.4 mm at $OT = 80C$.

The residual thermal hoop strain on the corrosion barrier decreases with increments in the operating temperature. The residual hydric hoop strain in the corrosion barrier is always compressive. This indicates improved infiltration performance of tanks operating saturated in water and at higher temperatures. This discussion leaves no doubt about the improved infiltration performance of vertical storage tanks operating at higher temperatures. However, it says not a word about the actual service life of the equipment. For information on the chemical life of the equipment, see chapter 10.

Example 4 – Determine the minimum operating temperatures for UD70 vertical storage tanks.

The total hoop strain on the corrosion barrier controls the tank design. The mode of failure is infiltration, and the allowable strain is 0.25% . The total hoop strain on the corrosion barrier should never exceed 0.25% .

From the equation for the total strains, we have

$$\epsilon_y = \left(0.017 \frac{PD}{t_{UD}} + 0.068(PT - OT) - 1220\Delta m \right) \times 10^{-4}$$

The minimum operating temperature OT is determined so that

$$\left(0.017 \frac{PD}{t_{UD}} + 0.068(PT - OT) - 1220\Delta m \right) \times 10^{-4} = 0.0025$$

Assuming the stress-free temperature $PT = 100C$ and $\Delta m = 0.01$, we have

$$OT = 0.25 \times \frac{PD}{t_{UD}} - 447C$$

The minimum operating temperature for empty tanks, $P = 0$, is

$OT = -447C$, meaning the tank can operate at 0 degrees Kelvin.

Chapter 7

Total strains in hoop-chop laminates

1 – Introduction: In this chapter, we compute the total strains of plies embedded in hoop-chop laminates. The computation combines the protocol developed in chapter 4 with the numerical matrices of chapter 3. Hoop-chop laminates combine plies of sand-filled resin, UD90, chopped fibers and neat resin. The critical ply is that of chopped fibers, that works as corrosion barriers in chemical service and weep barriers in sanitation pipes.

The assumed design conditions are:

- *The corrosion barrier of polyester resin is fully preserved*
- *The operating temperature is $OT = 25C$.*
- *The water pickup by polyester matrices is $\Delta m = 1.0\%$.*
- *The allowable weep strain is $\varepsilon_y = Tw/2 = 0.80/2 = 0.40\%$.*

The structural contribution from four distinct plies – UD90, sand core, chopped fibers and neat resin – complicates the design equations, which become too lengthy and unwieldy. To facilitate the mathematical manipulation, we develop the numerical examples in this chapter using predefined laminates.

2 – Thermal residual strains in hoop-chop laminates: The thermal resultants $[N^T]$ and the stiffness matrix $[A]$ of hoop-chop laminates are

$$\begin{aligned} [A] = & \begin{bmatrix} 100000 & 30000 & 0 \\ 30000 & 400000 & 0 \\ 0 & 0 & 350000 \end{bmatrix} \times t_{UD} + \begin{bmatrix} 77000 & 23000 & 0 \\ 23000 & 77000 & 0 \\ 0 & 0 & 27000 \end{bmatrix} \times t_{chop} \\ & + \begin{bmatrix} 66000 & 20000 & 0 \\ 20000 & 66000 & 0 \\ 0 & 0 & 23000 \end{bmatrix} \times t_{sand} + \begin{bmatrix} 33000 & 10000 & 0 \\ 10000 & 33000 & 0 \\ 0 & 0 & 11500 \end{bmatrix} \times t_r \\ \begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = & \begin{bmatrix} 4.73 \times t_{UD} + 2.50 \times t_{chop} + 1.12 \times t_{sand} + 2.58 \times t_r \\ 4.35 \times t_{UD} + 2.50 \times t_{chop} + 1.12 \times t_{sand} + 2.58 \times t_r \\ 0 \end{bmatrix} \times (OT - PT) \end{aligned}$$

The equilibrium thermal strains derive from the general equation

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = [A] \times \begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix}$$

The laminate thickness do not cancel out in this case, making the computations too lengthy. To avoid complex algebraic manipulations, we will do all computations using arbitrarily chosen tentative laminate constructions. To define an acceptable laminate, the computed total strains are set equal to the allowable values, and the entire procedure repeated until we reach a satisfactory solution. Usually three trials suffice to arrive at an acceptable laminate construction. .

3 – Hydric residual strains in hoop-chop laminates: The hydric resultant for hoop-chop laminates is

$$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = \begin{bmatrix} 19200 \times t_{UD} + 19000 \times t_{chop} + 12800 \times t_{sand} + 17200 \times t_r \\ 21400 \times t_{UD} + 19000 \times t_{chop} + 12800 \times t_{sand} + 17200 \times t_r \\ 0 \end{bmatrix} \times \Delta m$$

The equilibrium hydric strains are

$$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = [A] \times \begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix}$$

4 – Mechanical strains in hoop-chop sanitation pipes: Hoop-chop sanitation pipes usually work underground with the soil friction absorbing the longitudinal movement. The following conditions apply:

P is the internal pressure

D is the pipe diameter

$N_y = PD/2$ is the hoop force resultant

N_x is the axial force soil friction

$\epsilon_x = 0$ is axial strain

These are typical operating conditions for underground pipes. The computation of the equilibrium mechanical strains follows the classical protocol, using the mechanical force resultants and the laminate stiffness matrix [A].

For underground pipes, we have

$$[A] \times \begin{bmatrix} 0 \\ \epsilon_y^e \\ 0 \end{bmatrix} = \begin{bmatrix} N_x \\ \frac{P \times D}{2} \\ 0 \end{bmatrix}$$

$$\left(\begin{bmatrix} 100 & 30 & 0 \\ 30 & 400 & 0 \\ 0 & 0 & 35 \end{bmatrix} \times t_{UD} + \begin{bmatrix} 77 & 23 & 0 \\ 23 & 77 & 0 \\ 0 & 0 & 27 \end{bmatrix} \times t_{chop} + \begin{bmatrix} 66 & 20 & 0 \\ 20 & 66 & 0 \\ 0 & 0 & 23 \end{bmatrix} \times t_{sand} + \begin{bmatrix} 33 & 10 & 0 \\ 10 & 33 & 0 \\ 0 & 0 & 11.5 \end{bmatrix} \times t_r \right) \times 10^3$$

$$\times \begin{bmatrix} 0 \\ \epsilon_y^e \\ 0 \end{bmatrix} = \begin{bmatrix} N_x \\ PD/2 \\ 0 \end{bmatrix}$$

To compute the mechanical strains, the designer assigns arbitrary thicknesses to the four plies.

5 – Total strains: The total strains are obtained by adding the mechanical, thermal and hydric components

$$\begin{bmatrix} \text{Total} \\ \text{strain} \end{bmatrix} = \begin{bmatrix} \text{mechanical} \\ \text{strain} \end{bmatrix} + \begin{bmatrix} \text{residual} \\ \text{thermal strain} \end{bmatrix} + \begin{bmatrix} \text{residual} \\ \text{hydric strain} \end{bmatrix}$$

Example 1: Suggest an acceptable laminate construction for a hoop-chop underground pipe operating in the following conditions.

- The HDT of the polyester resin matrix is $HDT = 100C$
- The peak post-cure temperature is $PCT = PT = 100C$.
- The pipe carries water at room temperature $OT = RT = 25C$
- The operating pressure is $P = 10 \text{ bar}$.
- The pipe diameter is $D = 500 \text{ mm}$
- The water pick up is $\Delta m = 1.0\%$
- The weep threshold is $Tw = 0.80\%$ and the allowable strain is $0.80/2 = 0.40\%$.

The design criterion is weeping. The critical ply is the weep barrier of chopped fibers, not the UD plies of oil pipes discussed in chapter 5. The laminate construction is determined by trial and error until a solution is found that keeps the hoop strain in the weep barrier less than $0.80/2 = 0.40\%$.

Let us compute the total strains by tentatively setting $t_{chop} = 4.0 \text{ mm}$, $t_{UD} = 3.0 \text{ mm}$, $t_r = 0.6 \text{ mm}$ and $t_{sand} = 5.0 \text{ mm}$.

Entering the above ply thicknesses in the general equations, we have

$$\begin{aligned} [A] = & \begin{bmatrix} 100000 & 30000 & 0 \\ 30000 & 400000 & 0 \\ 0 & 0 & 35000 \end{bmatrix} \times 0.3 + \begin{bmatrix} 77000 & 23000 & 0 \\ 23000 & 77000 & 0 \\ 0 & 0 & 27000 \end{bmatrix} \times 0.4 \\ & + \begin{bmatrix} 66000 & 20000 & 0 \\ 20000 & 66000 & 0 \\ 0 & 0 & 23000 \end{bmatrix} \times 0.5 + \begin{bmatrix} 33000 & 10000 & 0 \\ 10000 & 33000 & 0 \\ 0 & 0 & 11500 \end{bmatrix} \times 0.06 \end{aligned}$$

$$[A] = \begin{bmatrix} 95780 & 28800 & 0 \\ 28800 & 185780 & 0 \\ 0 & 0 & 33490 \end{bmatrix} \text{ kg/cm}$$

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} 4.73 \times 0.3 + 2.50 \times 0.4 + 1.12 \times 0.5 + 2.58 \times 0.06 \\ 4.35 \times 0.3 + 2.50 \times 0.4 + 1.12 \times 0.5 + 2.58 \times 0.06 \\ 0 \end{bmatrix} \times (25 - 100)$$

$$\begin{bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{bmatrix} = \begin{bmatrix} -229.5 \\ -226.5 \\ 0 \end{bmatrix} \text{ kg/cm}$$

The equilibrium thermal strains are

$$\begin{bmatrix} N_x^T \\ N_y^T \\ 0 \end{bmatrix} = \begin{bmatrix} A_{xx} & A_{xy} & 0 \\ A_{yx} & A_{yy} & 0 \\ 0 & 0 & A_{ss} \end{bmatrix} \times \begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix}$$

$$\begin{bmatrix} -229.5 \\ -226.5 \\ 0 \end{bmatrix} = \begin{bmatrix} 95780 & 28800 & 0 \\ 28800 & 185780 & 0 \\ 0 & 0 & 33490 \end{bmatrix} \times \begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix} = \begin{bmatrix} -0.00215 \\ -0.00081 \\ 0 \end{bmatrix}$$

The above are the laminate equilibrium thermal strains in the global frame. For hoop-chop laminates, these are also the local strains of all plies. Strain rotations to the local frames are, therefore, not required in this case. The residual thermal strains on all plies in their local frames are

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix}_{UD}^{thermal} = \begin{bmatrix} -0.00215 - 45.0 \times 10^{-6}(25 - 100) \\ -0.00081 - 7.5 \times 10^{-6}(25 - 100) \\ 0 \end{bmatrix} = \begin{bmatrix} 0.00123 \\ -0.00025 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix}_{chop}^{thermal} = \begin{bmatrix} -0.00215 - 25.0 \times 10^{-6}(25 - 100) \\ -0.00081 - 25.0 \times 10^{-6}(25 - 100) \\ 0 \end{bmatrix} = \begin{bmatrix} -0.00028 \\ 0.00107 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix}_{sand}^{thermal} = \begin{bmatrix} -0.00215 - 13.0 \times 10^{-6}(25 - 100) \\ -0.00081 - 13.0 \times 10^{-6}(25 - 100) \\ 0 \end{bmatrix} = \begin{bmatrix} -0.00118 \\ 0.000165 \\ 0 \end{bmatrix}$$

To compute the residual hydric strains in the global frame, we substitute the thermal resultants for their hydric counterparts.

$$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = \begin{bmatrix} 19200 \times t_{UD} + 19000 \times t_{chop} + 12800 \times t_{sand} + 17200 \times t_r \\ 21400 \times t_{UD} + 19000 \times t_{chop} + 12800 \times t_{sand} + 17200 \times t_r \\ 0 \end{bmatrix} \times \Delta m$$

$$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = \begin{bmatrix} 19200 \times 0.3 + 19000 \times 0.4 + 12800 \times 0.5 + 17200 \times 0.06 \\ 21400 \times 0.3 + 19000 \times 0.4 + 12800 \times 0.5 + 17200 \times 0.06 \\ 0 \end{bmatrix} \times 0.01$$

$$\begin{bmatrix} N_x^H \\ N_y^H \\ N_{xy}^H \end{bmatrix} = \begin{bmatrix} 208.0 \\ 214.5 \\ 0 \end{bmatrix}$$

The hydric equilibrium strains are

$$\begin{bmatrix} 208.0 \\ 214.5 \\ 0 \end{bmatrix} = \begin{bmatrix} 95780 & 28800 & 0 \\ 28800 & 185780 & 0 \\ 0 & 0 & 33490 \end{bmatrix} \times \begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix} = \begin{bmatrix} 0.0019 \\ 0.0009 \\ 0 \end{bmatrix}$$

The residual hydric strains are

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix}_{UD}^{hydraulic} = \begin{bmatrix} 0.0019 - 0.18 \times 0.01 \\ 0.0009 - 0.04 \times 0.01 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.0001 \\ 0.0005 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix}_{chop}^{hydraulic} = \begin{bmatrix} 0.0019 - 0.19 \times 0.01 \\ 0.0009 - 0.19 \times 0.01 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.0000 \\ -0.0010 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix}_{sand}^{hydraulic} = \begin{bmatrix} 0.0019 - 0.15 \times 0.01 \\ 0.0009 - 0.15 \times 0.01 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.0004 \\ -0.0006 \\ 0 \end{bmatrix}$$

The mechanical strains in the global system are

$$[A] \times \begin{bmatrix} 0 \\ \epsilon_y^e \\ 0 \end{bmatrix} = \begin{bmatrix} N_x \\ P \times D \\ 2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 95780 & 28800 & 0 \\ 28800 & 185780 & 0 \\ 0 & 0 & 33490 \end{bmatrix} \times \begin{bmatrix} 0 \\ \epsilon_y^e \\ 0 \end{bmatrix} = \begin{bmatrix} N_x \\ (10 \times 50)/2 \\ 0 \end{bmatrix}$$

From which we have $\epsilon_y = 0.0013$ and $N_x = 38.7 \text{ kg/cm}$. The mechanical strains in the local systems of all plies are equal to those in the global system.

The total strains in each ply are

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix}_{UD} = \begin{bmatrix} 0 \\ 0.0013 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.0012 \\ -0.0003 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.0001 \\ 0.0005 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.13\% \\ 0.15\% \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix}_{chop} = \begin{bmatrix} 0 \\ 0.0013 \\ 0 \end{bmatrix} + \begin{bmatrix} -0.0003 \\ 0.0011 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.0000 \\ -0.0010 \\ 0 \end{bmatrix} = \begin{bmatrix} -0.03\% \\ 0.14\% \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix}_{sand} = \begin{bmatrix} 0 \\ 0.0013 \\ 0 \end{bmatrix} + \begin{bmatrix} -0.0012 \\ 0.0002 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.0004 \\ -0.0006 \\ 0 \end{bmatrix} = \begin{bmatrix} -0.08\% \\ 0.09\% \\ 0 \end{bmatrix}$$

The maximum total strain in the weep barrier of chopped fibers is 0.14%, in the hoop direction. This is much less than the allowable 0.40%, meaning the chosen laminate is too conservative. We should try a different laminate construction, perhaps by eliminating the sand core, or reducing the thickness of the UD plies. A satisfactory solution takes no more than three trials.

Example 2: Let us estimate the pressure rating of the hoop-chop pipe described in example 1.

We begin by computing the hoop mechanical strain component required to produce a total strain of 0.40% in the weep barrier of the pipe. The total hoop strain in the weep barrier is

$$\epsilon_y + 0.0011 - 0.0010 = 0.0040$$

The required mechanical hoop strain component is

$$\epsilon_y = 0.0039 \text{ (0.39\%)}$$

The hoop component of the mechanical strain connects with the operating pressure by the expression below, taken from the mechanical equation.

$$185780 \times \epsilon_y = \frac{P \times 50}{2}$$

Solving the above for $\epsilon_y = 0.0039$ gives the pressure rating $P = 29$ bar.

Example 3: Let us compute the rupture durability of the pipe described in example 1 operating at rated pressure. The total hoop strain in the UD90 fibers at the operating pressure $P = 39$ bar is

$$\epsilon_y = \epsilon_1 = 0.0039 - 0.0003 + 0.0005 = 0.0041$$

The thermal strain component is compressive in the fiber direction, which improves the durability. The hydric strain component is tensile and detrimental. On balance, the total strain in the fiber direction increases from a pure mechanical component of 0.39% to 0.41% after inclusion of the residual strains. The residual hydric and thermal strains may be important in designing composite laminates.

The question is.... How long will it take until the glass fibers fail? The answer to this question is in the part 2 of this book.

The underground pipes used in sewage service usually work at high burial depths and low pressures. These conditions require large ring stiffness (to resist the soil weight) and low tensile stiffness. To meet these requirements, most sewer pipes have thick sand cores, to provide ring stiffness, and thin fiber-reinforced thickness, just enough to handle the low pressure. Figure 1 shows the cross section of a typical laminate used in gravity sewage pipes.

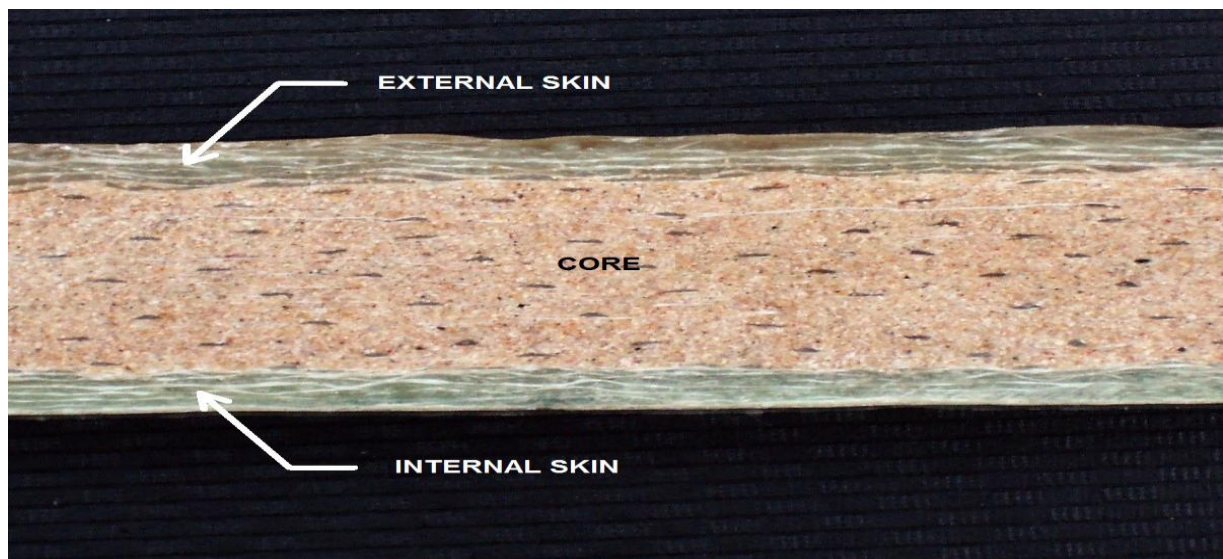


Figure 1
Wall cross section of a typical sewage pipe showing the large sand core between two thin inner and outer skins. The large sand core provides the required ring stiffness.

Example 4 – A consulting firm is evaluating the possibility of rerating an underground composite pipeline originally designed to operate in sewer service at room temperature and working pressure of 2 ± 1 bar. To increase the flow rate, the owner requires a new working pressure increased to $P = 4 \pm 1$ bar. The consulting firm should evaluate this possibility for a desired lifetime of 50 years.

Since this is an existing pipeline, no information is available regarding the ply thicknesses. The analysis will therefore take into consideration the laminate composition obtained from burnout tests.

The following information is available:

Pipe diameter $D = 1000 \text{ mm}$

Operating pressure $P = 4 \pm 1 \text{ bar}$

Pipe wall thickness $t = 19.0 \text{ mm}$

The pipeline operates underground

Stress-free temperature $PT = 100^\circ\text{C}$

Operating temperature $OT = 25^\circ\text{C}$

Water pickup by the resin $\Delta m = 0.01$

Resin content (res) = 28%

Chopped fibers content (chop) = 5%

Sand content (sand) = 68%

The first order of business is to establish the structural capability of the pipe. We do this using the formulas based on the laminate composition.

The laminate stiffness matrix is

$$[A] = \frac{3,12 \times t}{1,20 + 1,40(\text{res})} [2,33(\text{chop})[Q]_{\text{chop}} + 0,60(\text{sand})[Q]_{\text{sand}} + 0,83K[Q]_r]$$

Where

$t = 19.0 \text{ mm (1.9 cm)}$

$(\text{res}) = 0.28$

$(\text{chop}) = 0.05$

$(\text{sand}) = 0.68$

The laminate dryness is

$$K = (\text{res}) - \left[\frac{3}{7}(UD) + \frac{7}{3}(\text{chop}) + \frac{2}{8}(\text{sand}) \right]$$

$$K = (0.28) - \left[\frac{7}{3}(0.05) + \frac{2}{8}(0.68) \right] = -0.01$$

The dryness K is slightly negative, indicating pipes just barely dry compared to the standard.

Entering the data in the expression for the stiffness matrix, we have

$$[A] = \frac{3,12 \times 1.9}{1,20 + 1,40 \times 0.28} \left[2,33 \times 0.05 \begin{bmatrix} 77000 & 23000 & 0 \\ 23000 & 77000 & 0 \\ 0 & 0 & 27000 \end{bmatrix} + 0.60 \times 0.68 \begin{bmatrix} 66000 & 20000 & 0 \\ 20000 & 66000 & 0 \\ 0 & 0 & 23000 \end{bmatrix} - 0,83 \times 0.01 \begin{bmatrix} 33000 & 10000 & 0 \\ 10000 & 33000 & 0 \\ 0 & 0 & 11500 \end{bmatrix} \right]$$

$$[A] = \begin{bmatrix} 132652 & 40053 & 0 \\ 40053 & 132652 & 0 \\ 0 & 0 & 46299 \end{bmatrix} \text{ kg/cm}$$

The thermal force resultant is

$$[N^T] = \frac{3,12 \times t}{1,20 + 1,40(\text{res})} [2,33(\text{chop})[N^T]_{\text{chop}} + 0,60(\text{sand})[N^T]_{\text{sand}} + 0,83K[N^T]_r]$$

$$[N^T] = \frac{3,12 \times t}{1,20 + 1,40(res)} \left[2.33(chop) \begin{bmatrix} 2.5 \\ 2.5 \\ 0 \end{bmatrix} + 0.60(sand) \begin{bmatrix} 1.12 \\ 1.12 \\ 0 \end{bmatrix} + 0,83K \begin{bmatrix} 2.58 \\ 2.58 \\ 0 \end{bmatrix} \right] \times (OT - PT)$$

$$[N^T] = \frac{3,12 \times 1.9}{1,20 + 1,40 \times 0.28} \left[2.33 \times 0.05 \begin{bmatrix} 2.5 \\ 2.5 \\ 0 \end{bmatrix} + 0.60 \times 0.68 \begin{bmatrix} 1.12 \\ 1.12 \\ 0 \end{bmatrix} - 0,83 \times 0.01 \begin{bmatrix} 2.58 \\ 2.58 \\ 0 \end{bmatrix} \right] \times (25 - 100)$$

$$[N^T] = \begin{bmatrix} -202.97 \\ -202.97 \\ 0 \end{bmatrix} kg/cm$$

The hydric force resultant is

$$[N^H] = \frac{3,12 \times t}{1,20 + 1,40(res)} [2,33(chop)[N^H]_{chop} + 0,60(sand)[N^H]_{sand} + 0,83K[N^H]_r]$$

$$[N^H] = \frac{3,12 \times t}{1,20 + 1,40(res)} \left[2.33(chop) \begin{bmatrix} 19000 \\ 19000 \\ 0 \end{bmatrix} + 0.60(sand) \begin{bmatrix} 12800 \\ 12800 \\ 0 \end{bmatrix} + 0,83K \begin{bmatrix} 17200 \\ 17200 \\ 0 \end{bmatrix} \right] \times \Delta m$$

$$[N^H] = \frac{3,12 \times 1.9}{1,20 + 1,40 \times 0.28} \left[2.33 \times 0.05 \begin{bmatrix} 19000 \\ 19000 \\ 0 \end{bmatrix} + 0.60 \times 0.68 \begin{bmatrix} 12800 \\ 12800 \\ 0 \end{bmatrix} - 0,83 \times 0.01 \begin{bmatrix} 17200 \\ 17200 \\ 0 \end{bmatrix} \right] \times 0.01$$

$$[N^H] = \begin{bmatrix} 271.57 \\ 271.57 \\ 0 \end{bmatrix}$$

The thermal equilibrium strains are

$$\begin{bmatrix} -202.97 \\ -202.97 \\ 0 \end{bmatrix} = \begin{bmatrix} 132652 & 40053 & 0 \\ 40053 & 132652 & 0 \\ 0 & 0 & 46299 \end{bmatrix} \times \begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix} = \begin{bmatrix} -0.0012 \\ -0.0012 \\ 0 \end{bmatrix}$$

The hydric equilibrium strains are

$$\begin{bmatrix} 271.57 \\ 271.57 \\ 0 \end{bmatrix} = \begin{bmatrix} 132652 & 40053 & 0 \\ 40053 & 132652 & 0 \\ 0 & 0 & 46299 \end{bmatrix} \times \begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix} = \begin{bmatrix} 0.0016 \\ 0.0016 \\ 0 \end{bmatrix}$$

The residual thermal strains in the global system are.

$$\begin{bmatrix} \epsilon_x^r \\ \epsilon_y^r \\ \gamma_{xy}^r \end{bmatrix} = \begin{bmatrix} \epsilon_x^e - \alpha_1(OT - PT) \\ \epsilon_y^e - \alpha_2(OT - PT) \\ \gamma_{12}^e \end{bmatrix}$$

We proceed to compute the residual thermal strains on each ply

For the ply of chopped fibers, the thermal residual strains are

$$\begin{bmatrix} \epsilon_x^r \\ \epsilon_y^r \\ \gamma_{xy}^r \end{bmatrix}_{chop} = \begin{bmatrix} -0.0012 - 25.0 \times 10^{-6} \times (25 - 100) \\ -0.0012 - 25.0 \times 10^{-6} \times (25 - 100) \\ 0 \end{bmatrix} = \begin{bmatrix} 0.07\% \\ 0.07\% \\ 0 \end{bmatrix}$$

For the core ply, the thermal residual strains are

$$\begin{bmatrix} \epsilon_x^r \\ \epsilon_y^r \\ \gamma_{xy}^r \end{bmatrix}_{core} = \begin{bmatrix} -0.0012 - 13.0 \times 10^{-6} \times (25 - 100) \\ -0.0012 - 13.0 \times 10^{-6} \times (25 - 100) \\ 0 \end{bmatrix} = \begin{bmatrix} -0.02\% \\ -0.02\% \\ 0 \end{bmatrix}$$

The residual hydric strains in the global system are

$$\begin{bmatrix} \epsilon_x^r \\ \epsilon_y^r \\ \gamma_{xy}^r \end{bmatrix} = \begin{bmatrix} \epsilon_x^e - \beta_1 \Delta m \\ \epsilon_y^e - \beta_2 \Delta m \\ \gamma_{12}^e \end{bmatrix}$$

For the ply of chopped fibers, the residual hydric strains are

$$\begin{bmatrix} \epsilon_x^r \\ \epsilon_y^r \\ \gamma_{xy}^r \end{bmatrix}_{chop} = \begin{bmatrix} 0.0016 - 0.19 \times 0.01 \\ 0.0016 - 0.19 \times 0.01 \\ 0 \end{bmatrix} = \begin{bmatrix} -0.03\% \\ -0.03\% \\ 0 \end{bmatrix}$$

For the sand core ply, the residual hydric strains are

$$\begin{bmatrix} \epsilon_x^r \\ \epsilon_y^r \\ \gamma_{xy}^r \end{bmatrix}_{core} = \begin{bmatrix} 0.0016 - 0.15 \times 0.01 \\ 0.0016 - 0.15 \times 0.01 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.01\% \\ 0.01\% \\ 0 \end{bmatrix}$$

These are the residual strains, hydric and thermal, on all plies

Next, we compute the mechanical strains

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} 132652 & 40053 & 0 \\ 40053 & 132652 & 0 \\ 0 & 0 & 46299 \end{bmatrix} \times \begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix}$$

The soil friction prevents pipe expansion and contraction in the axial direction. Therefore, taking $\epsilon_x = 0$, we have

$$\begin{bmatrix} N_x \\ \frac{P \times D}{2} \\ 0 \end{bmatrix} = \begin{bmatrix} 132652 & 40053 & 0 \\ 40053 & 132652 & 0 \\ 0 & 0 & 46299 \end{bmatrix} \times \begin{bmatrix} 0 \\ \epsilon_y^e \\ \gamma_{xy}^e \end{bmatrix}$$

Solving the above, we obtain

$$\epsilon_y = \frac{P \times D}{2 \times 132652} = 3.77 \times 10^{-6} \times P \times D$$

$$N_x = 0.151 \times P \times D$$

Finally, the total strains are

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix}_{chop} = \begin{bmatrix} 0 \\ 3.77 \times 10^{-6} \\ 0 \end{bmatrix} \times P \times D + \begin{bmatrix} 0.07\% \\ 0.07\% \\ 0 \end{bmatrix} + \begin{bmatrix} -0.03\% \\ -0.03\% \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix}_{sand} = \begin{bmatrix} 0 \\ 3.77 \times 10^{-6} \\ 0 \end{bmatrix} \times P \times D + \begin{bmatrix} -0.02\% \\ -0.02\% \\ 0 \end{bmatrix} + \begin{bmatrix} 0.01\% \\ 0.01\% \\ 0 \end{bmatrix}$$

We can now compute the pipe pressure rating. The pressure rating of sanitation pipes is usually determined by limiting the maximum strain in the weep barrier to $\frac{1}{2}$ the weep threshold. Assuming a typical weep threshold of 0.80%, the design strain would be

$$\text{design strain} = \frac{0.80\%}{2} = 0.40\%$$

Such a large design strain is perfectly acceptable for sanitation pipes in which the sand core plays a minor part in the overall tensile stiffness. This is not the case in this example, where the pipe composition is 68% sand, 5% chopped fibers and no UD plies. In cases like this, it is safer to design the pipes for core failure, instead of weeping. The rationale is that once the core fails, or cracks, the pipe's tensile stiffness drops significantly and the chopped fibers by themselves may not suffice to control the weep failure. In other words, the failure of the weak sand core precipitates the collapse of the chopped plies. Therefore, the critical ply in this pipeline is the sand core, not the chopped fibers.

From this perspective, the design strain in this case should not exceed $\frac{1}{2}$ the rupture strain of the sand core, assumed 0.30%. Therefore, the allowable strain in this case should be

$$\text{allowable strain} = \frac{0.30\%}{2} = 0.15\%$$

Taking the above as the design strain, the pipe's pressure rating is

$$3.77 \times 10^{-6} \times P \times D - 0.0002 + 0.0001 = 0.0015$$

$$P = \frac{424.40}{D}$$

The pipe diameter is $D = 1000 \text{ mm} = 100 \text{ cm}$

$$P = \frac{424.40}{100} = 4.3 \text{ bar}$$

The pressure rating, $P = 4.3 \text{ bar}$, is slightly less than the proposed maximum working pressure $P = 4 + 1 = 5 \text{ bar}$. Based on this, and the desired lifetime of 50 years, the consulting firm rejected the pipeline for the new intended working condition. In their view, the pipe's rating is too close to the working pressure and they had no idea what to say about its durability.

The durability issue is a topic for later in this book. For a continuation and final solution to this problem, please go to example 2 in chapter 16.

Appendix

Residual mechanical strains

We end this chapter discussing a detail not mentioned in chapter 4. The reader will remember that in developing our protocol to compute the mechanical strains – see chapter 4 – we emphatically denied the existence of residual mechanical strains. This simplification was necessary to facilitate the presentation that would otherwise become too complex. In reality, however, laminates constrained by external reactive forces develop residual mechanical strains. Since this is an exception, rather than a rule, we have decided to exclude the residual mechanical strains from our protocol and compute them separately every time they arise. The residual mechanical strains are then added to the strains computed by the protocol. The following numerical example clarifies this abstruse statement.

Example 6 – Suppose a pressurized underground pipeline. The friction with the soil prevents any axial movement of the pipes and the mechanical axial strain is $\epsilon_x = 0$. This strain condition is used to compute the reactive axial force N_x on the pipe, which takes the exact value required to make $\epsilon_x = 0$. This situation occurred frequently in the numerical examples discussed in this chapter. There is no dispute that the axial mechanical strain is zero in this case. However, we should acknowledge the presence of the residual mechanical axial strain arising from the reactive force N_x preventing the free Poisson contraction of the pipe.

Let us compute the residual mechanical strains for the underground pipeline discussed in example 1. The equilibrium mechanical strains and the reactive force N_x are

$$\begin{bmatrix} 93800 & 28200 & 0 \\ 28200 & 183800 & 0 \\ 0 & 0 & 32800 \end{bmatrix} \times \begin{bmatrix} 0 \\ \epsilon_y^e \\ 0 \end{bmatrix} = \begin{bmatrix} N_x \\ 10 \times 50 \\ 2 \\ 0 \end{bmatrix}$$

From which we have $\epsilon_y = 0.136\%$ and $N_x = 38.4 \text{ kg/cm}$. The equilibrium axial strain is $\epsilon_x = 0$.

The free mechanical strains that would occur in the absence of the reactive force N_x are

$$\begin{bmatrix} 93800 & 28200 & 0 \\ 28200 & 183800 & 0 \\ 0 & 0 & 32800 \end{bmatrix} \times \begin{bmatrix} \epsilon_x^e \\ \epsilon_y^e \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \times 50 \\ 2 \\ 0 \end{bmatrix}$$

Solving the above, we have

$$\epsilon_y = 0.143\%$$

$$\epsilon_x = -0.043\%$$

By definition, the residual mechanical strain is the difference between the equilibrium and the free mechanical strain.

$$\begin{bmatrix} \epsilon_y^r \\ \epsilon_x^r \end{bmatrix} = \begin{bmatrix} 0.136 - 0.143 \\ 0 - (-0.043) \end{bmatrix} = \begin{bmatrix} -0.007\% \\ +0.043\% \end{bmatrix}$$

Again, by definition, the total mechanical strains are obtained by adding the free and the residual strains components.

$$\begin{bmatrix} \epsilon_y^{total} \\ \epsilon_x^{total} \end{bmatrix} = \begin{bmatrix} 0.143 - 0.007 \\ -0.043 + 0.043 \end{bmatrix} = \begin{bmatrix} +0.136\% \\ zero \end{bmatrix}$$

As we see, the total mechanical strains remain unchanged at $\epsilon_y = 0.136\%$ and $\epsilon_x = 0$. The residual axial strain, however, is $\epsilon_x = +0.043\%$. This strain results from the reactive force N_x counteracting the Poisson contraction that would otherwise occur. This example illustrates a situation when the equilibrium axial strain is zero, and the residual axial strain is not zero.

As discussed in the part 2 of this book, the higher hoop strains control the load-dependent weep and strain-corrosion durability of underground sanitation pipes. The smaller axial residual strains are not relevant and the simplified protocol developed in chapter 4 is valid in all practical cases.

Part 2

The part 2 of this book deals with the durability of composite equipment in industrial service. We start with a brief description of the eight modes of equipment failure, followed by the detailed derivation of the eight models to predict their durability in each case. To clarify the concepts, we introduce many carefully designed numerical examples alongside the discussions.

The durability of industrial composites equipment usually revolves around the estimation of their deterioration from aggressive chemicals or mechanical loads. The chemical and mechanical lives are by far the principal and most pressing durability issues facing the designers of composites for industrial application. Other modes of deterioration, while also of interest, are not as important.

The part 2 of this book reflects my efforts to develop and present the numerical models to predict the durability of composites in all cases, with emphasis on the chemical and mechanical lives. The solutions of the chemical and load-dependent durability problems require accurate knowledge of the laminate moduli and the total ply strains, as computed in the part 1 of this book.

Chapter 8

The five modes of failure.

1 – Introduction – The deteriorations leading to long-term failures of composite equipment fall in two categories, those that are load-dependent, and those that are chemical-dependent. The computation of the chemical-dependent durability is relatively simple and straightforward. By contrast, the high complexity of the protocols to predict the load-dependent cases may require some computational effort.

The fitness for use defines the mode of failure of the equipment. Let us consider, as illustration, the cases of pressurized pipelines and wind blades. The pipelines fail by weeping while the wind blades fail by loss of stiffness. In both cases, the fitness for use defines the mode of failure (weeping or loss of stiffness). The modes of failure set the criteria for the measurement of durability.

The equipment failures from load-dependent degradation involve the accumulation of small micro-cracks in the laminate, usually invisible to the naked eye. The large visible damages (mostly delamination) develop later, after the equipment has exceeded its service life. These large damages have no practical relevance. This book will offer little or no comments on large posthumous damages, like delamination.

In all cases, the failure of the critical ply determines the fitness for use and durability of the entire equipment. The critical ply, not the laminate itself, defines the failure criteria. Every ply has its own allowable level of accumulated damage, determined for each mode of failure. Equipment failure occurs when the critical ply accumulates damages above the specified allowable level, known as the failure threshold.

2 – The five modes of failure – The estimation of the durability of any equipment requires knowledge of the following

- *The mode of failure*
- *The degradation mechanism*
- *The critical ply*
- *The failure criterion*

In all cases, the equipment fails when the critical ply accumulates a damage level defined by a predetermined criterion. There is a criterion for every mode of failure. For example:

- *In applications where long-term deflection is critical, as in wind blades, the mode of failure is loss of stiffness, the degradation mechanism is cyclic fatigue, and the failure criterion is the acceptable level of accumulated damage in the critical ply.*
- *In pressurized pipes for fluid transmission, the mode of failure is weeping, the degradation mechanism is cyclic fatigue, and the failure criterion is the acceptable level of accumulated crack densities in the critical ply.*
- *In chemical service, the mode of failure is infiltration, the degradation mechanism is chemical attack and the failure criterion is the aged infiltration threshold of the critical ply.*
- *In other cases, the mode of failure could be an objectionable fading of the initial color and the failure criterion would be the deviation from an agreed upon standard.*

There is a trend in academic circles to design for durability with basis on the long-term rupture, taking as leading indicators the gross structural deterioration of the laminate. The design for long-term rupture is not relevant in industrial service. The fitness for use of industrial equipment relates to other modes of failure, such as infiltration, weeping and loss of stiffness. The design of composite equipment based on the ultimate strength, just prior to rupture, is unrealistic in most industrial service.

By definition, the durability of any equipment is the length of time until the accumulated damages render it unfit for service. The fitness for use is the sole determinant of the mode of failure and the failure criteria. This chapter introduces the five modes of failure applicable in industrial service, identifies the critical plies and discusses the failure criteria.

The five modes of failure of industrial equipment are.

- ***Infiltration failure*** is the level of accumulated damages that allows the fast ingress of aggressive chemicals in laminates. This mode of failure may result from a weakening of the laminate by chemical attack or by the accumulation of crack damages from cyclic loads. In both cases, failure occurs when the level of damage in the critical ply allows the fast ingress of chemicals in the laminate. This topic is addressed in chapters 10 and 14
- ***Weep failure*** is the level of accumulated damages that allows the leakage of fluids. This mode of failure results from the accumulation of crack damages from cyclic loads. Failure occurs when the level of damage in the critical ply allows the formation of pathways for the passage of fluids. This is a load-dependent mode of failure, governed by crack densities from cyclic loads. See chapter 14.
- ***Stiffness failure*** is the level of accumulated damages that cause unacceptable loss of laminate stiffness. This is a load-dependent mode of failure, governed by crack densities from cyclic loads. See chapter 14.
- ***Rupture failure*** is the level of accumulated fiber damages from strain-corrosion and/or cyclic fatigue. This mode of failure is a mixture of load-dependence and water attack on the glass fibers. For details, see chapters 11 and 15.
- ***Anomalous failure***, characterized by the growth of large, non-osmotic, water-filled blisters in the laminate. The anomalous blisters originate and grow in pre-existing de-

lamination usually present in the sand core of sanitation pipes. The discussion of this topic is in chapter 18.

The above five modes exhaust all known causes of failure in industrial laminates.

The main causes of failure in industrial equipment are

- *Mechanical loads, mostly cyclic*
- *Chemical attack*
- *Strain-corrosion*
- *Abrasion*
- *Impact*
- *Others*

Table 1 shows the five modes of failure with their main causes and failure mechanisms.

<i>Mode of failure</i>	<i>Cause</i>	<i>Mechanism</i>
<i>Infiltration</i>	<i>High crack density Laminate deterioration</i>	<i>Cyclic fatigue Chemical attack</i>
<i>Weeping</i>	<i>High crack density</i>	<i>Cyclic fatigue</i>
<i>Loss of stiffness</i>	<i>High crack density</i>	<i>Cyclic fatigue</i>
<i>Rupture</i>	<i>High crack density</i>	<i>Cyclic fatigue Fiber strain-corrosion</i>
<i>Anomalous</i>	<i>Temperature or pressure oscillation</i>	<i>Diffused water accumulated in voids</i>

Table 1

The five modes of failure with their main causes and failure mechanisms.



Long-term exposure to UV light destroys the resin matrix in the outer layers of laminates, while leaving the fibers intact. The shielding from the remaining fibers makes this a self-limiting deterioration, not included as a mode of failure.

3 – Crack densities and ply strains. The load-dependent failure of composites equipment results from the accumulation of micro-cracks in their critical plies. The micro-cracks grow in number – not in size – eventually reaching a critical failure density. To compute the equipment durability, we need to understand the relation between ply strains and crack densities.

The number of cycles that grow the initially small crack densities to their critical values defines the equipment durability. The difficulty with this definition is the impossibility to quantify the invisible crack densities. We bypass this problem by postulating a **direct correspondence** between the crack densities and the applied strains. Low applied strains produce small initial crack densities that require a large number of cycles to grow and fail the equipment, as captured by the regression equation.

$$\log(\epsilon) = A_c - G_c \log(N)$$

This equation indicates that the small damage from the applied peak strain ϵ requires N cycles to grow and reach the critical crack density. This explains why laminates under low cyclic strains sustain higher damages at failure than those under higher strains.

Example 1: *Explain the differences between applied peak strains and damages*

There are four ply damages of interest in the study of the load-dependent durability of composites equipment.

1. **Failure threshold.** *This is the crack density, or damage, that fails the ply in the first cycle, $N = 1$. It corresponds to the short-term strength of the ply.*
2. **Critical crack density.** *This is the damage state that fails the ply at N cycles under the applied strain*
3. **Applied strain.** *The small initial state of damage from the applied peak strain requires N cycles to grow and reach the critical crack density.*
4. **Fatigue threshold.** *This is the crack density at the ply formation. It corresponds to the least possible damage state incurred by the ply.*

The fundamental fatigue threshold is a topic for later discussion.

Example 2: *Explain the differences between short-term strengths, failure thresholds and design strains.*

The short-term strengths and the failure thresholds are the same thing. Both are ply properties measured in short-term quasi-static lab tests.

The design strains derive from the application of a short-term safety factor to the failure thresholds.

$$\text{design strain} = \frac{\text{Failure threshold}}{\text{Short term SF}}$$

The product standard AWWA C950 for composite pipes used in water transmission, suggests a short-term design factor $SF = 1.8$. When we say, for instance, that the weep threshold of a polyester ply of chopped fibers is 0.80%, the design strain would be $0.80\%/1.8 = 0.45\%$. The above short-term SF has no relation with the long-term SF computed from the unified equation. See chapter 16.

4 – Critical plies and their failure thresholds – The two critical plies of practical significance in the analysis of composite equipment are those made of unidirectional (UD) fibers or chopped fibers. The failure of the critical ply terminates the useful life of the equipment. The obvious concept of critical ply is universally accepted and used in the short-term analysis of composites. What we do in this book is simply extend this well-accepted short-term concept to the long-term.

As discussed later in this book, the computation of the load-dependent durability of equipment requires the use of a special unified equation and its inputs. Among these inputs are the four short-term strengths (one for each mode of failure) of the critical ply. These fundamental short-term ply strengths have received special names and symbols in this book. See table 2.

<i>Failure mode</i>	<i>Short-term strength</i>	<i>Symbol</i>
<i>Infiltration</i>	<i>Infiltration threshold</i>	<i>Ti</i>
<i>Weeping</i>	<i>Weep threshold</i>	<i>Tw</i>
<i>Loss of stiffness</i>	<i>Stiffness threshold</i>	<i>Ts</i>
<i>Rupture</i>	<i>Rupture threshold</i>	<i>Tr</i>

Table 2

Showing four modes of failure and their corresponding short-term strengths. The term “rupture strain” is the same as “rupture threshold”. The failure thresholds are properties of the critical plies, i.e., those governing the laminate durability.

Figure 1 ranks these thresholds in relation to the operating strain. The failure thresholds are short-term ply strengths, i.e., strains that cause immediate ply failure.

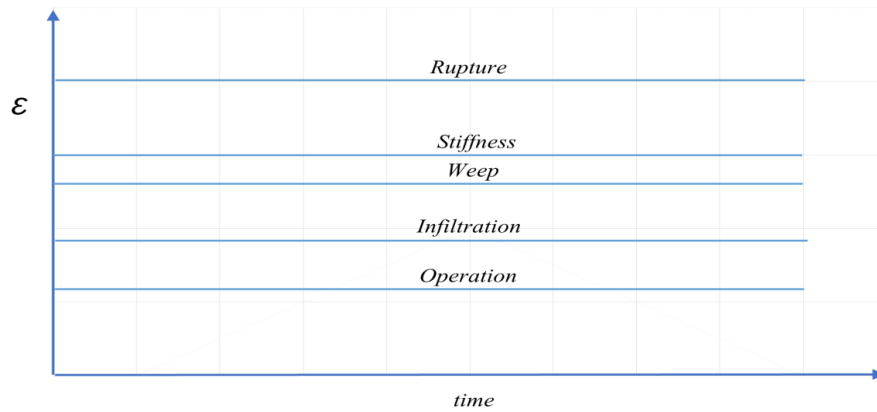


Figure 1

The four failure thresholds are short-term ply strengths that do not change with time. The failure sequence is first infiltration, followed by weep and lastly by rupture.

There are four failure thresholds, one for each mode of load-dependent failure.

- The **infiltration threshold** is the short-term strain marking the onset of crack densities high enough to allow the fast ingress of chemicals in the laminate. The infiltration threshold separates the “slow diffusion ingress” from the “fast infiltration ingress” of aggressive chemicals in the laminate. Laminates operating above the infiltration threshold have low chemical lives.
- The **weep threshold** is the short-term strain marking the onset of crack densities high enough to allow the passage of fluids through the laminate wall. The weep failure involves higher crack densities than those in infiltration.
- The **stiffness threshold** is the short-term strain corresponding to the onset of delamination and loss of laminate stiffness.
- The **rupture threshold** is the short-term rupture strength of the critical ply.

In discussing the failure of composite equipment, we focus primarily on the two fundamental critical plies, the corrosion barrier of chopped fibers and the structural UD plies. It is instructive to say a few words about these plies, emphasizing their differences and common features.

- The UD plies consist of many closely packed parallel continuous glass filaments. These plies develop large cracks when loaded transverse to the fibers. The transverse cracks grow in the resin matrix, along the fibers, increasing in length and in number in response to increments in the transverse load. Eventually the crack density becomes high enough to allow equipment failure by infiltration, followed by weeping and loss of stiffness.
- By contrast, the chopped fiber plies consist of a collection of widely separated and randomly dispersed bundles of short fibers. The short fibers in the randomly dispersed bundles are parallel and closely packed, exactly like those in the UD plies. At the fiber level, there is no difference between the architecture of UD plies and chopped bundles.

The foregoing explanation indicates that the UD and chopped plies share identical fiber patterns at the bundle level, both having close packed parallel filaments. Given this sim-

ilarity, one might conjecture that, perhaps, these plies might have equal infiltration and weep thresholds.

As it turns out, the above conjecture is correct only for the infiltration threshold. We justify this by the equal fiber array at the bundle level. We have reached an important conclusion:

The infiltration threshold of transverse loaded UD plies and chopped plies have the same value, controlled by fiber-resin bonding and resin matrix toughness.

Several authors support the correctness of the above statement. See chapter 8 of the book “Introdução aos Compósitos Industriais”.

The suggested conjecture, however, does not hold for the weep thresholds. This results from the wide separation and random orientation of the short bundles of chopped fibers not allowing the formation of large transverse cracks as in UD plies. This leads to the following conclusions:

Chopped fibers have higher weep thresholds than transverse loaded UD plies.

The resin toughness controls the weep threshold of chopped plies, with little influence from the fiber-resin adhesion.

4.1 – Longitudinal failure thresholds of UD plies. When loaded in the fiber direction, the UD plies lose stiffness from fiber rupture and fiber-resin debonds, with little resin cracks. Therefore, the infiltration and weep thresholds of UD plies tensioned in the fiber direction have no significance. The rupture threshold – the short-term tensile strength – of UD plies in the fiber direction is $Tr = 3.0\%$ regardless of the resin matrix.

Table 3 lists the failure thresholds of UD plies loaded in the fiber direction.

<i>Failure thresholds of UD plies loaded in the fiber direction</i>	
<i>Infiltration</i>	<i>Not relevant</i>
<i>Weep</i>	<i>Not relevant</i>
<i>Stiffness</i>	<i>Fiber-controlled</i>
<i>Rupture</i>	<i>$Tr = 3.0\%$</i>

Table 3

Rupture threshold of UD plies loaded in the fiber direction.

4.2 – Transverse failure thresholds of UD plies. Transverse loaded UD plies develop many large cracks running parallel to the fibers. The formation of the first cracks marks the infiltration failure. When embedded in laminates, the transverse loaded UD plies

stretch beyond the infiltration threshold, developing a larger number of cracks that eventually coalesce with those on adjacent plies to allow the passage of fluids. This is the transverse weep threshold of the UD ply. Beyond the weep threshold, the crack density continues to grow until stabilizing at a maximum value – known as CDS, for *Characteristic Damage State*. At this point the UD plies start to delaminate, marking the stiffness threshold. Further transverse tensioning will eventually lead to the ply rupture.

We have just described the four failure thresholds of transverse loaded UD plies. In figure 2 we have assumed the stiffness and rupture thresholds to be equal, coinciding with the CDS.

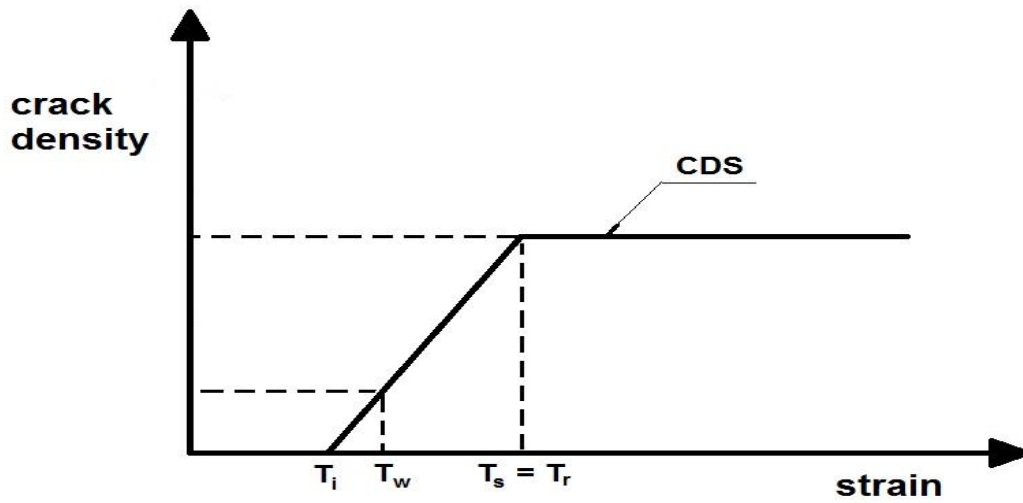


Figure 2

Crack densities and the failure thresholds of transverse-loaded UD plies. The stiffness and rupture thresholds correspond to the same state of damage, $T_s = T_r$.

The infiltration threshold of transverse loaded UD plies is the strain that generates the first cracks. The weep threshold is the transverse strain at the onset of weeping. From this discussion, it is reasonable to suspect the infiltration and weep thresholds of transverse loaded UD plies to be very close. In other words, the transverse crack density at the onset of weeping should be just slightly higher than that at the onset of infiltration.

Table 4 suggests some failure thresholds for transverse loaded UD plies.

	<i>Suggested failure thresholds for transverse-loaded UD plies</i>	
	<i>Polyester</i>	<i>Vinyl ester</i>
<i>Infiltration</i>	$T_i = 0.40\%$	$T_i = 0.50\%$
<i>Weep</i>	$T_w = 0.45\%$	$T_w = 0.55\%$
<i>Stiffness</i>	$T_s = 0.70\%$	$T_s = 0.90\%$

<i>Rupture</i>	$Tr = 0.70\%$	$Tr = 0.90\%$

Table 4

The stiffness and rupture thresholds of transverse-loaded UD plies correspond to the same state of damage. The infiltration and weep thresholds of transverse loaded UD plies differ by a small value.

4.3 – Failure thresholds of chopped fiber plies. The plies of chopped fibers do not develop large cracks visible to the naked eye. This invisibility is a source of dispute in the measurement of the threshold values.

The infiltration threshold of chopped plies is the strain corresponding to the onset of the first cracks. The detection of the invisible cracks requires special techniques, such as acoustic emission. For details, see chapter 14 of this book. The suggested values for the infiltration thresholds of chopped plies are $Ti = 0.40\%$ for polyester and $Ti = 0.50\%$ for vinyl ester resin matrices. These are the same values for the infiltration threshold of transverse loaded UD plies, as explained earlier.

The weep threshold of chopped plies is the crack density (measured as strain) corresponding to the onset of weeping. Visual observation is the only method available to detect the onset of weeping. For details, see chapter 14 in this book. The suggested weep thresholds for these plies are $Tw = 0.80\%$ for polyester and $Tw = 1.10\%$ for vinyl ester matrices.

The stiffness and rupture thresholds of chopped plies have little practical relevance. The rupture thresholds display very wide dispersions, ranging from 1.0% to 3.0%. In this book we adopt $Tr = 1.50\%$ for polyesters and $Tr = 2.50\%$ for vinyl ester matrices.

Table 5 lists the suggested failure thresholds for plies of chopped fibers.

	<i>Suggested failure thresholds for chopped fiber plies</i>	
	<i>Polyester</i>	<i>Vinyl ester</i>
<i>Infiltration</i>	$Ti = 0.40\%$	$Ti = 0.50\%$
<i>Weep</i>	$Tw = 0.80\%$	$Tw = 1.10\%$
<i>Stiffness</i>	-----	-----
<i>Rupture</i>	$Tr = 1.50\%$	$Tr = 2.50\%$

Table 5

Stiffness failures are not relevant for plies of chopped fibers. The rupture thresholds vary within wide ranges

5 – Short-term and long-term analysis. We define two safety factors SF for the mechanical load-dependent durability of composites equipment in industrial service.

The short-term SF is

$$SF = \frac{\text{Failure threshold}}{\text{operating peak strain}}$$

The long-term SF comes from the unified equation discussed in chapter 16.

$$\left(\frac{R \times \varepsilon \times SF}{S_s} \right)^{\frac{1}{G_s}} + \left(\frac{(1-R) \times \varepsilon \times SF}{S_c} \right)^{\frac{1}{G_c}} + \left(\frac{R \times (1-R) \times \varepsilon^2 \times SF^2}{S_s \times S_c} \right)^{\frac{1}{G_{sc}}} = 1.00$$

The current product standards recognize only the short-term SF. The computation of the long-term SF is one of the main objects of this book. For details, see the discussions of the unified equation in chapter 16

The rest of this chapter is dedicated to the critical plies and their short-term strengths, or failure thresholds.

Example 3 – Solvent deterioration was not included in the list of the modes of failure. Explain why.

The fast solvent diffusion of solvents in composite laminates usually leads to short time failures. Solvent failures, when they happen, are short-term processes. The impermeable pipes, discussed in chapter 21, provide a complete solution to the short-term solvent failure problem.

Example 4 – The five modes of failure listed in this book excludes creep. Why is that so?

There are two reasons.

- *The low operating temperature of composite equipment, usually 120C, are not conducive to creep of glass fibers.*
- *The low operating strain of composite equipment, usually below 0.1%, are not conducive to creep.*

Industrial equipment operating under low strains and low temperatures do not creep.

There are several short-term failure criteria relating the applied ply strains to their thresholds. The most convenient is the maximum-strain criterion, which consists in taking a direct relation of the applied ply strain components and the corresponding failure thresholds.

$$\frac{\text{failure threshold}}{\text{ply strain}} = SF$$

The maximum-strain criterion produces four short-term safety factors SF, one for each mode of failure.

Other short-term failure criteria mentioned in the literature are the maximum-stress, and the failure equations proposed by Tsai-Wu and Tsai-Hill. These criteria require conver-

sion of the ply strain components into stress components. Although primarily derived for rupture failure, the Tsai-Wu and Tsai-Hill quadratic equations can also predict other modes of failure, simply by substituting the four failure thresholds – expressed as stresses – for the short-term ply strengths. As discussed earlier, the failure thresholds are conceptually identical to the short-term ply strengths. The quadratic equations produce four short-term safety factors, one for each mode of failure.

The classical failure criteria described in the literature are fully compatible with the failure threshold concept introduced in this book.

The reader should keep in mind, however, that the above failure criteria – maximum-strain, maximum-stress, Tsai-Wu and Tsai-Hill – apply to short-term failures and, as such, cannot predict durability. The analysis for long-term failures requires a completely different approach, based on the unified equation. The object of this book is the computation of the long-term safety factors using the unified equation. The short-term safety factors, when discussed at all in this book, uses the maximum-strain criterion.

Example 5 – *Consider a critical ply of chopped fibers. Identify its load-dependent modes of failure, degradation mechanisms, failure criteria and methods of failure analyses.*

Modes of failure: *In chemical service, infiltration. In sanitation service, weeping. Loss of stiffness and rupture are usually associated with UD plies, not with chopped fiber plies.*

Degradation mechanism: *In chemical service, the degradation comes from chemical attack. In load-dependent service, the damage mechanism is the increase of crack density from cyclic fatigue loads*

Failure criteria: *In chemical service, the aged infiltration threshold. In sanitation service, the weep threshold. The stiffness and rupture thresholds are rarely associated with chopped plies.*

Short-term methods of failure analysis: *Maximum-strain, maximum-stress, Tsai-Wu and Tsai-Hill.*

Long-term method of failure analysis: *The unified equation, discussed in chapter 16.*

The object of this book is to present solutions to the durability of industrial laminates, taking into account the five identified modes of failure. The arguments developed in the rest of the part II of this book are for the most part straightforward and easy to understand. The exception, perhaps, is the analysis of the load-dependent durability that requires some intellectual effort.

Chapter 9

Temperature, moisture and resin toughness

1 – Introduction: This chapter deals with the effects of resin toughness, fiber sizing, environmental moisture and operating temperature (OT) on the short-term failure thresholds of plies. The intuitive and qualitative approach taken in this chapter avoids as much as possible the abstrusive details that shed no light on the subject. The reader will find some unavoidable details in the appendices. The topic of resin-glass inter-phase/interface is barely touched, just enough to convey the basic ideas.

The discussion that follows will highlight two points:

- *The strong and expected positive effect of the resin toughness on the failure thresholds. Tougher resins always improve the load-dependent durability of composites equipment.*
- *The surprisingly small and negligible effect of the environmental moisture and operating temperature on the failure thresholds. The environment has no appreciable effect on the load-dependent durability of composites equipment.*

The small positive effect of the environmental moisture and operating temperature brings a tremendous simplification to the prediction of the load-dependent durability of composites equipment. The assurance of improved performance – however small – implies in the failure thresholds measured in dry plies at room temperature being safe to use at any temperature and moisture situation. This offers a significant simplification to the entire process.

The reader should not erroneously assume from the previous statement that the environmental moisture and temperature have no effect on the load-dependent durability. Far from it. The careful engineer knows that, while improving the failure thresholds, increased moisture and operating temperature also affect the thermal and hydric residual strains. This was the main topic of discussion in the part 1 of this book

Before entering the discussion of specifics, we take a moment to recall an important point discussed in chapter 8 regarding UD and chopped fiber plies.

The fiber array in the short bundles of randomly dispersed chopped plies is identical to that of the long bundles of UD plies. The fiber-resin bonding and the resin cracking at the fiber are

therefore equal in both plies. The discussions involving fiber array, resin-fiber bonding and resin cracking are therefore valid for both UD and chopped fiber plies.

With this important point in mind, we proceed with our discussion. We start with the effects on the UD plies. The effects on plies of chopped fibers will follow.

2 – Effects of temperature and moisture on the failure thresholds of UD plies: As explained in chapter 1, some ply properties – like the expansion coefficients and the moduli – are not overly sensitive to changes in the environmental temperature and moisture. Is this statement also true for the failure thresholds?

In other words.... Are the failure thresholds, measured in plies at the room temperature and in a dry atmosphere, valid at any temperature and moisture?

Catherine Woods and Walter Bradley answered this question in a remarkable paper titled “Effect of seawater on the interfacial strength of E glass/graphite epoxy”. See ref. 18. In this paper, the authors carefully measured and reported the micro-strains that developed on the UD plies of laminates subjected to changes in temperature and water intake. The conclusions that follow derive mostly from the data and findings reported by these authors, assuming the square fiber array shown in figure 1.

The resin shrinkage from crosslinking and cooling down pulls the UD fibers together, placing the points B (figure 1) in compression. At the same time, the surrounding fibers prevent the free shrinkage of the resin towards the point A, placing the resin mass within the dotted circle under tensile strains. At room temperature, and in a dry environment, the resin-rich region in the dotted circle A is in tension, while the regions around the points B are in compression.

Suppose now we immerse this UD ply, initially dry and at room temperature, in hot water. In this new environment, the resin matrix absorbs and saturates with water, while the ply takes a higher equilibrium temperature. The discussions that follow for UD plies are valid also for the randomly dispersed bundles of chopped fibers.

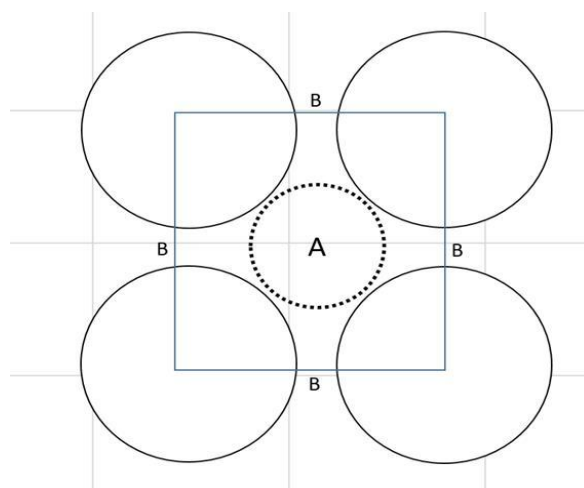


Figure 1

The square array of fibers indicates the resin tensioned in the dotted circle A and compressed at the points B. These residual strains are relieved with water intake and increments in the operating temperature. The above fiber array applies to both UD plies and the bundles of chopped fiber plies.

The resin shrinkage from cooling down and crosslinking places the regions at the points B in compression, thereby increasing the resin-fiber bond. Increasing the operating temperature, or the water intake, would expand the resin and reduce the residual compression at the points B. This is detrimental to fiber-resin bonding. We have our first conclusion:

Increased water intake and higher operating temperature facilitate resin-fiber debonding in both UD and chopped fiber bundles.

These fiber-resin debonds – noted as tiny micro-cracks running along the fiber-resin interface – are commonplace events in composite laminates. These harmless cracks run along the fibers, without linking to form pathways for the infiltration of molecules. At most, their presence increases the diffusion rate of chemicals in the laminate. We have our second conclusion:

The fiber-resin debonds have no effect on the infiltration threshold of UD or chopped fiber bundles.

The infiltration of chemicals takes place in resin cracks that develop in the dotted circle A. Let us check the resin strains in this dotted circle. As we have seen, the resin shrinkage from crosslinking and thermal cooling puts this region in tension, which is bad for cracking. However, increments in the operating temperature or in the water intake will expand the resin and alleviate this condition. This is good for the infiltration threshold. We have our third conclusion:

Increments in the water intake and in the operating temperature improve the infiltration thresholds of UD and chopped fiber plies.

In other words, increments in the operating temperature OT or in the water intake Δm alleviate the tensile strain in the dotted circle A, thereby delaying the onset of cracking there. This situation improves the infiltration and weep thresholds.

Table 1 summarizes these conclusions.

	<i>Increased water intake</i>	<i>Increased operating temperature</i>
<i>Fiber-resin bonding</i>	<i>Decreases</i>	<i>Decreases</i>
<i>Infiltration and weep thresholds (resin cracks in region A)</i>	<i>Increases</i>	<i>Increases</i>
<i>Rupture threshold</i>	<i>Increases</i>	<i>Increases</i>

Table 1

Changes in operating temperature and water intake expand/contract the resin in the same way and have similar effects. Anything said about temperature increase is also applicable to water pickup. This table indicates that the failure thresholds measured on dry UD plies at room temperature are safe to use at any higher temperature and moisture. The errors from this simplification are conservative and safe for weep and infiltration failures, while detrimental for resin-fiber debonds. Resin-fiber debonds, however, is not a relevant mode of laminate deterioration.

3 – An important practical application: The practical situation discussed in this section involves the weep failure of UD55 oil pipes operating at several temperatures under static loads.

Figure 2 shows two ASTM D 2992 B static (not cyclic) weep regression lines for oil pipes tested at 25C and at 80C. From the figure, we see the 80C line clearly extrapolating to a lower HDB value than the 25C line. This unquestionable experimental evidence seems to indicate that pipelines for high temperatures should have large thicknesses to compensate for the lower HDB value. The cost penalty associate with this apparently correct conclusion is enormous, not only in terms of increased materials usage, but mostly in terms of the formidable quality control and qualification testing required at the higher operating temperature.

The arguments developed earlier are in strong opposition to the above conclusion. First, we strongly reject the extrapolated HDB as a valid predictor of weep failure. The weep threshold T_w is the only valid indicator of weep failure. Second, as shown in table 1, the weep threshold T_w increases at higher operating temperatures. This improved T_w should reflect in better pipe performance at higher temperatures, in direct opposition to the classic HDB conclusion.

We have reached an interesting deadlock. On the one hand, the currently accepted static HDB concept, adopted by giant authorities like ASTM, API and ISO, indicates lower weep performance at higher temperatures. On the other hand, the weep threshold concept, proposed in this book, indicates otherwise.

We proceed to describe an experiment that throws some light on this disagreement.

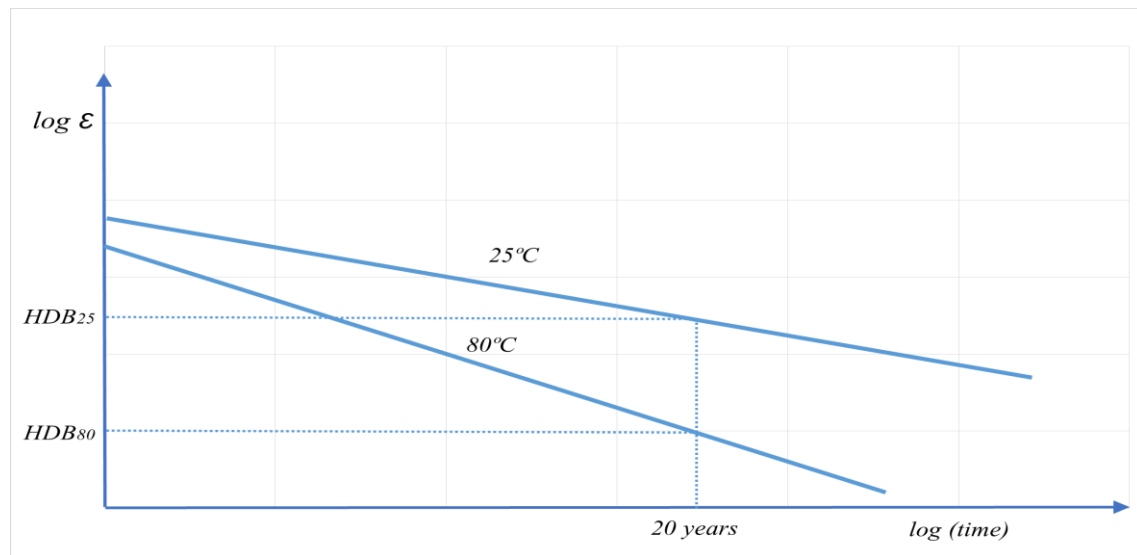


Figure 2

The slopes of classic static weep lines of UD55 pipes increase with the operating temperature. This unimpeachable experimental conclusion extrapolates to lower HDBs at higher temperatures, falsely indicating a lower pressure rating. The weep threshold concept, however, indicates better pipe performance at higher temperatures.

In 1994, a group of French scientists published some interesting experimental short-term weep test results on oil pipes. The reported results covered identical pipe specimens made of different resins and tested at several temperatures. Furthermore, the pipes had no weep barrier of chopped fibers (oil pipes) and the strains were correctly measured transverse to the UD55 fibers. The only variables were the resin toughness and the test temperature.

The experiments quantified the effects of resin toughness and operating temperature on the short-term weep failure of identical pipes under static loading.

Note: The short-term hoop strain that weeps water-filled pipes is an important parameter known as the “pipe weep threshold”, represented in this book as A_w . See chapter 13.

Figure 3 summarizes the test results. The ply thresholds (infiltration and weep) were not a part of the original paper, since such concepts were unknown back in 1994. The ply thresholds T_w and T_i shown in figure 3 are of my creation, introduced to help clarify the concepts. A close inspection of figure 3 indicates:

- *As expected, brittle pipes have lower infiltration thresholds T_i*
- *The weep thresholds T_w are also lower for brittle pipes, as expected*
- *The short-term weep pressures, however, are higher for brittle pipes. This is not expected*

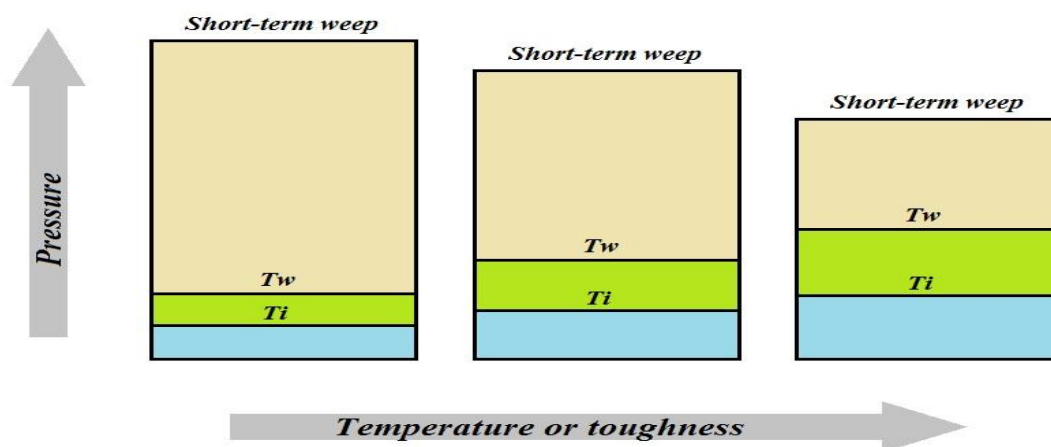


Figure 3

The infiltration and weep thresholds are ply properties that increase with increasing resin toughness and operating temperature. The short-term weep pressures, however, are pipe (not ply) properties that decrease with these changes.

Figure 3 shows the failure thresholds increasing with the resin toughness or the operating temperature, as they should. However, the decreasing short-term weep pressures in tougher or hotter pipes are not expected.

The point requiring clarification in figure 3 is...

Why pipes tested at higher temperatures, or made of tougher resins, weep at lower pressures than those tested at lower temperatures or made of brittle resins?

This is a surprising result. We would expect the weep strength of the pipes to follow the same trend observed in the infiltration and weep thresholds. The explanation for this apparent paradox is in the appendix 1 of this chapter. The conclusion from this important experiment are:

- *The brittle pipes (brittle resins or lower temperatures) develop many small cracks that link up to form tortuous pathways ending mostly in blind alleys that hamper the free passage of water. This explains the need for higher pressures, giving a false impression of improved pipe performance. The true indicator of better performance is the weep threshold T_w that increases in tougher pipes.*
- *The tougher pipes (tougher resins or higher temperatures) develop few long cracks that link up to form straight pathways that facilitate the passage of water. This explains the need for lower pressures, conveying a false impression of poorer pipe performance*

As we see, the pipes tested at lower temperatures have higher short-term weep strengths than those tested at higher temperatures. This implies in the false conclusion that hotter pipes have poorer long-term weep performance.

The discussion in this section involved static pressures.

4 – Cyclic loadings: In this section, we discuss the effects of environmental conditions and resin toughness on the cyclic durability of composites equipment. The arguments that follow are applicable to all cyclic load-dependent failure modes and, although focused on UD plies, is valid for chopped plies as well.

As explained in chapter 6, the cyclic failure of plies involves the steady growth of initially small crack densities to their critical values. The regression equation to predict the number of cycles to fail the critical ply is

$$\log(\epsilon) = A_c - G_c \log(N) \quad (1)$$

Where ϵ is the peak strain and N is the number of cycles to failure. Equation (1) is general, holding for all cyclic loading and all modes of failure.

In the previous sections, we demonstrated that increments in resin toughness, operating temperature and moisture slightly increase the failure thresholds. In this section, we will see the effects of these environmental conditions on the cyclic regression parameters A_c and G_c in equation (1).

By taking $N = 1$ in equation (1), we obtain

$$A_c = \log(\text{failure threshold})$$

The static A_s and the cyclic A_c parameters have the same value, defined as the logarithm of the short-term strength of the ply. Therefore, the A_c parameter in equation (1) increases in response to changing environmental conditions.

We have our first two conclusions:

- *Increments in the operating conditions (temperature and moisture) have a small positive effect on A_c .*

- *The resin toughness has a strong and easily quantifiable effect on A_c .*

We next turn our attention to the cyclic slope parameter G_c . As discussed in chapter 13, the slope G_c is expressed as

$$G_c = \frac{\log(T/T_0)}{\log N_0}$$

Where

T is the short-term failure threshold

T₀ and N₀ are the coordinates of the long-term fatigue threshold.

Given that the environmental conditions have negligible effect on the above thresholds, we offer a third conclusion:

- *The cyclic slopes G_c are not affected by the operating conditions.*

The cyclic slopes G_c are clearly affected by the failure thresholds T , increasing with increments in the resin toughness. We have a fourth conclusion:

- *The resin effect on the cyclic slope G_c is easily quantifiable through the short-term failure thresholds T .*

These four conclusions lead to the following simplifications:

1. *The environmental conditions – temperature and moisture – have no effect on the cyclic regression parameters A_c and G_c . The regression equations, for both plies and pipes, measured at room temperature in a dry atmosphere hold well in all operating conditions.*
2. *The effect of the resin toughness on the short-term failure thresholds affects the intercept A_c and the slope G_c of the cyclic regression lines.*

The above simplifications are fundamental in the study of load-dependent cyclic durability of composite laminates. They simplify the analysis and avoid the high cost involved in developing a plethora of expensive cyclic regression equations at different operating conditions. For a detailed discussion of this topic, see chapter 13.

For a list of the short-term failure thresholds of commercial plies, see tables 3 through 5 in chapter 8. These failure thresholds are ply properties that do not change with the environment.

Example 1 – *Compare the cyclic growth of crack densities in tough and brittle pipes.*

Composite pipes fail in ductile fatigue, i.e., they develop large crack densities that grow and eventually coalesce to failure. Ductile fatigue does not develop large a crack characteristic of brittle failure.

Let us compare the development of crack densities in brittle and tough pipes.

Brittle pipes develop extremely large crack densities that coalesce to form tortuous pathways that slows the passage of water. Tough pipes develop small crack densities that coalesce to form straight pathways that eases the passage of water. The conclusions are:

- *Brittle pipes require high pressures to negotiate the complex pathways and weep.*
- *Tough pipes require low pressures to negotiate the straight pathways and weep.*

The cracking process starts sooner, and the crack densities grow faster in brittle than in tough pipes. In the specific case of UD55 pipes, in which the cracks grow along the fibers, brittle pipes develop long and tortuous pathways that slows the passage of water. This explains the high weep pressures of brittle pipes.

Example 2 – *Compute the cyclic growth rate of the macro-crack openings in embedded UD plies.*

The ply thickness and the global laminate stiffness control the crack openings in embedded UD plies. The ply thickness is constant. The laminate global stiffness is also constant. Therefore, the crack openings do not change in embedded UD plies under cyclic loadings. The cracks under cyclic loads grow in length, not in opening.

Example 3 – *Table 2 reports the measured static weep regression parameters $A_s = A_w$ and G_s of epoxy UD55 oil pipes under 2:1 loadings at different temperatures. From the tabulated parameters, we write the static weep equations (to predict the times to weep) at each tested temperature.*

At 25C

$$\log \epsilon = -0.096 - 0.030 \times \log (\text{hours})$$

At 65C

$$\log \epsilon = -0.032 - 0.073 \times \log (\text{hours})$$

At 82C

$$\log \epsilon = +0.087 - 0.102 \times \log (\text{hours})$$

The above weep equations have the intercepts $A_s = A_w$ growing from -0.096 at 25C to $+0.087$ at 82C, in direct contradiction with our conclusion in this chapter. Our conclusion claims a decrease, not an increase, of the pipe weep threshold with increments in the operating temperature. Let us explain this contradiction.

There is only one explanation. The reported equations refer to different pipes, with different resins, different constructions and, most important, different wall thickness.

Appendix 1

The weep mechanism

5 – The weep mechanism in UD pipes. The crack densities in composites may grow in two ways.

- *First, in response to increments in static loads. This is the “static” growth.*
- *Second, in response to cyclic loads. This is the “fatigue” growth.*

The following discussion applies to weep failure of water-filled UD pipes under static loads.

The UD plies in pipes under 2:1 low static pressure develop small stationary penny-shaped cracks in planes perpendicular to their thickness and parallel to the fibers. Once formed, such penny-shaped cracks grow to a certain size and become stationary as long as the pressure does not change. Pressure increments to a higher constant value will increase both the number and the size of the cracks to a different stable level. Thus, both the crack density and their size increase in response to increments in the applied static pressure.

This process continues until the penny-shaped cracks reach the upper and lower adjacent plies, at which point they cease to grow in the thickness direction of the ply. Beyond this point, further pressure increases will increase the number of cracks, not their size in the thickness direction. The crack growth continues, however, in the fiber direction, turning the initially circular penny-shaped micro-cracks into elongated elliptical macro-cracks. At some point, the density of elongated cracks becomes large enough to allow the passage of water from one ply to the next. The transverse strain corresponding to this event marks the onset of infiltration. This transverse strain is the infiltration threshold T_i .

Further pressure increments will increase the crack density and eventually allow the formation of pathways through the pipe wall. This marks the onset of weep, and the corresponding transverse strain is the weep threshold T_w .

From this description, we see that the weep threshold T_w of UD plies should be slightly higher than the infiltration threshold T_i . Both T_w and T_i are ply properties dependent on the resin toughness. The ply thresholds control the pipe failures.

If we insist on increasing the pressure, the crack density becomes so large as to weep the pipe in the short-term. This is the short-term weep strength of the pipe, also called the pipe weep threshold A_w . The weep threshold A_w depends on the resin toughness and the pipe thickness.

Note. There are three cases of self-similar crack growth in composites. The first is ply delamination, not discussed in this book. The second is the crack growth along the UD fibers, de-

scribed above. Both cracks grow in the resin, between parallel fibers. The third and last case of self-similar crack growth is laminate strain-corrosion, discussed in chapter 11.

The preceding discussion highlighted the difference between the pipe weep threshold A_w (discussed in chapter 13) and the ply weep threshold T_w . The pipe weep threshold A_w is the hoop strain that weeps pipe in the short-term. The ply weep threshold T_w is the strain that marks the onset of the weeping process. For details, see chapters 13 and 14.

The short-term weep tests conducted by the French group in 1994 measured the pipe weep threshold A_w . Let us say a few words about A_w . Higher pressures increase the length, opening and density of the transverse cracks, facilitating the water advance and shortening the times to weep. Higher pressures imply in shorter weep times. Eventually, the test pressure and crack density are so high as to cause short-term weeping. The hoop strain corresponding to the short-term weeping of the pipe, designated as A_w , is substantially higher than the ply weep threshold T_w . Figure 3 illustrates this.

The weep threshold A_w is a pipe feature, dependent on its wall thickness and construction. In the part 2 of chapter 13, we use both the ply weep threshold T_w and the pipe weep strength A_w to compute the weep durability of pipes. The reader will be surprised to see how easy it is to compute the weep durability of pipes from the weep thresholds.

We now have the required background to explain the puzzling results in figure 3. The conditions most favorable to low short-term weep pressures in water-filled pipes are

- *High crack densities*
- *Long cracks.*

At the same strain level, experimental evidence and careful reasoning indicate that

- *Brittle pipes develop many short cracks*
- *Tough pipes develop few long cracks.*

The crucial question is.... Which condition produces lower short-term weep pressures, few long cracks or many short cracks? The experiments conducted by the French group clearly indicate that few long cracks weep pipes at lower pressures. The following paragraph gives a reasonable explanation for this.

The many short UD cracks in brittle pipes interact to form tortuous pathways ending mostly in blind alleys. At the same strain level, the advancing water takes a long time to find its way through the complex maze of many short cracks ending in blind alleys. Brittle pipes require higher pressures (strains) to weep than tougher pipes. The straight pathways formed by few long cracks reduces the weep pressure in tougher pipes. This situation is analogous to driving a car through densely populated areas, where the travel time is shorter through a long expressways than through many short interconnecting streets.

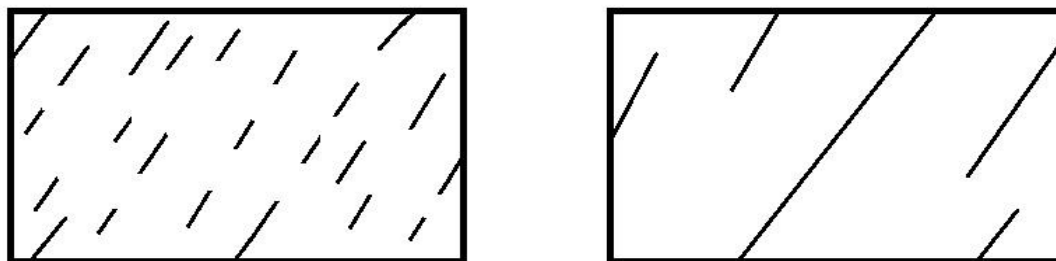


Figure 7

Cracks in UD plies. The few long cracks in tough plies (right) intercept many similar long cracks in adjacent plies. The opposite occurs with the many short cracks in brittle plies (left). The long cracks in tough UD plies provide direct pathways for the advancing water, resulting in lower short-term weep pressure.

We complete our discussion with a visual comparison of the crack patterns that develop in the UD plies of oil pipes of different resin toughness – or operating at different temperatures. The comparison is for equal pipe specimens under equal stationary pressure. The only variables are the resin toughness or the operating temperature. Figure 7 shows the cracks formed in tough and brittle UD plies running in the fiber direction. The tough UD plies on the right develop fewer and longer cracks than the brittle plies on the left. From the foregoing discussion, the tough pipes weep sooner than the brittle ones. This explains the bizarre experimental results reported by the French researchers in 1994.

In conclusion, all else being equal, tougher pipes weep sooner, showing regression lines that plot below those of brittle pipes. The same conclusion holds for operating temperatures, that show hotter pipes weeping sooner and plotting below colder ones. This explains the steeper slopes and the lower HDBs of tougher – or hotter – pipes shown in figures 2 and 8.

However, as explained in chapter 14, the extrapolated HDB values are not good predictors of long-term static weep failures. The true indicator of long-term static weep performance is the weep threshold, which is higher for tougher resins or hotter pipes. Figure 8 shows the details.

The ply weep threshold indicates improved static pipe performance at higher temperatures, in direct contrast with the false indications of the static weep regression lines and their HDB values. Such a compelling argument leads to the inescapable conclusion.

- *The static weep performance of oil pipes increases with the operating temperature, in direct contrast with the predictions by the ASTM D2992 B test protocol.*

This compelling argument carries enormous economic significance. The possibility to qualify all pipes at room temperature only would bring the following benefits.

- *Immediate cessation of all expensive and time-consuming qualification and quality control tests required at high temperatures.*
- *Pipes qualified for 25C static service can handle temperature upsets with no concerns.*
- *The heat distortion temperature (HDT) of the resin matrix is the only limitation to the pipe's upper working temperature.*

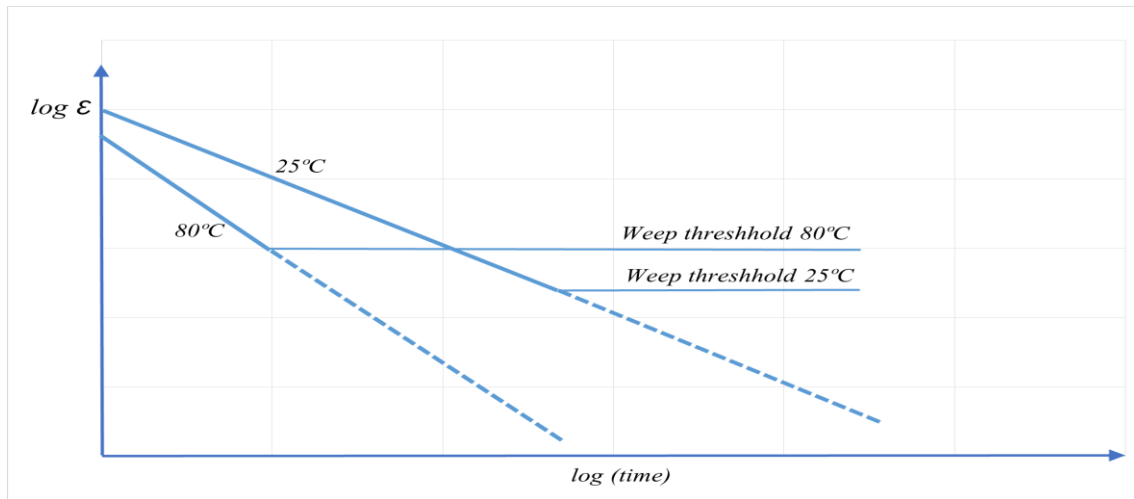


Figure 8

The static weep lines of pipes at lower temperatures plot above those at higher temperatures. This undisputed experimental fact falsely indicates poorer pipe performance at higher operating temperatures. By contrast, the true indicator of long-term weep performance, the weep threshold, improve at higher temperatures. This compelling argument suggests that pipes qualified for static room temperature service can safely operate at higher temperatures.

Example 5 – This numerical example discusses a few weep test results reported in the literature for UD55 epoxy oil pipes.

Table 2 shows measured static weep parameters A_s and G_s for UD55 epoxy oil pipes at three temperatures. The static slopes increase from $G_s = 0.030$ at 25C to $G_s = 0.102$ at 82C. A similar slope variation occurs in pipes of different resin toughness. This is in agreement with our previous discussions.

Temperature	$A_s = A_w$	G_s	HDB (25 years)	HDB (50 years)	Hoop weep threshold for polyester UD55 plies
25 C	- 0.096	0.030	0.55%	0.54%	1.00%
65 C	- 0.032	0.073	0.38%	0.36%	> 1.00%
82 C	0.087	0.102	0.35%	0.32%	> 1.00%

Table 2

Temperature dependence of the static weep parameters $A_s = A_w$ and G_s for UD55 epoxy oil pipes. The hoop weep thresholds listed in the table are those for UD55 polyester plies under 2:1 loading. The weep thresholds for 2:1 UD55 epoxy plies are unknown to the author.

The slope increase observed with increments in the operating temperature is in agreement with our previous discussion. We do not question the relative positions and slopes of the above regression lines. What we question is the validity of the extrapolated HDB as a weep predictor. The extrapolated HDB is meaningless.

Let us compute the extrapolated HDB for 25 years and 50 years at the reported temperatures. The following equation applies

$$\log \epsilon = A_s - G_s \times \log(\text{hours})$$

For 25 years at 25C

$$\log(HDB) = -0.096 - 0.030 \times \log(25 \times 365 \times 24)$$

$$HDB = 0.55\%$$

For 25 years at 65C

$$\log(HDB) = -0.032 - 0.073 \times \log(25 \times 365 \times 24)$$

$$HDB = 0.38\%$$

For 25 years at 82C

$$\log(HDB) = 0.087 - 0.102 \times \log(25 \times 365 \times 24)$$

$$HDB = 0.35\%$$

Similar computations lead to the HDB values at 50 years. The results are in table 2.

The high temperature lines extrapolate to low HDBs values. This undisputed experimental fact has led designers to derate composite pipes for high temperature service. This is in direct opposition to the concept of weep thresholds that actually improve at higher temperatures

We do not have the weep thresholds for epoxy UD plies. Their values, however, should be higher than those for polyester, shown in table 2. The weep threshold concept indicates that composite pipes have essentially the same long-term weep performance, regardless of the operating temperature.

The conflict between extrapolated HDB values and weep thresholds will surface again in chapter 14.

6 – The weep mechanism in chopped fiber pipes. In the preceding section, we discussed the effects of resin toughness and operating temperature on the failure thresholds of UD55 pipes. Let us now do the same for pipes of chopped fibers.

At the fiber level, the bundles of chopped fibers and continuous UD plies have identical arrays, indicating equal infiltration thresholds for both plies. As for the weep thresholds, there is a great difference. Let us see.

- *The continuous UD plies allow the development of long cracks that crossover to form weeping pathways immediately after formation of the first cracks. The weep threshold T_w of UD plies is, therefore, just slightly higher than the infiltration threshold T_i .*
- *This is not the case for chopped fibers. The wide separation and random orientation of the short bundles in the plies of chopped fibers do not allow the formation of large cracks that crossover to allow the passage of water. The weep threshold of chopped fibers comes from the coalescence of crack densities in the resin matrix, which makes them significantly higher than the infiltration threshold.*

Let us proceed to discuss the effects of the resin toughness and the environment on the failure thresholds of chopped plies. The infiltration threshold, measured at the onset of the first cracks, obviously increases with increments in resin toughness, temperature and moisture. This is the same situation discussed earlier for continuous UD plies. The resin-controlled weep threshold of chopped plies also improves in response to increased

plasticization from increments in resin toughness, operating temperature and moisture. Therefore, our previous conclusions hold for both chopped and UD plies.

As for the short-term pressure to weep, the argument for chopped fibers is identical to that for UD plies. Figure 9 shows the many short coalesced cracks in brittle chopped plies (left) compared with the few long cracks in tough plies. Note the irregular crack pattern in the brittle ply, forming many branches ending up mostly in blind alleys. This explains why tougher chopped fiber pipes weep at lower pressures than brittle ones. Furthermore, as explained before, the pipe performance measured by the ply weep threshold T_w improves with increments in resin toughness and at higher operating temperatures.

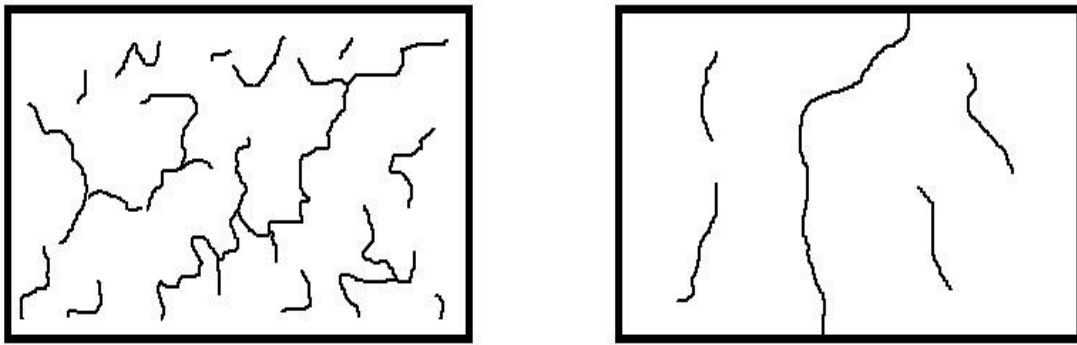


Figure 9

Side view of chopped fiber plies, showing the crack patterns in brittle and tough pipes. The brittle pipe on the left develop many short cracks that coalesce to form blind alleys. The tough pipe on the right develop few long cracks that form direct pathways for the water. This is the same argument used for UD plies.

Appendix 2

Improving the failure thresholds

In this appendix, we discuss a few ideas to improve the failure threshold by increasing the resin-glass bonding. The arguments will develop around the concepts of interface and interphase, illustrated in figure 10. The interface is the glass-resin contact surface. The interphase is a thin transition layer formed by blending the resin matrix with the fiber sizing. The interphase forms a thin transition cushion around the fibers.

Both the interface and the interphase have considerable effect on the fiber-resin debonds. In fact, they essentially control the onset of resin cracks leading to the infiltra-

tion and weep thresholds. The glass-resin bonding strength, as well as the toughness of the resin matrix have enormous importance in the technology of composites.

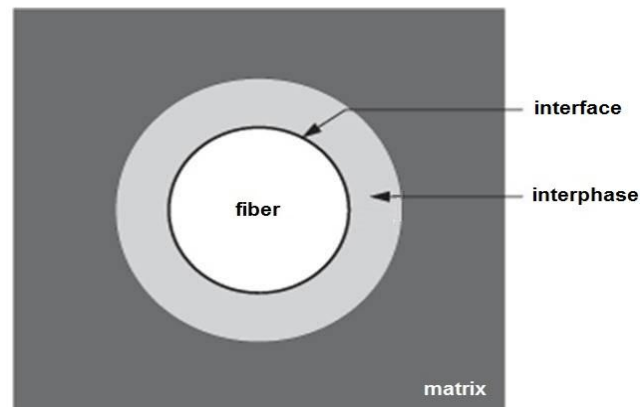


Figure 10

The interface and interphase control the onset of cracks leading to the infiltration threshold. The interphase and the resin matrix control the rate of crack growth. .

The following discussion is limited to static transverse-loaded UD plies. The cyclic loadings will be discussed later.

The total transverse strain (mechanical + residual) initiates the fiber-resin debonds that drives the cracking of the interphase and the resin matrix. In figure 11, we have modelled the interphase as springs, to better illustrate its role in transferring stresses from the resin matrix to the fiber interface.

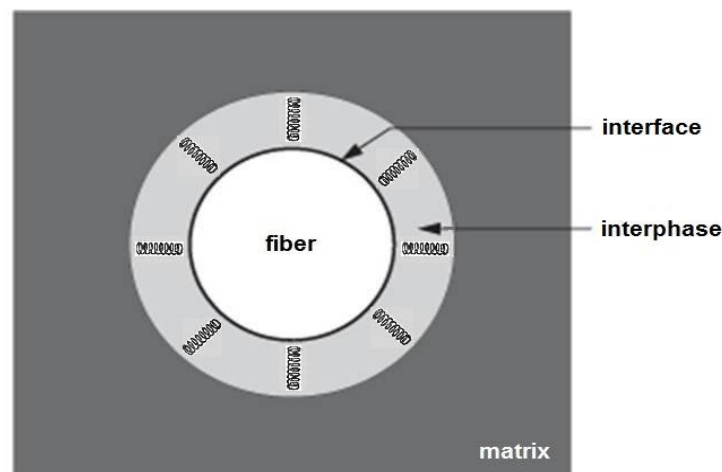


Figure 11

The interphase modeled as springs to illustrate its role in carrying stresses from the resin matrix to the fiber interface.

The arguments that follow focus the resin in the dotted circle A and around the points B. See figure 1. The decisive importance of the total strains at these locations is apparent. The tensile strains within the dotted circle A control the resin cracks leading to weep and stiffness failures. The compressive strains at the points B initiate and grow resin-fiber debonds. We seek answers to the following question:

What are the best resin matrices and fiber sizings to improve the infiltration, stiffness and weep thresholds of UD plies?

The answers to this apparently complex question are surprisingly simple.

- The film former in the sizing should have high elongation and low stiffness to avoid premature rupture and transfer the least possible stress to the interface. To achieve this, it should be readily soluble in the resin matrix and provide a low stiffness, tough interphase. Figure 8 models the flexible interphase as low stiffness springs. There is good experimental evidence supporting this idea. Excellent fiber-resin adhesion and large infiltration thresholds have been reported in transverse loaded UD plies with flexible film formers of polyester bisphenol resins. See references 19, 20 and 22.
- The interface should have the best possible adhesion to the glass fibers, perhaps by increasing the silane loading, or using hydroxyl-rich film formers.
- The resin matrix should have the highest toughness possible, to accommodate large strains before cracking. The resin modulus should not be too low, since this would be detrimental to the HDT. These attractive features are found in vinyl ester resins, that combine high toughness and high HDT.
- The resin matrix should also have good adhesion to the glass fibers. Excellent glass adhesion is one of the reasons for the distinct superiority of hydroxyl-rich vinyl ester resins vis-à-vis polyesters.

Reducing the ply thickness and the glass loading also improve the infiltration, stiffness and weep thresholds of UD plies. These approaches, however, are detrimental to the equipment cost.

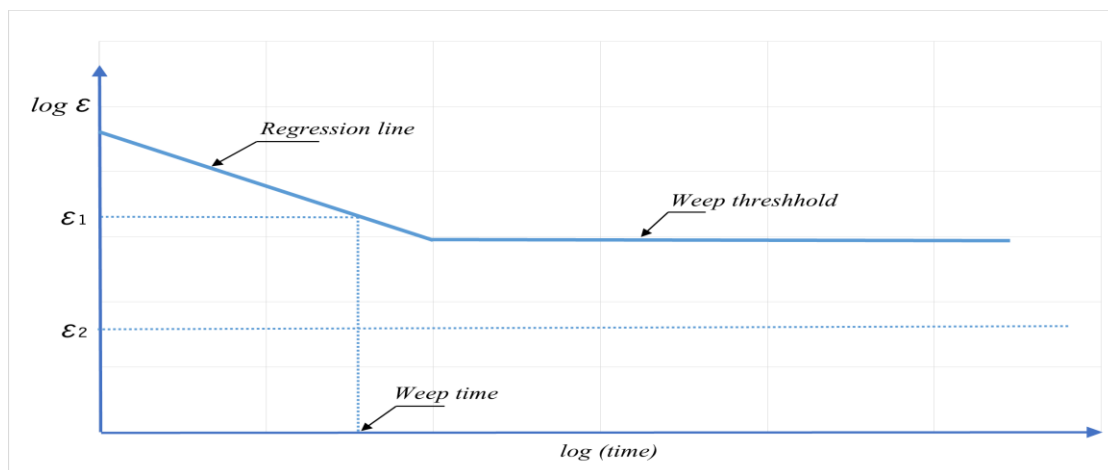


Figure 12

Static weep envelope of typical 1:0 pipes illustrating the failure times for two static strains ϵ_1 and ϵ_2 . The high strain ϵ_1 weeps the pipe at the time predicted by the regression line. The low strain ϵ_2 never weeps the pipe, since it is less than the weep threshold.

Figure 12 shows a typical 1:0 static weep line defining the failure envelope of composite pipes. Note the sloped regression part of the weep line turning horizontal on reaching the ply weep threshold. The sloped part of the regression line comes from pressure testing water-filled pipes in accordance with the ASTM D 2992B protocol. The horizon-

tal weep threshold line is a property of the critical ply. Figure 12 describes 1:0 static weep tests.

Still in figure 12, the pressure/strains above the weep threshold fail the pipe in the times predicted by the sloped part of the regression line. Furthermore, pressure/strains below the threshold never weep the pipe. As examples, consider the two static strains ϵ_1 and ϵ_2 shown in figure 12. The strain ϵ_1 , higher than the weep threshold, intercepts the sloped regression line at the indicated failure weep time. The strain ϵ_2 , lower than the weep threshold, never intercepts the failure envelope and never weeps the pipe. Figure 12 indicates that static weep failures never happen at strains below the weep threshold of the critical ply.

Appendix 3

Transverse cracks in UD plies

The transverse crack densities are different in embedded versus isolated UD plies. Isolated plies rupture at the formation of the first transverse crack, while embedded plies develop multiple transverse cracks. In embedded UD plies, the multiple long cracks grow in the resin matrix along the fiber direction. This crack growth, however, does not go on forever. There are at least two mechanisms arresting the crack growth in the fiber direction of UD fibers:

- *First, the stress intensity factor K_I decreases with increasing crack length. For details, see any book on fracture mechanics. Eventually, as the cracks become too long in comparison with the ply thickness, the crack growth stops as K_I vanishes.*
- *Second, there is the mutual shielding of crack tips approaching each other from opposite directions. As the crack tips approach each other, the high stress fields first combine to accelerate the growth. Later, as the tips pass each other, the stress field drops, and the crack growth stops. This shielding effect is illustrated in figure 13*

Figure 13 illustrates the arrest process observed in a trunk of eucalyptus tree left for years to dry in the sun. Observe the arrest of the approaching cracks close enough to have their tips shielded. The strong shielding prevents the development of large cracks in brittle plies. The opposite thing happens in tough plies. Of course, the same ideas apply to different temperatures as well. This argument explains why brittle and cold UD plies develop many short cracks, when compared to tough and hot ones.



Figure 13

The long transverse macro-cracks along UD fibers stop their growth as they approach each other from opposite directions. The photo shows cracks developed in a tree trunk left to dry in the sun.

Appendix 4

The glass-resin interphase

In the fabrication process, the liquid resin matrix dissolves the sizing on the fibers, forming a thin interphase layer around them. The interphase layer is a blend of the fiber sizing with the resin matrix. All phenomena involving the immediate glass-resin vicinity, such as fiber blooming, fiber debonding, etc., involve the interphase, not the resin matrix itself. As examples, the chemical attack on the glass fibers depends on the solubility of the aggressive product in the interphase, not in the resin. Furthermore, the interphase controls the onset of cracking that triggers the infiltration failure.

The fiber-resin interphase has never been properly studied and characterized.

CHAPTER 10

PART 1

INTRODUCTION TO CHEMICAL DURABILITY

1 – Introduction: The engineering community is well aware of the long service lives of composite equipment in a variety of harsh environments. It takes no more than a brief tour of any modern pulp bleaching or chlorine-alkali facility to realize the importance of composites in applications where metal corrosion is a major issue. However, in spite of this remarkable proven-in-use durability, there is still no reliable method to predict the chemical durability of composite equipment. The many studies and test methods developed over the years, while certainly useful to establish chemical suitability, are not adequate to quantify durability. This is understandable in view of the overwhelming complexity of the problem. The many combinations of resin types, working temperatures and other process variables, coupled with a myriad of aggressive chemicals, make this prediction a most daunting endeavor.

In chapter 8, we identified the five modes of failure of composite equipment, all leading to a particular lifespan. This book will present the protocols to compute the durability associated with each of those modes of failure, especially the ones related to mechanical loads and chemical exposure. The load-dependent problem is addressed in chapter 16, with the introduction of the various forms of the unified equation. The chemical-dependent durability is dealt with in this chapter.

2 – Three types of chemicals: The chemical damage of laminates is associated with the penetrated depth, together with the deterioration of the fibers and the resin matrix. The damaging potential of chemicals relates strongly to their reactivity and solubility in resin matrices.

Other things being equal, the higher the reactivity of the chemical, the slower it penetrates in laminates. This results from the fast capture and immobilization of highly reactive molecules by the resin matrix. As a rule, the composite equipment exposed to highly reactive chemicals suffer considerable and shallow damage, while those in contact with less reactive products usually display lesser and deeper damage.

For the discussion of chemical durability, it is useful to classify the industrial chemicals based on their ability to react with resin matrices. Although important, the solubility of the chemical is not a classification criterion.

- **Highly reactive chemicals.** The fast immobilization of the reactive molecules slows down their penetration, producing sharp and well-defined advancing fronts. Highly reactive chemicals can take extremely long times to saturate composite laminates.
- **Medium reactive chemicals.** The small immobilization rate of less reactive molecules allows them to penetrate faster and deeper, displaying various degrees of diffuse advancing fronts. The mild chemicals may saturate composite laminates in relatively short times.
- **Non-reactive chemicals.** The chemicals in this group do not react with resin matrices, displaying fast diffusing rates that saturate the entire laminate. The shape, size and polarity of the penetrating molecules control their rate of advance, which is essentially friction-dependent. The most important industrial non-reactive chemicals are water and solvents that, as a rule, display high penetrating rates, saturating entire laminates in short times.

Figure 1 shows the concentration profiles of chemicals in laminates immersed in high and a medium reactive environment. The highly reactive chemicals display flat and clearly identifiable advancing fronts, in contrast to the diffuse and non-identifiable advancing fronts of mild species.

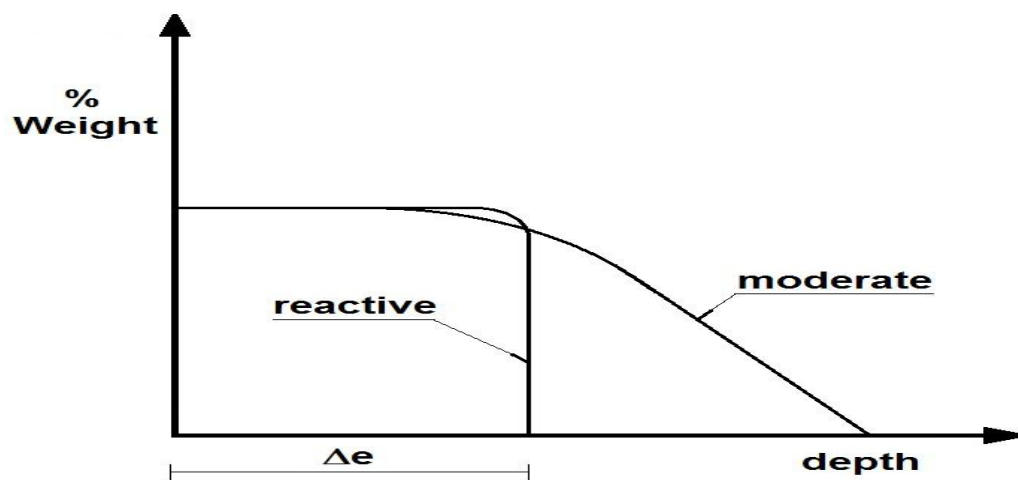


Figure 1

The flat advancing fronts of highly reactive chemicals allow the clear identification and measurement of the penetrated depth Δe . The diffuse fronts of mild chemicals do not allow this measurement.

Table 1 summarizes these statements

	Highly reactive chemicals	Medium reactive chemicals	Unreactive solvents
Penetration rate	Slow	Moderate	Fast
Time to penetrate the entire laminate	Long	Short to moderate	Saturate the entire laminate
Damage	High	Moderate	No chemical damage
Advancing front	Flat	Diffuse	Saturate the entire laminate
Metrics of durability	Tensile stiffness Penetrated depth	Tensile stiffness	Excessive swelling

Table 1.

Chemical classification to predict the durability of composites. The highly aggressive chemicals take long times to fully penetrate and saturate the equipment wall. The mild reactive chemicals, on the other hand, can penetrate and saturate the entire equipment wall in relatively short times. In all cases, the penetrated material suffers a continued chemical deterioration that slowly reduces its tensile stiffness and failure thresholds.

3 – Penetration of reactive chemicals. Figure 2 shows the concentration profiles of three highly reactive chemicals in typical laminates designed for chemical service. The chemicals are identified by the numerals 1, 2 and 3 and different shades of grey. The figure shows the concentrations lower in the resin-rich liner, higher in the corrosion barrier of chopped fibers and highest in the resin-poor structural UD plies. This reflects the tendency of chemicals to accumulate in voids and at the glass-resin interface. In all situations, the actual chemical concentrations in the resin itself is the same, regardless of their location in the liner or anywhere else. The reader will understand that molecules of water and chemicals accumulate and fill up the penetrated laminate voids and resin-glass debonds.

Six points need attention in figure 2.

1. *The reaction with the resin matrix immobilizes and slows down the rate of penetration, producing flat and well-defined advancing fronts. Flat fronts are characteristic of highly reactive chemicals. Other things being equal, the higher the reactivity of the penetrating chemical, the flatter is the advancing front.*
2. *The presence of resin-rich liners have no effect on the concentration profiles. The only benefit of liners is a small retardation in the chemical advance. Since most liners crack in highly aggressive chemicals, their protection is more a fiction than a fact. For details, see the work of Regester in the appendix 1 of this book.*
3. *The chemical damage to laminates depends on (a) the operating temperature, (b) the concentration profile of the aggressive chemical, (c) the penetrated depth and (d) the resin-chemical interaction.*
4. *The penetrated depth depends on the reactivity and solubility of the chemical solution. Medium reactive and highly soluble chemicals may quickly saturate the entire laminate.*
5. *The concentration profile of product 3 indicates partial penetration of the structural plies. The computation of the chemical life of the equipment (object of this chapter) takes into consideration the presence of aggressive chemicals in the structural laminate.*
6. *The concentration profile of product 3 increases from a low value in the liner to a maximum in the structural plies. This is in line with the increased presence of voids in laminates with high fiber contents.*

At this point, the reader should be aware that

- *The concentration of chemicals in laminates increases with the operating temperature, void content and their solubility in the resin matrix.*
- *The penetrated depth varies with the exposure time and operating temperature. The solubility and reactivity of the chemical molecules in the resin matrix are also important.*
- *The chemical activity decreases with time, reflecting the depletion of available active sites. The decrease in active sites occurs both in the resin and in the penetrating chemical.*
- *In all cases, the chemical deterioration slowly reduces the tensile stiffness and the infiltration threshold of the penetrated material. The increasing strain from the lower stiffness eventually equals the degraded infiltration threshold, at which point the equipment fails.*

The concentration profiles of less reactive chemicals are similar to those in figure 2, except the advancing fronts are diffuse and not clearly identifiable.

Eventually, all aggressive chemicals advance past the corrosion barriers to penetrate the structural plies. The maintenance crews usually replace penetrated corrosion barriers in an attempt to protect the structural plies. This is a current practice in the industry, which I do not concur. My recommendation is not to replace the corrosion barrier, letting the chemicals penetrate the equipment as they would. The penetrated equipment should remain in operation until very near its failure, at which time the maintenance crew performs an external (not internal) intervention to restore its fitness for service. The computation of the times to chemical failure – the chemical durability – is the topic of this chapter.

Note: Some resins give long service lives not because of their superior chemical resistance, but because of their lower diffusivity to aggressive chemicals. This explains the superior chemical performance of highly cross-linked resins and the outstanding effect of post-cure.

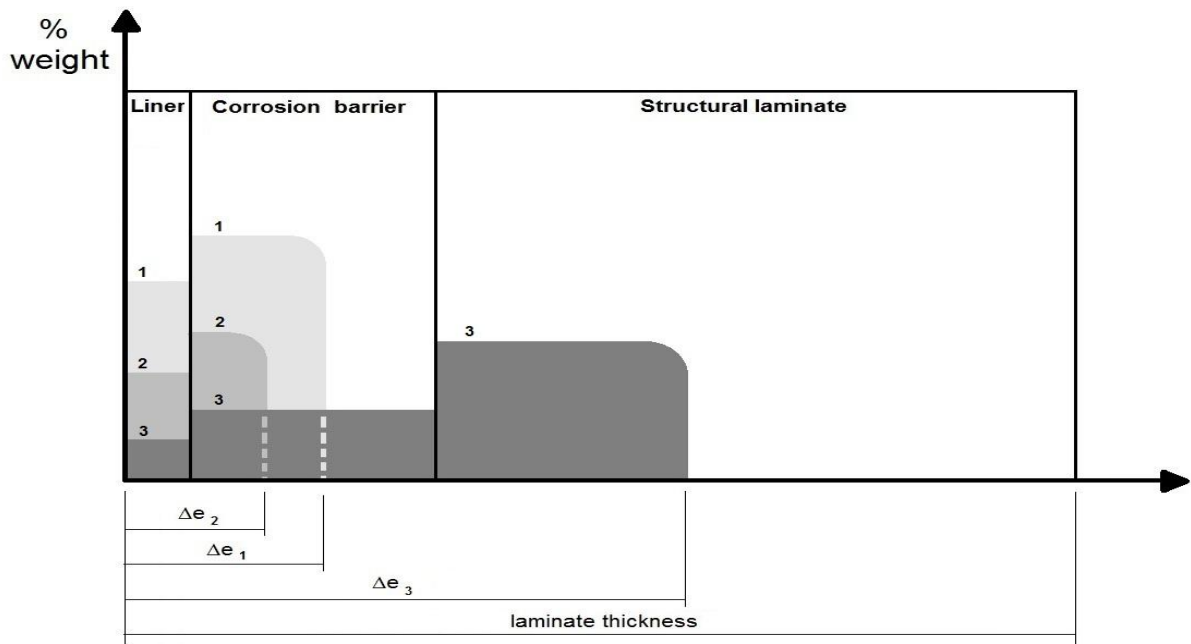


Figure 2

Concentration profiles of three highly reactive chemicals in composite laminates. The flat advancing fronts of highly reactive chemicals allow measurement of the penetrated depths Δe . The concentration profiles of less reactive chemicals are similar, except the advancing fronts are diffuse and not identifiable.

4 – Chemical durability. The attainment of the aged infiltration threshold defines the chemical life of composite equipment under external mechanical loads. The chemical activity causes a loss of stiffness that gradually increases the strain in the equipment. The operating strain should never exceed the aged infiltration threshold. The flooding of aggressive chemicals observed above the aged infiltration threshold quickly fails the equipment. We have four conclusions:

- *Below the aged infiltration threshold, the chemical deterioration process is independent of the external load. This allows an enormous simplification.*
- *The operating strain increases with the loss of tensile stiffness.*
- *Equipment failure occurs when the residual stiffness eventually reaches the aged infiltration threshold.*
- *The critical stiffness define the chemical durability of composite equipment.*

The foregoing discussion defines the chemical durability of composite equipment as the time for the reactive chemical to

- *Penetrate the laminate*
- *Reduce its original tensile stiffness to a critical value determined by the applied load and the aged infiltration threshold.*

The aged infiltration threshold is a safe, simple, reliable, consistent and logical failure criterion to define the critical stiffness and the chemical durability of composite equipment. Let us continue with this explanation.

5 – The chemical durability equation. All composite equipment in chemical service should operate below their aged infiltration thresholds. This is an essential requirement to prevent quick loss

of stiffness and assure long chemical durability. Failure occurs when the increasing strain from the reduced stiffness eventually equals the aged infiltration threshold. Above the aged infiltration threshold, the fast chemical ingress reaches the weep threshold in a short time. Beyond the weep threshold, the deterioration is even faster, leading to imminent rupture. We have an important conclusion:

The critical tensile stiffness, defined by the external load and the deteriorated infiltration threshold, is the ultimate metric of the chemical durability of composites equipment. The weep and rupture failures follow immediately after the onset of infiltration.

The definition of the residual tensile stiffness is

$$(Residual\ stiffness) = (E \times e)_{res}$$

Where E and e are the tensile modulus and the laminate thickness respectively. The subscript “res” denotes the residual values of these quantities, as they deteriorate with time.

The equation to link the residual stiffness to the time in service is.

$$\log\left(\frac{(E \times e)_{res}}{(E \times e)_0}\right) = C \times \log(time\ in\ service)$$

Where $(E \times e)_0$ is the original laminate stiffness.

This equation relates the initial and residual stiffness to the time in service. Failure occurs when the residual stiffness equals the critical value

$$(E \times e)_{res} = (E \times e)_{cri}$$

Where the subscript “cri” indicates the critical residual stiffness. Entering this in the above, we obtain the chemical durability equation governing the life of composite equipment in stable environments.

$$\log\left(\frac{(E \times e)_{cri}}{(E \times e)_0}\right) = C \times \log(durability)$$

Or, expressed in terms of the tensile modulus

$$\log\left[\frac{E_{cri}}{E_0}\right] = C \times \log(durability)$$

The parameter C is

$$C = \frac{\log\left(\frac{E_{res}}{E_0}\right)}{\log(inspection\ time)}$$

Which gives the final form of the durability equation

$$\log(durability) = \log(inspection\ time) \times \frac{\log\left[\frac{E_{cri}}{E_0}\right]}{\log\left(\frac{E_{res}}{E_0}\right)}$$

The initial stiffness is readily available from the manufacturer’s data book or by inspecting the aged equipment. The residual modulus is available from tests performed on the equipment itself

at the inspection time. The critical stiffness derives from the known external load and the laminate properties. The only unknown in the above equation is the chemical durability.

6 – The critical tensile modulus. The external loading N and the deteriorated infiltration threshold define the critical tensile stiffness of the equipment.

$$(E \times e)_{cri} = \frac{N}{\left[\begin{array}{c} \text{infiltration} \\ \text{threshold at failure} \end{array} \right]}$$

Where N is the external mechanical force per unit length.

This equation acknowledges that failure occurs when the strain from the external force N equals the aged infiltration threshold. The external force N is available in all cases. For cylindrical pipes and tanks of diameter D under internal pressure P , the force N in the hoop direction is

$$N_y = \frac{P \times D}{2}$$

To estimate the value of the aged infiltration threshold, we assume it to deteriorate linearly with the tensile modulus. Therefore, at failure, we have

$$\left[\begin{array}{c} \text{infiltration} \\ \text{threshold at failure} \end{array} \right] = T_i \times \frac{E_{cri}}{E_0}$$

Where

T_i is the pristine infiltration threshold
 E_{cri} is the tensile modulus at failure
 E_0 is the original tensile modulus

The critical tensile stiffness is

$$(E \times e)_{cri} = \frac{N}{T_i} \times \frac{E_0}{E_{cri}}$$

For cylindrical structures of diameter D under pressure P , the above equation simplifies to

$$(E \times e)_{cri} = \frac{P \times D}{2 \times T_i} \times \frac{E_0}{E_{cri}}$$

In non-abrasive services, where the laminate thickness “ e ” is preserved, we have

$$E_{cri} = \frac{N}{e \times T_i} \times \frac{E_0}{E_{cri}}$$

Which gives the critical tensile modulus

$$E_{cri} = \sqrt{\frac{N \times E_0}{e \times T_i}}$$

Where

e is the original thickness
 T_i is the pristine infiltration threshold
 $(E)_{cri}$ is the critical tensile modulus

N is the known external force per unit length
 E_0 is the original tensile modulus

In non-abrasive services, the tensile modulus replaces the tensile stiffness. The critical modulus is readily available from known quantities.

Example 1. Explain the processes leading to the chemical failure of composites equipment in stable non-abrasive and abrasive service.

In non-abrasive service, the original laminate thickness is preserved. The reactive chemical diffuses into the laminate, slowly degrading its tensile modulus and causing an increase in the operating strain. Eventually the decreasing tensile modulus reaches a critical value causing the strain to reach the deteriorated infiltration threshold. At this point, the equipment fails.

In abrasive service, the erosion process removes the damaged penetrated material, decreasing the laminate thickness in the process. The decreased thickness cause the laminate strain to increase, eventually reaching the pristine infiltration threshold of the remaining laminate. Again, the attainment of the infiltration threshold marks the point of equipment failure.

In non-abrasive services, the residual modulus and the aged infiltration threshold define the durability.

In abrasive service, the residual thickness and the pristine threshold of the remaining laminate define the durability.

Example 2. Compute the critical modulus of composite equipment in chemical service.

From Hooke's law, we have

$$\frac{N}{e} = E_0 \times \epsilon$$

Entering the above in the equation for the critical modulus

$$E_{cri} = \sqrt{\frac{N \times E_0}{e \times T_i}}$$

$$E_{cri} = \sqrt{\frac{\epsilon}{T_i}} \times E_0$$

Assuming $\epsilon = 0.001$ and $T_i = 0.005$, the critical modulus is

$$E_{cri} = \sqrt{\frac{0.001}{0.005}} \times E_0$$

$$E_{cri} = 0.45 \times E_0$$

This interesting rule establishes that well designed equipment of DK411 have critical tensile modulus equal to 45% of the initial modulus.

5 – Durability in solvents: Non-reactive solvents diffuse rapidly into resin matrices, quickly saturating entire laminates. Their non-reactive nature implies in mechanical damages from swelling, not from chemical action. The fast diffusion of solvents implies in quick responses measured in short-term testing of immersed laminates. The behavior of equipment in the presence of solvents is therefore a short-term issue requiring no durability analysis. The study of the chemical durability of composites equipment is a topic for reactive chemicals.

In some cases, the resin swelling from the absorption of non-reactive solvents may be high enough to cause mechanical degradation of laminates. This degradation may be so high as to preclude the use of composites in structural service, even at room temperature. This is the case of ethanol, where its 14.0% absorption in the resin weakens the laminates to a point that prohibits their use in structural service at room temperature. Composite pipes cannot handle ethanol in pressure applications.

As a rule, composite materials do not work well in the presence of solvents.

6 – The effect of water. Water is a very special solvent.

- *First, its small and polar molecule penetrates and fits almost anywhere.*
- *Second, its low solubility (1.0%) in resin matrices has no relevant effect on the laminate properties.*
- *The small water molecules penetrate all plies, being present everywhere at all times in the entire laminate, both in the resin matrix and at the glass-resin interface.*
- *The water presence is unavoidable in composite laminates, even in dry atmospheres.*

Water is a penetrating species that reaches all points in all plies, being always present everywhere in all laminates. The effect of water on the durability of composite equipment is a topic for chapters 9, 15 and 18. In chapter 9, we argue in favor of its positive effect on the weep and infiltration thresholds. In chapter 15, we discuss its negative effect on the rupture durability of laminates. Chapter 18 describes the water effect on the so-called anomalous failure.

The laminate damages from solvents other than water are short-term phenomena not addressed in this book. The following example summarizes all we have to say about solvents and their effect on durability.

Example 3 – Explain the effect of solvents on the durability of composites.

The non-reactive solvents readily diffuse into resin matrices, rapidly saturating all plies. The penetrated solvents plasticize the resin matrices, lowering their Heat Distortion Temperature – HDT. As a simple example, we mention the modest water pickup of 1.0%, that reduces the resin HDT by a significant 10C. The penetrated solvents also reduce the laminate mechanical properties. The above effects may range from moderate and acceptable, as in the case of water, to excessive and unacceptable.

The applicability of composites in solvent service depends on the solvent solubility in the resin, measured in resin castings at room temperature. The decision to allow or deny direct contact of solvents with composites requires nothing more than a simple short-term immersion test of resin castings at room temperature. This is a short-term issue, outside the scope of this book.

Water is a special solvent that do harm in two ways. First, like all solvents, it lowers the resin HDT and degrades the laminate mechanical properties, as explained. Second, it promotes a continuous and sustained strain-corrosion attack on the glass fibers leading to long-term laminate rupture under static loads. Water is the only known chemical capable of (a) fully penetrate and (b) strain-corrode the glass fibers in all plies.

Appendix 1

Debonds and chemical resistance of glass fibers

In my efforts to understand the chemical durability of composites, I met two difficulties not satisfactorily discussed in the published literature. This appendix is an attempt to clarify these issues.

- *The published literature proposes **fiber-resin debonds** as a major booster of chemical infiltration, fast laminate deterioration and short chemical lives. Since such debonds are present in all laminates, at all times, chemical infiltration seems to be an unavoidable and ubiquitous problem that would destroy the concept of infiltration threshold and lead to extremely short chemical lives.*
- *The available literature also highlights the known **poor chemical resistance of glass fibers** as a major factor in the chemical durability of composite equipment. This true statement, if not properly explained, creates the false impression of extremely short chemical durability.*

These true assertions are in opposition to the proven long lifespans of composite equipment in a wealth of aggressive chemicals. This appendix is an attempt to clarify these issues.

Fiber-resin debonds. We start with the fiber-resin debonds. As explained elsewhere, chemical infiltration occurs above the infiltration threshold, in paths formed by coalesced resin cracks. The fiber-resin debonds do not coalesce to form pathways, since they develop along the fibers, not in the resin itself. While debonds may affect laminate permeability, and certainly increase the concentration of chemicals at the fiber surface, their contribution to the infiltration process is negligible.

We therefore dismiss the ubiquitous and unavoidable resin-fiber debonds, as they have no appreciable effect on the infiltration threshold.

Poor chemical resistance of glass fibers. The poor chemical resistance of glass fibers is undeniable, as demonstrated by many experiments.

- *Bare glass fibers show quick dissolution when immersed in most commonly used industrial chemicals. For details, see the work of Kimab (ref. 32).*
- *Creep-rupture tests of laminates in aggressive solutions confirm the extremely poor chemical resistance of glass fibers. For details, see the work of Mark Greenwood (ref. 3).*

The tests conducted on bare fibers are especially scary, as they show extremely fast chemical deterioration and complete fiber destruction after short exposures. These bare fiber tests, however, are meaningless, since the presence of chemicals at the surface of fibers embedded in laminates is extremely low, far lower than in the test solutions. Again, we can discard the scary results from direct immersion of bare glass fibers in chemicals.

However, the fibers embedded in creep-tested laminates also show poor performance in spite of no direct contact with the full strength solution. These tests are not easy to justify and dismiss. For details, see the work of Regester in the appendix 1 of this book.

What follows is an attempt to explain the poor performance of immersed laminates in aggressive creep tests. The justification for such poor performance comes from the test itself that requires straining the specimens far above their infiltration and sometimes their weep thresholds. The high crack densities that develop in this situation allow the fast ingress of the full strength solution that quickly deteriorates the exposed fibers. In the presence of aggressive chemical, no composite equipment should operate above the infiltration threshold, lest their chemical lives be too short.

Chapter 10

Part 3

Chemical durability in abrasive service

1 – Introduction. In this part 3 of chapter 10, we compute the durability of composite equipment in environments combining abrasive and corrosive deterioration. The abrasive action removes part of the penetrated material, thus reducing the equipment thickness and significantly increasing the chemical activity.

We illustrate the method with a numerical example of an actual elbow that ruptured after 19 years of continuous service carrying an acidic solution of chlorine dioxide at 85C. We do not have any measurement of the residual stiffness of the laminate. The only data point available is the rupture at 19 years of continuous service.

Example 1. Explain the rupture failure of an elbow at 19 years of continuous service carrying an acidic solution of chlorine dioxide at 85C. The following information is available.

$D = 150 \text{ mm}$ (elbow diameter)

$R_c = 450 \text{ mm}$ (Radius of the elbow)

$P = 5 \text{ kg/cm}^2$ (Operating pressure)

$OT = 85C$ (Operating temperature)

Time to rupture = 19 years

Laminate construction: chopped fibers

$E = 70\,000 \text{ kg/cm}^2$ (Original tensile modulus)

$\epsilon = 0.10\%$ (design strain)

$T_i = 0.50\%$ (Infiltration threshold for DK 411 chopped fibers)

$T_w = 1.10\%$ (Weep threshold for DK 411 chopped fibers)

$T_r = 2.50\%$ (Rupture threshold for DK 411 chopped fibers)

We will compute the durability to infiltration and weep based on the given data and the informed rupture failure at 19 years. Failure will come from the combined action of chemical attack and erosion, meaning stiffness deterioration from both loss of modulus and thickness. We will assume full allocation of deterioration to the residual stiffness. The failure thresholds maintain their pristine values.

We solve problem using the chemical durability equation for each mode of failure. We will compute an infiltration life, a weep life and a rupture life.

$$\log \left[\frac{(E \times e)_{cri}}{(E \times e)_0} \right] = C \times \log \left(\begin{array}{c} \text{durability for each} \\ \text{mode of failure} \end{array} \right)$$

The critical tensile stiffness varies with the mode of failure. We will compute the elbow's original tensile stiffness as well as its critical stiffness for the three relevant modes of failure, namely infiltration, weep and rupture. The ageing parameter C derives from the only experimental data point available, namely rupture at 19 years.

$$C = \frac{\log \left(\frac{(E \times e)_{res}}{(E \times e)_0} \right)}{\log(\text{exposure time})}$$

$$C = \frac{\log\left(\frac{16\,500}{406\,000}\right)}{\log(19 \times 12)} = -0.59$$

The durability for each mode of failure is

$$\log\left(\frac{\text{durability for each}}{\text{mode of failure}}\right) = \frac{1}{C} \times \log\left[\frac{(E \times e)_{cri}}{(E \times e)_0}\right]$$

The general expression to compute the hoop tensile stiffness of elbows is

$$E \times e = \frac{P \times R}{2 \times \epsilon} \times \left(\frac{2 \times R_c - R}{R_c - R}\right)$$

For the elbow object of this analysis, we have

$$P = 5 \text{ kg/cm}^2$$

$$R = 75 \text{ mm}$$

$$E = 70\,000 \text{ kg/cm}^2$$

$$R_c = 450 \text{ mm}$$

$$E \times e = \frac{5 \times 75}{2 \times \epsilon} \times \left(\frac{2 \times 450 - 75}{450 - 75}\right) = \frac{406}{\epsilon} \frac{\text{kg}}{\text{cm}^2} \times \text{mm}$$

The tensile stiffness depends on the strain.

We use the design strain $\epsilon = 0.001$ to compute the original hoop tensile stiffness.

$$(E \times e)_0 = \frac{406}{0.001} \frac{\text{kg}}{\text{cm}^2} \times \text{mm}$$

$$(E \times e)_0 = 406\,000 \frac{\text{kg}}{\text{cm}^2} \times \text{mm}$$

We use the pristine infiltration threshold $T_i = 0.005$ to compute the critical infiltration stiffness.

$$(E \times e)_{cri}^{infilt} = \frac{406}{0.005} \frac{\text{kg}}{\text{cm}^2} \times \text{mm}$$

$$(E \times e)_{cri}^{infilt} = 82\,500 \frac{\text{kg}}{\text{cm}^2} \cdot \text{mm}$$

We compute the critical weep tensile stiffness in the same way, entering the pristine weep threshold $T_w = 0.011$ in the general equation

$$(E \times e)_{cri}^{weep} = \frac{406}{0.011} \frac{\text{kg}}{\text{cm}^2} \times \text{mm}$$

$$(E \times e)_{cri}^{weep} = 37\,000 \frac{\text{kg}}{\text{cm}^2} \cdot \text{mm}$$

The same reasoning applies to the critical rupture stiffness, entering the pristine rupture threshold $T_r = 2.5\%$ in the general equation.

$$(E \times e)_{cri}^{rup} = \frac{406}{0.025} \frac{\text{kg}}{\text{cm}^2} \times \text{mm}$$

$$(E \times e)_{cri}^{rup} = 16\,500 \frac{\text{kg}}{\text{cm}^2} \cdot \text{mm}$$

We have just computed the critical tensile stiffness of the elbow for the three relevant modes of failure. In doing so, we assumed the pristine values of the failure thresholds. This assumption implies in allocating the full chemical damage to the modulus.

We have all information needed to compute the durability for each mode of failure.

$$\log \left(\begin{array}{c} \text{durability for} \\ \text{rupture failure} \end{array} \right) = \frac{1}{C} \times \log \left[\frac{(E \times e)_{cri}^{rup}}{(E \times e)_0} \right]$$

$$\log \left(\begin{array}{c} \text{durability for} \\ \text{rupture failure} \end{array} \right) = \frac{-1}{0.59} \times \log \left[\frac{16\,500}{406\,000} \right]$$

$$\left(\begin{array}{c} \text{durability for} \\ \text{rupture failure} \end{array} \right) = 228 \text{ months (19 years)}$$

$$\log \left(\begin{array}{c} \text{durability for} \\ \text{weep failure} \end{array} \right) = \frac{1}{C} \times \log \left[\frac{(E \times e)_{cri}^{weep}}{(E \times e)_0} \right]$$

$$\log \left(\begin{array}{c} \text{durability for} \\ \text{weep failure} \end{array} \right) = \frac{-1}{0.59} \times \log \left[\frac{37\,000}{406\,000} \right]$$

$$\left(\begin{array}{c} \text{durability for} \\ \text{weep failure} \end{array} \right) = 58 \text{ months (4.8 years)}$$

$$\log \left(\begin{array}{c} \text{durability for} \\ \text{infiltration failure} \end{array} \right) = \frac{1}{C} \times \log \left[\frac{(E \times e)_{cri}^{infi}}{(E \times e)_0} \right]$$

$$\log \left(\begin{array}{c} \text{durability for} \\ \text{infiltration failure} \end{array} \right) = \frac{-1}{0.59} \times \log \left[\frac{82\,500}{406\,000} \right]$$

$$\left(\begin{array}{c} \text{durability for} \\ \text{infiltration failure} \end{array} \right) = 15 \text{ months (1.2 years)}$$

Table 2 summarizes these results

Mode of failure	Critical stiffness (kg/cm ²) × mm	Durability (months)
Infiltration	82 500	15
Weeping	37 000	58
Rupture	16 500	228
Initial stiffness	406 000	-----

Table 2
Critical stiffness and durability for each mode of failure

Our computation indicates the onset of infiltration at 15 months, with weep failure starting at 58 months. Rupture occurs, as stated in problem, at 228 months. The present analysis shows a very small durability for infiltration failure. This is not correct, as explained in the next numerical example.

Note: The straight pipe segments in the present pipeline were not subject to abrasive action and have significantly longer chemical life. To confirm this, we can measure their residual tensile stiffness to compute the parameter C , as explained in the part 2 of this chapter.

Example 2. We will revisit the previous example, assuming likely values for the parameter C .

The parameter C changes smoothly along the time in service, assuming higher values prior to infiltration, medium values between infiltration and weep, and lower values after weep. The determination of the actual values of C requires measurements of the residual tensile modulus, as stated earlier. We did not do this.

Table 3 assumes a few hypothetical values of C .

We will use a piecemeal approach to compute the durability for each of the three critical intervals listed in table 3.

To obtain the piecemeal durability, we use the known equation

$$\log \left[\frac{\text{lower critical stiffness}}{\text{upper critical stiffness}} \right] = -C \times \log (\text{piecemeal durability})$$

$$\log (\text{piecemeal durability}) = \frac{-\log \left[\frac{\text{lower critical stiffness}}{\text{upper critical stiffness}} \right]}{C}$$

$$\log (\text{piecemeal durability for infiltration}) = \frac{-\log \left[\frac{82\,500}{406\,000} \right]}{0.308}$$

$$(\text{piecemeal durability for infiltration}) = 176 \text{ months (14.7 years)}$$

$$\log (\text{piecemeal durability for weep}) = \frac{-\log \left[\frac{37\,000}{82\,500} \right]}{0.224}$$

$$(\text{piecemeal durability for weep}) = 36 \text{ months (3.0 years)}$$

$$\log (\text{piecemeal durability for rupture}) = \frac{-\log \left[\frac{16\,500}{37\,000} \right]}{0.291}$$

$$(\text{piecemeal durability for rupture}) = 16 \text{ months (1.9 years)}$$

Critical interval	Upper and lower critical stiffness	Durability parameter	Time to failure in each interval	Accumulated durability
Start to Infiltration	$(E \times e)_0 = 406\,000$ $(E \times e)_{cri} = 82\,500$	$C = 0.308$	176 months	176 months
Infiltration to weep	$(E \times e)_i = 82\,500$ $(E \times e)_{cri} = 37\,000$	$C = 0.224$	36 months	$176 + 36 = 212 \text{ months}$
Weep to rupture	$(E \times e)_i = 37\,000$ $(E \times e)_{cri} = 16\,500$	$C = 0.291$	16 months	$212 + 16 = 228 \text{ months}$
Overall	$(E \times e)_0 = 406\,000$ $(E \times e)_{cri} = 16\,500$	$C = 0.590$	228 months	228 months

Table 3

Piecemeal durability in each interval

This numerical example shows things as they are, with infiltration dominating the chemical durability. The onset of weep occurs at 212 months, 16 months prior to rupture. We have a buffer interval of 16 months between the onset of weep and rupture.

Example 3. We end this chapter computing the design structural thickness of the elbow.

The design structural thickness of the elbow is

$$70\,000 \times e = 406\,000$$

$$e = 5.8 \text{ mm}$$

This is the design structural thickness. The total thickness is the sum of the structural thickness and the corrosion barrier

$$e = 5.8 + 2.5 = 8.3 \text{ mm}$$

The actual reported thickness (7.0 mm) is slightly less than the computed design value (8.3 mm). This small difference is not relevant.

Chapter 10

Part 4

Chemical durability of composite linings

1 – Introduction. The high tensile stiffness of steel or concrete substrates control the strains on composite linings. The constant strain precludes the use of the residual tensile stiffness to measure the durability of linings, as we did for full composites equipment. In discussing the chemical durability of linings, we take into account four facts:

- *The almost certain full saturation of the lining*
- *The constant strain on the lining, independent of its deterioration.*
- *The shielding effect, giving low chemical presence at the lining-substrate interface*
- *The linings never lose their protective function.*

The high substrate stiffness assures a constant and low strain on linings, always less than the infiltration threshold. Furthermore, the shielding effect assures a low chemical presence in the lining itself as well as at the surface of the substrate. As long as the lining stays in place, the only concern is the corrosive effect from the low concentration aggressive chemical at the substrate surface.

From the discussions in the part 1 of this chapter, we can expect different behavior from linings, depending on the reactivity of the stored chemical.

<i>Aggressive chemical</i>	<i>Mild chemical</i>
<i>Long time to reach the substrate</i>	<i>Short time to reach the substrate</i>
<i>Flat advancing front</i>	<i>Diffuse advancing front</i>
<i>Substantial damage to the substrate</i>	<i>Mild damage do the substrate</i>

Highly aggressive chemicals may be harmful to substrates, even in the low concentrations resulting from the shielding effect. The durability in such cases is the time for the chemical to traverse the lining and do harm to the substrate. We can compute this time – the durability – as we did for full composite equipment, using the penetrated depth in place of the residual tensile stiffness.

$$\log(e) = C \times \log(\text{durability})$$

Where

e is the lining thickness.

C is an experimental parameter

This equation is useful only in highly aggressive chemicals, with flat and easy to measure advancing fronts. The diffuse fronts observed in mild chemicals defeats all attempts to estimate the durability of linings. Besides, the times to reach the substrates are short in mild chemicals. The thing to do in linings, regardless of chemical activity, is observe and monitor the substrate deterioration.

2 – Chemical presence in plies. Figure 1 shows the concentration profile of sulfuric acid in a fully penetrated protective lining on a steel substrate. The acid concentration in the outside solution is $C_0 = 25\%$ and the operating temperature is $OT = 100^\circ\text{C}$. The acid ions slowly diffuse from the solution into the lining, eventually reaching the steel substrate. In its advance, the acid saturates the resin matrix and fills out all laminate voids. At equilibrium, the average acid concentration in the plies of the lining is

$$\left[\begin{array}{c} \text{Acid} \\ \text{concentration} \end{array} \right] = \frac{\Sigma(\text{Acid accumulated in voids} + \text{acid taken up by the resin})}{\text{volume}}$$

Where

$$\text{Voids} = \text{air voids} + \text{resin micro-cracks} + \text{resin-fiber debonds}$$

At equilibrium, the outside solution has the same concentration as the solution in the voids.

$$C = C_0 \times (AV + MC + DB) + S \times C_0 \times R$$

Where

C is the average acid concentration in any chosen ply

C₀ is the acid concentration in the outside solution

S is the acid solubility in the resin

AV is volume fraction of air voids in the ply

MC is the volume fraction of micro-cracks in the ply

DB is the volume fraction of debonds in the ply

R is the volume fraction of resin in the ply

This equation allows the computation of the average acid concentration in each ply.

Example 1 – Explain the data reported in figure 1 for each ply of the lining of brittle polyester bisphenol-A fumarate immersed in 25% sulfuric acid at 100 °C.

We use the equation just derived.

$$C = C_0 \times (AV + MC + DB) + S \times C_0 \times R$$

For the liner

C = 0.91% is the reported acid concentration in the liner

C₀ = 25% is the acid concentration in the outside solution

AV = 0, since the volume fraction of air voids in neat resins is negligibly small.

MC = 0, since no liner cracking was reported

DB = 0, since neat resins have no fibers

R = 1.0 is the volume fraction of neat resins

Entering the above in the equation, we have

$$0.91 = 25 \times (0 + 0 + 0) + S \times 25 \times 1$$

From which we have

$$S = 0.037 \text{ (3.7\%)}$$

This is the solubility of sulfuric acid in the tested resin at 100 C

For the corrosion barrier of chopped fibers

C = 4.88% is the reported acid concentration in the corrosion barrier

Co = 25% is the acid concentration in the outside solution

AV = 0, since the volume fraction of air voids in well-consolidate plies is too small.

MC = unknown. Should not be zero in plies of chopped fibers at 100 C

DB = unknown

R = 0.84 is the volume fraction of resin in plies of chopped fibers

S = 0.037 is the acid solubility in the resin at 100 C

Entering the above in the general equation

$$4.88 = 25 \times (0 + MC + DB) + 0.037 \times 25 \times 0.84$$

From which we have

$$MC + DB = 0.164 = 16.4\%$$

This is the combined volume fraction of the resin micro-cracks and the resin-glass debonds in chopped fibers of bisphenol-A fumarate at 100C. Such a high volume fraction indicates that brittle bisphenol-A fumarate resins may not be adequate for use in 25% sulfuric acid solutions at 100C.

For the primer of neat resin at the substrate surface

The thin primer of neat resin at the substrate surface behaves exactly as the liner. This indicates a very low acid concentration at the substrate.

The important message from figure 1 is the significant depletion of the acid presence at the substrate, vis-à-vis its concentration in the outside solution. For the numerical example herein discussed, the lining barrier depresses the outside acid concentration from $Co = 25.0\%$ to a mere 0.91% at the surface of the substrate. This reduction in the concentration of aggressive chemicals at the substrate is the ultimate justification for the protective action of paints and linings.

As for other protective actions of linings, we repeat what we have already said in the part 1 of this chapter 10

- 1. The resin-chemical interaction slows down the rate of penetration. Other things being equal, the higher the reactivity of the chemical, the lower is the rate of advance.*
- 2. The presence of a resin-rich liner has no effect on the concentration of chemicals in the subsequent layers. The only benefit from the liner is the retardation of the saturation. For further details, see the work of Regester in the appendix 1 of this book.*

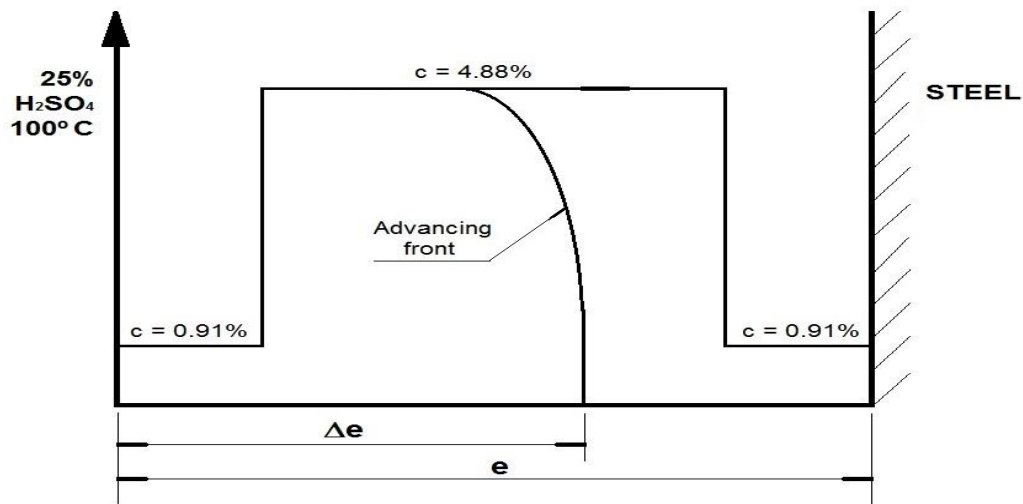


Figure 1

The 25% outside solution penetrates and eventually saturates the entire lining. The figure shows two stages of acid advance, before and after reaching the substrate. The chemical concentration in the lining depends on the void content in each ply, which explains the low ($c = 0.91\%$) value for the void-free liner of neat resin, vis-à-vis the high ($c = 4.88\%$) value for the plies of chopped fibers. The neat resin primer in contact with the substrate depresses the chemical concentration there. Data from Regester. See the appendix 1 of this book

2 – Chemical durability of linings. Given enough time, the aggressive chemical will reach the substrate and saturate the entire lining. This event may or may not define the lifespan of the lining. Let us explain.

In mild chemicals, the shielding effect may depress its presence to values low enough as to give full protection to the substrate. This favorable situation explains the long durability of paints and thin protective linings in mild environments. We should keep in mind the long protection by thin paintings in a variety of mild chemicals.

However, in highly aggressive environments, the depressed chemical concentration may still be too high for the substrate. In such cases, full penetration marks the end of the lining life. The maintenance crew takes immediate action to replace the compromised lining before the aggressive chemical harms the substrate.

Example 2. Compare the chemical durability of linings with the durability of full composite equipment.

We assume two scenarios:

Scenario 1. The shielding effect depresses the chemical concentration to values low enough to do protect the substrate. In such cases, the durability of the liner may be very long, longer than that of the full composite equipment.

Scenario 2. The chemical depression from the shielding effect is not enough to assure adequate protection to the substrate. In such cases, the lining durability depends on its thickness, being usually less than that of full composites equipment. Furthermore, we should also take into account the difficulty to make quality linings in the field.

The above example indicates that the durability of linings may be a concern in the presence of aggressive environments. To compute the chemical durability in such cases, we use the same equation derived earlier for full composite equipment, replacing the penetrated depth for the loss of stiffness.

$$\log(\Delta e) = C \times \log(\text{inspection time})$$

Where Δe is the penetrated depth and C is an experimental parameter. As explained in the earlier parts of this chapter, C is not constant.

The durability measure the time to full penetration of the lining

$$\log(e) = C \times \log(\text{durability})$$

$$\log(\text{durability}) = \frac{1}{C} \times \log(e)$$

$$\log(\text{durability}) = \log(\text{inspection time}) \times \frac{\log(e)}{\log(\Delta e)}$$

This equation applies to stable environments, at constant temperature and chemical concentration. The penetrated depth derives from measurements taken at the inspection time.

3 – Measuring the penetrated depth in linings. The measurement of the penetrated depth is not always possible and, when possible, not reliable. Any attempt to make measurements of the penetrated depth in laminates is inherently inaccurate. The best way to measure penetrated depths in linings is on replica linings immersed in the environment for that purpose. In the absence of such replicas, we can apply the unreliable adherence test on the equipment itself, as escribed in the sequence.

- *Grind the lining surface to remove all visually penetrated material.*
- *The grinding stops on reaching apparently sound, non-penetrated material.*
- *The first adherence test will most likely fail, indicting the presence of contaminants at the tested depth.*
- *The grinding and adherence test routine is repeated.*
- *This routine stops on reaching contamination-free material*
- *The first layer of contamination-free material defines the penetrated depth Δe .*

With penetrated depth, we immediately compute the durability

The residual life comes from discounting the time served from the computed durability

$$\text{residual life} = (\text{durability}) - (\text{time in service})$$

Example 3. Determine the residual life of a lining at 10 years of service. The following information is available.

<i>Inspection time</i>	<i>Penetrated depth</i>
<i>120 months (10 years)</i>	<i>$\Delta e = 3.0 \text{ mm}$</i>
<i>The original lining thickness is $e = 3.5 \text{ mm}$</i>	

We use the durability equation

$$\log(\text{durability}) = \log(\text{inspection time}) \times \frac{\log(e)}{\log(\Delta e)}$$

$$\log(\text{durability}) = \log(120) \times \frac{\log(3.5)}{\log(3.0)}$$

$$(\text{durability}) = 235 \text{ months (19 years)}$$

The residual life after 10 years of continuous service is

$$\text{residual life} = (19) - (10) = 9 \text{ years}$$

We close this chapter with two numerical examples.

Example 4 – *Explain the effect of thickness in the durability of composite linings.*

We make the following assumptions:

- *All chemicals eventually penetrate and saturate the lining*
- *The shielding effect reduces the chemical concentration at the substrate surface*
- *The aggressive chemical loses strength over time*

These assumptions define the thickness of linings:

The low strength and small concentration of mild chemicals will most likely have no effect on the substrate, regardless of the lining thickness. This explains the excellent protective history of thin film paints in mild environments.

Aggressive chemicals, however, despite their low strength and small concentration, may still cause damage to substrates. In aggressive chemical service, the lining thickness defines the chemical durability.

The following table lists the above conclusions.

<i>Chemical aggressiveness</i>	<i>Lining thickness</i>
<i>Very aggressive</i>	<i>High above 5.0 mm</i>
<i>Mildly aggressive</i>	<i>Low Below 3.0 mm</i>
<i>The computation of the chemical life of linings is feasible only in highly aggressive chemicals, those with neat and flat advancing fronts. The slow advance of aggressive chemicals gives excellent durability to thick composite linings.</i> <i>The less aggressive chemicals display diffuse advancing fronts not amenable to measurement. In such cases, we monitor the substrate to observe if the shielding effect assures enough protection. Often times it does, since mild chemicals in low concentrations and activities are not harmful.</i>	

Example 5 – *Describe the maintenance procedures in composite linings.*

The discussion in this chapter covered the topic of durability, not maintenance of linings. My recommendation for maintenance is to fully remove and replace old linings.

Appendix 1

Damages and manufacturing defects in resin-rich liners

The protection afforded by resin-rich liners in highly aggressive chemicals is at best temporary. It is true that intact liners retard the advance of chemicals in laminates. However, it is also true that they crack in the presence of aggressive chemicals. Furthermore, as shown by Regester in the appendix 1 of this book, the liner presence does not depress the concentration of aggressive chemicals in laminates, therefore having little effect on its chemical durability.

In conclusion, although potentially good to extend the chemical life of laminates, the resin-rich liners are not an overriding necessity. Given this background, we ask what to make of damaged or defective resin-rich liners. Should such damages and defects be repaired?

The most common manufacturing defects and damages in newly manufactured resin-rich liners are:

- *Cuts, gouges, foreign contamination, voids, pits, entrapped air, abrasion and porosity. These defects are restricted to the liner itself and do not penetrate the fiber reinforced corrosion barrier. They have little effect on the chemical durability and need no repair.*
- *The only liner defects requiring attention are hairline cracks from impact, or excessive deformation. These liner cracks – in fact any crack – need repair, lest they seed the onset of strain-corrosion. For details, see chapter 11.*

We can safely ignore all liner defects from entrapped air, encapsulated insects or other foreign objects like debris, droppings from ceilings and loosened bristles from laminating brushes. Likewise, we can also ignore damages like blisters and manufacturing flaws such as pits, voids, porosity and others. These liner defects and flaws are not overly detrimental to the chemical durability of the equipment. In fact, we can safely ignore all damages and manufacturing defects limited to the liner itself, and not penetrating the corrosion barrier. Such defects pose no threat.

However, the presence of hairline cracks in liners may be dangerous. Malmo (ref. 9), Norwood and Millman (ref. 5) investigated the effect of cracked gelcoats on substrates of chopped fibers. Although directed to gelcoats, the work developed by these investigators is fully applicable to liners as well. The following statement captures the essence of their work.

All hairline cracks observed in liners, regardless of cause – excessive deformation, cyclic or impact loads – have their origin in the underlying corrosion barrier of chopped fibers. The small and imperceptible damages in the corrosion barrier transfer to the unreinforced liner as visible hairline cracks. The presence of hairline cracks in liners can potentially seed a strain-corrosion situation.

Going one-step further, the presence of hairline cracks in liners could be early warnings of potentially dangerous strain-corrosion situations. For details, see chapter 11. The use of dye penetrants helps the identification of hairline cracks in the liner that, in my view, require repair.

In conclusion, hairline cracks are the only liner damages in need of repair. All other damages and defects in liners are tolerable.



The penetrant dye on the left indicates the presence of impact hairline cracks. The moderate hairline cracking on the right are probably due to excessive deformation. Both cases need intervention to eliminate the risk of strain-corrosion.

Appendix 2

Damages and manufacturing defects in corrosion barriers

We proceed to discuss damages and manufacturing defects in corrosion barriers. The most common manufacturing defects observed in newly fabricated corrosion barriers are dry spots, entrapped foreign materials and voids.

Dry spots and entrapped foreign materials are easy to spot and correct. As a rule, the manufacturers identify and eliminate all these gross defects before presenting their equipment for inspection. Dry spots and entrapped foreign materials should not be a matter of great concern to inspectors

The air voids observed in corrosion barriers start as small bubbles of entrapped air that flatten when rolled out in the manufacturing process. As they flatten out, these air bubbles change into disc-shaped flaws parallel to the equipment surface. The large diameter of these flat flaws make them look scary and ominous when observed broadside. However, when observed edgewise they are in fact extremely thin and not scary at all. These flat air occlusions are similar to delamination, large in area and small in thickness. Both defects – flat air voids and delamination – have a disc-shaped geometry, run parallel to the equipment surface and are extremely thin.

Let us see just how dangerous these flat air occlusions really are.

From the discussions in this chapter, we recall that the chemical durability of exposed equipment is the time for the aggressive chemicals to penetrate and deteriorate the stiffness of their laminates. Suppose now the presence of a flat void in the path of the advancing chemical. The penetrating chemicals reach this defect at the same time that they reach all parts of the corrosion barrier located at the same depth. The thin flat void offers no resistance to the advancing chemical that immediately jumps ahead by the extremely small quantity corresponding to its thickness. From this, we have a conclusion:

Air voids reduce the effective thickness of the corrosion barrier by a quantity equal to their own thicknesses. Therefore, the extremely thin occlusions of entrapped air have a negligible effect on the chemical durability.

In addition to the above, the voids may also provide sites for the development of ugly and frightful osmotic blisters in linings and corrosion barriers. These blisters, however, affect the already penetrated material, with no effect on the plies ahead. The osmotic blisters and penetrated depths are past events with no effect on the future residual life.

In conclusion, the air voids present in corrosion barriers take the form of extremely thin air bubbles with a small detrimental effect on durability. Although ugly and scary, they are acceptable and require no repair.

Example 5. *Let us consider the entrapment of a large air bubble in a 2.0 mm corrosion barrier. The rolling out process flattens the air bubble into a thin penny-shaped air void 0.10 mm thick. The presence of this void reduces in $0.10/2.00 = 5\%$ the effective thickness of the corrosion barrier. This reduction is negligibly small.*

The presence of voids (air bubbles) are not overly detrimental to the chemical durability of composites equipment. The exception to this statement is the anomalous failure in delaminations, discussed in chapter 18.

The reader should compare the discussion in this appendix with the maximum allowable sizes of defects in liners and corrosion barriers mandated by ASTM D 2567. This standard limits the maximum allowable size of “spherical” voids to 3.0 mm and a maximum density of 4 voids/in². To begin with, the air bubbles in corrosion barriers are flat, never spherical. Besides, it is hard to imagine 3.0 mm spherical voids fitting in 2.0 mm thick corrosion barriers. The important feature in assessing air voids is their thickness, not their diameters.

In closing, the reader will keep in mind that composite laminates do not develop large cracks. These damages are typical of homogeneous materials, like resin-rich liners, never in fiber-reinforced corrosion barriers.

Appendix 3

The ASTM C 581 test protocol for chemical suitability

In this chapter, we introduced a quantitative method to compute the chemical durability of composite equipment. This approach is a major advance from the old qualitative ASTM C 581 test for the chemical suitability of corrosion barriers. The major differences between the two methods are:

- *The new method we have just presented quantifies the chemical durability from measurements of the tensile stiffness of the equipment exposed to real operating conditions.*
- *The old ASTM method establishes chemical suitability by immersing coupons of chopped strand in representative lab solutions. The qualitative determination of suitability comes from classical indicators of deterioration, such as Barcol hardness, visual appearance and retention of flexural modulus*

Table 1 summarizes the differences.

	<i>ASTM C 581 test method</i>	<i>Proposed new method</i>
<i>Scope</i>	<i>Evaluates suitability</i>	<i>Predicts durability</i>
<i>Criteria</i>	<i>Hardness retention Coupon appearance. Medium appearance Retention of flexural modulus</i>	<i>Failure defined by the aged infiltration threshold</i>
<i>Coupons</i>	<i>Two 450 g/m² chopped glass mats with resin-rich liner on both surfaces.</i>	<i>Tests performed on the equipment itself</i>
<i>Test medium</i>	<i>Lab solutions representative of operating conditions</i>	<i>Real operating conditions</i>
<i>Procedure</i>	<i>Coupons immersed in representative lab solutions. Specimens tested for flexural modulus at regular intervals.</i>	<i>Residual tensile stiffness measured at two exposure times.</i>
<i>Calculation</i>	<i>Regression line of hardness and flexural modulus as a function of time</i>	<i>Corrosion parameters K and C computed from measurements of the residual tensile modulus at two different times</i>
<i>Interpretation</i>	<i>Laminate is suitable if flexural modulus and hardness extrapolate to acceptable values Other indicators are the visual appearances of the coupons and the medium.</i>	<i>The chemical durability is computed from the equations $\log(\Delta e) = K + C \times \log(\text{durability})$ $\log(\Delta S) = K + C \times \log(\text{durability})$ Visual appearances and hardness retention are not relevant.</i>

Table 1

Comparison of the two methods used to evaluate the chemical performance of corrosion barriers, linings and full composite equipment in highly aggressive chemicals. The classical ASTM C 581 method is an indicator of suitability – not a predictor of durability.

Chapter 11

Strain-corrosion failure

1 – Introduction: There are three active strain-corrosion processes in composite laminates, all leading to long-term damage of strained resins or fibers in the presence of chemicals.

- **Global strain-corrosion** from water attack on all fibers in the entire laminate. The deterioration process starts at fiber defects tensioned in the presence of water, creating many fiber breaks. The fiber breaks eventually coalesce to rupture the laminate with no visible display of deterioration.
- **Local strain-corrosion** from chemical attack on a few tensioned fibers in small patches of the laminate. The deterioration starts in the fibers, evolving into a single, long and clearly visible crack that eventually fails the laminate. The chemicals involved in the fiber deterioration usually do not affect the resin, leaving the surrounding laminate intact.
- **Resin strain-corrosion** is not relevant in the study of durability of composite equipment, since resin cracks do not grow in laminates. The notorious and structurally harmless mud-cracks observed in exposed resin-rich liners is an example of resin strain-corrosion. We will have nothing to say in this book about these irrelevant environmental stress-cracks.

The strain-corrosion rupture failures observed in laminates are associated with fiber, not resin, deterioration. The global strain-corrosion from water attack is the topic of chapter 15. In this chapter, we discuss the local strain-corrosion failure from chemicals other than water.

2 – The mechanism of local strain-corrosion: There are three processes to explain the penetration of chemicals in sound laminates.

- Below the infiltration threshold, we have a slow diffusion-controlled penetration process described in chapter 10. The chemical lives of equipment strained below the infiltration threshold are very large.
- Above the infiltration threshold, the large crack densities causes fast penetration. The chemical lives of equipment strained above the infiltration threshold are short.
- Above the weep threshold, the extra-large crack density allows the immediate chemical inundation of the laminate. The chemical lives of equipment strained above the weep threshold are very short.

The above scenarios, involving the entire laminate, leads to chemical failures from loss of tensile modulus and deterioration of the infiltration threshold, as discussed in chapter 10. In this chapter, we will discuss the local chemical attack on strained fibers at laminate flaws. The strained fibers at laminate defects provide the sites for the initiation of the local strain-corrosion process.

The local strain-corrosion mechanism is like follows:

Imagine a tensioned laminate with a small defect under its surface. The aggressive chemical diffuses through the thin surface skin to encounter the strained fibers at the defect. The combination of strain and chemical presence quickly destroys the fibers, producing many micro-cracks that eventually merge to grow into a single dominant large crack.

The chemical attack takes place on the strained fibers, not necessarily on the resin. The fiber failure forms a large crack that grows perpendicular to the laminate and transverse to the strain direction. The crack grows across the laminate, cutting the fibers, leaving behind two smooth surfaces. The smooth surfaces of strain-corrosion cracks are in sharp contrast to the jagged, bushy appearance observed in the normal rupture of laminates. The process continues as described, with the corroded fibers breaking in sequence to grow a large and deep crack. Eventually the remaining thickness cannot sustain the external load and we have rupture of the laminate.

The remaining laminate is not strain-corroded and ruptures in the normal way, forming the usual jagged and bushy fibers. The strain-corrosion ruptures of laminates are easy to identify. Their distinctive features are

- *Long and deep cracks*
- *Cutting across the laminate*
- *Smooth surfaces*
- *Absence of jagged or bushy fibers.*

The above is a fair description of the process leading to local strain-corrosion of laminates. The fiber degradation mechanism described here also applies to global strain-corrosion, with water in place of the aggressive chemical.

The differences between global and local strain-corrosion are:

- *Global strain-corrosion develops no visible cracks. By contrast, local strain-corrosion develops a single, smooth, deep and large crack in the laminate.*
- *Global strain-corrosion results from water attack on all fibers. Local strain-corrosion comes from chemical attack on a few localized fibers*
- *Local strain-corrosion occurs mostly in the presence of bending loads*

In all cases, the cracks grow in response to the combined action of strains and fiber corrosion.



The presence of fibers does not allow resin cracks to grow in laminates. The strain-corrosion of resin matrices is, therefore, not a concern. In extremely aggressive chemicals, the strain-corrosion of resin-rich liners causes the so-called “environmental stress- cracking”.

3 – Sudden in-service ruptures. The composite effect denies the possibility of laminate cracks growing across fibers. This extremely important effect explains the outstanding fatigue and impact resistance of laminates. Now, here comes the twist. In the strain-corrosion process just described, we have the cracks growing across the fibers. The explanation for this contradiction is as follows.

The cracks in the local strain-corrosion process grow in response to fiber ruptures from chemical action. It is as if a chemical razor cut the fibers. The growing crack exposes fresh fibers that hold the growth for a while until they too strain-corrode and fail. This unchecked process develops large cracks perpendicular to the laminate, cutting through the fibers in apparent violation of the fundamental composites effect. What happens then to the outstanding toughness and fatigue resistance of laminates? Well.... these outstanding features simply cease to exist in the neighborhood of strain-corrosion cracks. This leads to the following statement:

Most in-service sudden ruptures of composite structures have their origin in local strain-corrosion.

Excepting gross operational errors and other rare accidents, we can safely assume all cases of premature in-service sudden laminate ruptures resulting from local strain-corrosion

4 – Local strain-corrosion and bending loads: The preceding discussions explained the strain-corrosion process to the simultaneous action of aggressive chemicals and tensile strains. We made no mention of the all-important bending loads and their central role in the local strain-corrosion process.

Let us explain why bending loads are so special in the process of local strain-corrosion.

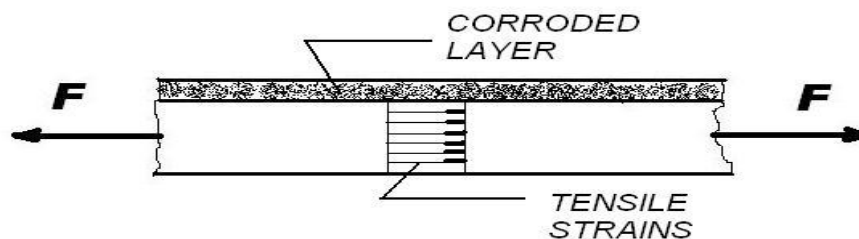


Figure 1A

Under tensile loads, the chemical solution causes a uniform degradation that slowly advance in the laminate. The tensile strains have a uniform distribution in the entire laminate.

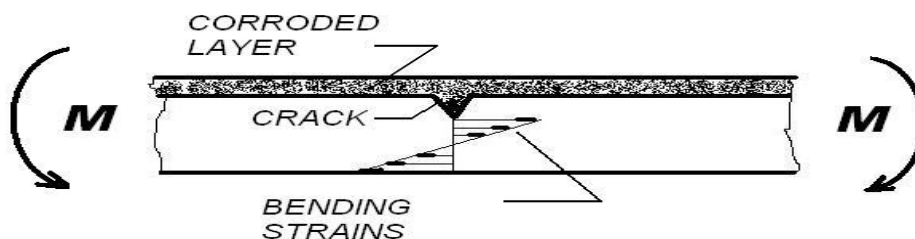


Figure 1B

The outside solution causes a uniform degradation of the exposed surface. However, unlike in the tensile case, the bending strains do not have a uniform distribution in the laminate. The large bending strain near the laminate surface accelerates the strain-corrosion process there.

Figure 1A shows the depth penetrated by aggressive chemicals in tensile loaded laminates. The uniform strain distribution causes laminate failure by chemical attack, as explained in chapter 10.

Figure 1B shows the same laminate under a bending load. We identify two particulars in bent laminates.

- *The bending strains are larger at the exposed surface than at the center of the laminate. The large bending strain starts the strain-corrosion process.*
- *The bending strains grow larger as the crack deepens, eventually rupturing the laminate.*

The presence of large bending strains at the laminate surface triggers the initiation of many small cracks. A dominant crack soon emerges, starting the local strain-corrosion process. It can be shown mathematically (see any book on fracture mechanics) that bending loads give higher strain intensity factors in surface cracks than tensile loads. As an example, when asked to break a pencil, we instinctively do it in bending, not in tension. The strain-corrosion phenomenon is prevalent in bending loads and, as a rule, does not happen in tensile loads.

There is, however, one rare instance of local strain-corrosion associated with tensile loads. Such a situation occurs in the presence of exceptionally large pre-existing surface cracks running perpendicular to the strain direction. This scenario favors local crack growth by tensile strain-corrosion, as in the bending case discussed above. See figure 2.

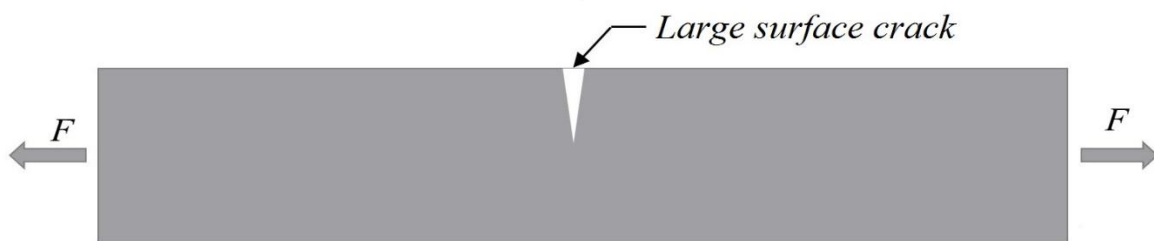


Figure 2

Tensile strain-corrosion develops in the presence of large pre-existing surface cracks transverse to the strain direction. That is the reason for not allowing hairline cracks in liners.

Example 1 – Explain the differences between tensile and bending local strain-corrosion.

The small surface flaws seeding local strain-corrosion usually hide beneath healthy liners, being difficult to spot. The bending strain-corrosion process starts underneath healthy liners, in the absence of visible flaws. The three requirements to get the bending strain-corrosion process going are

- *Presence of a flaw beneath the laminate surface*
- *A bending strain at the flaw*
- *Presence of an aggressive chemical.*

The absence of visible surface flaws makes the early detection of bending strain-corrosion practically impossible. The best strategy to minimize the risk of strain-corrosion is to increase the laminate thickness in those areas subjected to high bending moments. This is the procedure followed in product standards like ASME RTP1, which demand increased thickness in the knuckle transition of storage tanks.

By contrast, the tensile strain-corrosion requires the presence of large surface cracks clearly visible to the inspector.

Summarizing this section, the local strain-corrosion process requires the presence of both aggressive chemicals and bending loads. One important practical application involving local

strain-corrosion is underground sanitation pipes. Most of the discussions that follow will center on the local strain-corrosion of underground sanitation pipes.

5 – Strain-corrosion cracks: The strain-corrosion cracks start as invisible flaws that grow under bending loads in the presence of chemicals. These cracks are usually long and deep, with smooth surfaces cutting across the fibers and perpendicular to the laminate. The laminate failure develops in two stages. In the first stage, the crack growth slowly reduces the local load-bearing cross-section of the laminate. In the final stage, the remaining cross-section ruptures in the usual way, forming jagged and bushy surfaces.

The strain-corroded fibers fail by chemical action, not from mechanical stresses. The aggressive chemical cuts the fibers locally, like a sharp razor, forming smooth crack surfaces with no jagged, frayed and bushy fibers. Furthermore, the failed laminate retains a pristine look, indicating absence of chemical attack on the resin. The unexperienced inspector sees in front of him a failed laminate with a long and deep, smooth-walled crack, surrounded by a perfectly healthy laminate. The pristine look of the resin shows no sign of chemical attack. Where did such a large crack come from? What caused it? This puzzling and mysterious crack is impossible to explain by the uninitiated. The unexperienced inspector may hack his brain and never figure out an explanation. See figure 3a.

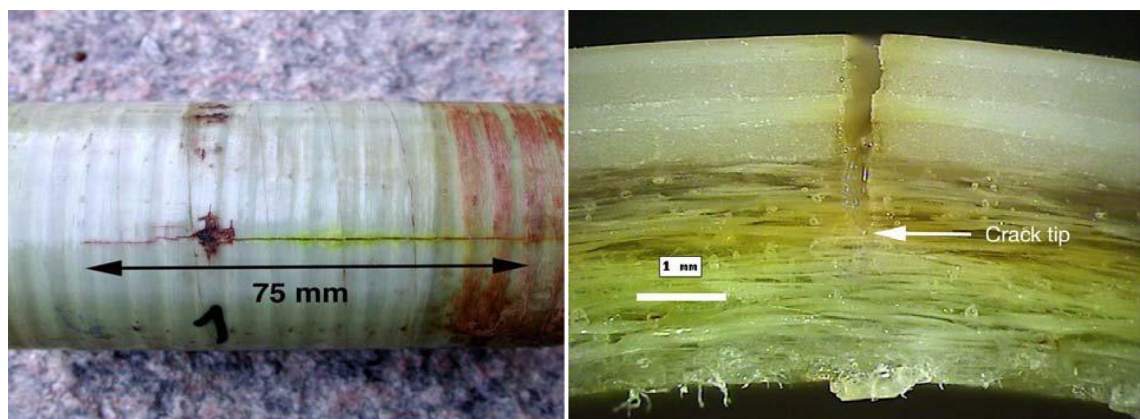


Figure 3a

Local strain-corrosion. The inspector sees a long, deep and smooth-walled crack surrounded by a perfectly healthy laminate. The pristine look of the resin shows no sign of chemical attack. Where did such a large crack come from?

Example 2 – Figure 3b shows a piece of pipe failed by strain-corrosion in a pulp bleaching plant while carrying a solution of acidic chlorine. The chlorine does not react with the glass fibers and therefore cannot cause rupture by strain-corrosion. The acid, however, quickly corrodes glass fibers. The pipeline installation and operation followed strict and correct procedures. Still, one – just one – short pipe segment failed forming the large and ugly longitudinal crack shown in figure 3b. How can we explain such a thing?

This case is a rare instance of tensile strain-corrosion failure. The corrosion process started in hairline liner cracks that locally magnified the tensile strain, setting the stage for the acidic strain-corrosion of the fibers.

The rupture failure evolved like this.

1. *The surface mud-cracks observed on the liner result from the chlorine attack on the resin. This is a normal and expected situation, with nothing to do with the problem at hand.*

2. *The presence of hairline cracks in the liner locally magnified the tensile strain field.*
3. *The acidic solution attacked the strained fibers at the crack tip, starting the strain-corrosion process.*
4. *The crack grew from the inside of the pipe, locally reducing the load-bearing wall thickness.*
5. *Eventually the operating pressure failed the thin remaining wall thickness*
6. *The remaining outer plies of the pipe, not in contact with the acid, broke in the usual way, showing the typical jagged and bushy fibers.*
7. *The chlorine solution had no part on the strain-corrosion process. The acid is the sole culprit.*

The chemicals in this case are not overly aggressive to the resin. If the strain-corrosion process had not developed, this failed pipe would have had an extremely long chemical life.



Figure 3b

Illustrating the tensile strain-corrosion crack in the pipe segment described in example 2. The crack shows neat and smooth surfaces, instead of the jagged, splintered and bushy fibers typical of normal ruptures. Except for some mud-cracks, the chlorine attack on the corrosion barrier is barely perceptible. The strain-corrosion process resulted from the acid attack on the fibers underneath hairline cracks.

6 – Predicting the time to local strain-corrosion rupture: The local strain-corrosion process starts when the aggressive chemical reaches a strained defect underneath the laminate surface. The time to rupture is the sum of two times, one for the chemical to reach the defect and another for the crack to grow. The following equation applies.

$$[\text{rupture time}] = [\text{penetration time}] + [\text{crack growth time}]$$

Ignoring the penetration time, we have

$$[\text{rupture time}] = [\text{crack growth time}]$$

The next logical step is to derive a static regression line to predict the strain-corrosion rupture time, exactly as we did for the creep-rupture failures in chapter 15. Unfortunately, this is not possible in this case, since the strains at the crack tips are not constant. We have a conclusion

There is no regression line to predict the local strain-corrosion rupture times. The tensile strains at the crack tips increase with the growing crack, invalidating any attempt to generate valid regression lines.

Example 2 – In chapter 15, we derive creep-rupture lines to predict global strain-corrosion failure times. How is that possible for global, and not for local, strain-corrosion?

We recall that global strain-corrosion failures come from the accumulation and coalescence of fiber ruptures in the entire laminate. In this process, the laminate modulus and strain do not change. The constant global strain allows the derivation of static regression lines. The constancy of the strain is a requirement in the derivation of static regression lines.

Example 3 – As discussed later in this chapter, there are currently two accepted ASTM test methods to develop strain-corrosion lines for deflected sanitation pipes. How could that be?

First, let us see how ASTM defines strain-corrosion failure. According to their test methods, failure is

“The sudden decrease observed in the bending stiffness of the tested pipe specimens”

This definition applies to crack initiation, not rupture. In fact, the two ASTM test methods and their corresponding regression lines, predict the **onsets of crack growth**, not the times to rupture. The two ASTM regression equations have the form.

$$\log(\epsilon) = A_{sc} - G_{sc} \times \log(\text{hour}) \quad (1)$$

Where

ϵ is the constant strain at the **onset of crack formation**

Hour is the time to the **onset of crack formation**

A_{sc} and G_{sc} are experimental parameters related to the glass resistance to chemical attack

The ASTM strain-corrosion equations predict the times to crack initiation, not the times to rupture.

7 – Strain-corrosion threshold: The computation of the load-dependent and resin-dominated equipment durability develops around three failure thresholds, namely weep, infiltration and stiffness. This raises the question....

Is there a strain-corrosion threshold as well?

The answer to this question is maybe.... The strain-corrosion ruptures come from fiber, not resin, deterioration. However, we should not lose sight of the fact that, in order to corrode the fibers, the chemicals should first penetrate the resin. This leads us to a careful examination of figure 4, which is the same figure 6 of chapter 14, repeated here for convenience.

Figure 4 shows creep-rupture lines for aggressive and mild chemicals. A close observation of figure 4 indicates that:

- Below the infiltration threshold, the strain-corrosion process is barely perceptible, even in aggressive chemicals. This is because, the small micro-crack densities below this threshold shield the underlying fibers from the full strength chemical solution
- Above the infiltration threshold, the high micro-crack densities allow partial access of the full strength chemical to the fibers, accelerating the strain-corrosion process. This results in fast strain-corrosion failures from aggressive chemicals above the infiltration threshold.
- In mild chemicals, strains above the infiltration threshold have little effect on strain-corrosion.
- Above the weep threshold, the extremely high crack densities allow inundation of the laminate with the full strength chemical, leading to fast strain-corrosion even in mild chemicals. This is a consequence of the notoriously poor chemical resistance of the glass fibers.

This leads to the following conclusions

- The infiltration threshold of the surface ply serves as a putative strain-corrosion threshold in aggressive chemical services

- *The weep threshold of the surface ply serves as a putative strain-corrosion threshold in mild chemical services*

In the sanitation industry, it is customary to designate the extrapolated long-term strain-corrosion strengths as either $\varepsilon = S_b$ (strain-corrosion in water) or $\varepsilon = S_c$ (strain-corrosion in urban sewer). From the foregoing discussion, and taking into account the mild aggressiveness of urban sewer, we have

$$S_c = T_w \quad (\text{Sewer service. } S_c \text{ is the weep threshold } T_w \text{ of the surface ply})$$

$$S_b = S_s \quad (\text{Water service. } S_b \text{ is the long-term static strength } S_s \text{ of the surface ply})$$

The equality of the long-term strain-corrosion strengths S_c (sewer service) and S_b (water service) to known failure thresholds have two important commercial consequences:

- *The pipe manufacturers of sanitation pipes no longer need to do the cumbersome, time consuming and expensive ASTM tests to measure these strengths.*
- *At the same time, these equalities establish a solid, coherent and reliable justification for the long-term strengths of underground pipes.*

Note: From figure 4, the long-term strain-corrosion strength of laminates in aggressive chemicals is the infiltration threshold of the surface ply,

$$S_c = T_i.$$

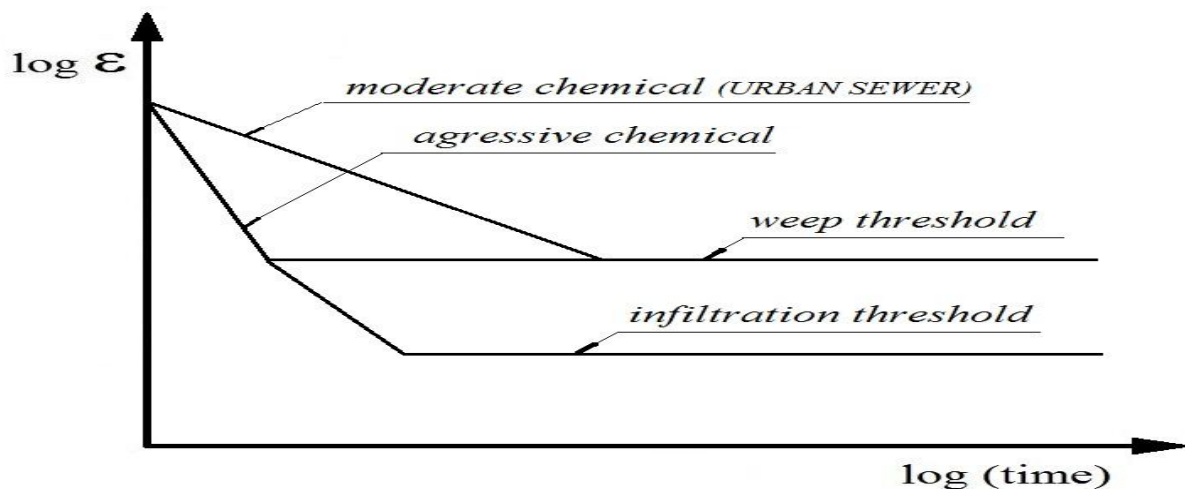


Figure 4

The creep-rupture lines discussed in the appendix 1 of chapter 14 are representative of the strain-corrosion lines. In both cases, the glass fibers fail while tensioned in the presence of chemicals. This is the same figure 6 of chapter 14.

There are many compelling evidences in support of the above conclusions. As an example, we mention the test results published by Hogni Jonsson on sanitation pipes deflected for 30 years in 5% sulfuric acid. The published results, shown in figure 5, indicate a sloped strain-corrosion line turning horizontal on meeting the weep threshold. Figure 5 certainly suggests the existence of a strain-corrosion threshold for sanitation pipes, equal to the weep threshold of their polyester corrosion barrier.

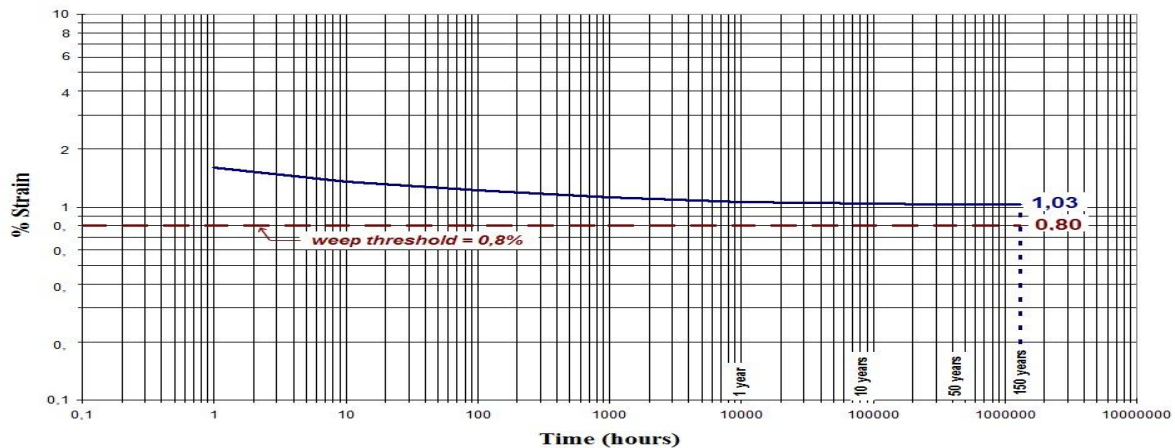


Figure 5

The long-term strain-corrosion tests of sanitation pipes in 5% sulfuric acid shows a flattening of the regression line at the weep threshold. The flattened line suggests a weep threshold of 1.03%, versus our conservative suggestion of $T_w = 0.80\%$. (Drawing by Gabriel Gonzalez).

8 – Strain-corrosion in underground pipes: Underground composite pipes find use in the transmission of non-aggressive chemicals, like urban sewer, water and a few mild industrial effluents. Fear of soil contamination precludes the use of composite underground pipes in aggressive industrial chemicals.

The critical plies and the likely locations of strain-corrosion failures in underground pipes are marked in figure 6.

- **Outer UD plies.** The outermost UD ply is strain-corroded by soil moisture at the locations marked as 1 and 3 in the figure. The long-term strain-corrosion strength of this ply is the same as the long-term tensile rupture strength S_s of UD plies immersed in water, discussed in chapter 15. In both situations, the statically tensioned UD fibers fail in the presence of water. For details, see the comments in example 3 below. As explained in the text, it is usual to refer to the long-term strength of plies in water service as S_b . Therefore, $S_b = S_s$.
- **Inner chopped fibers.** The innermost ply of chopped fibers is strain-corroded by acid (sewer service) or water (water service) or whatever mild chemical that it may carry. The corrosion process occurs at the pipe locations marked as 2 and 4 in the figure. In water service, the S_b value of the inner weep barrier is higher than that of the outer UD fibers, implying in $S_b = S_s$. In mild chemical service, however, the long-term strength of the inner ply is $S_c = T_w$.
- The situation for $S_c = T_i$ does not occur, since underground pipes never carry highly aggressive chemicals.

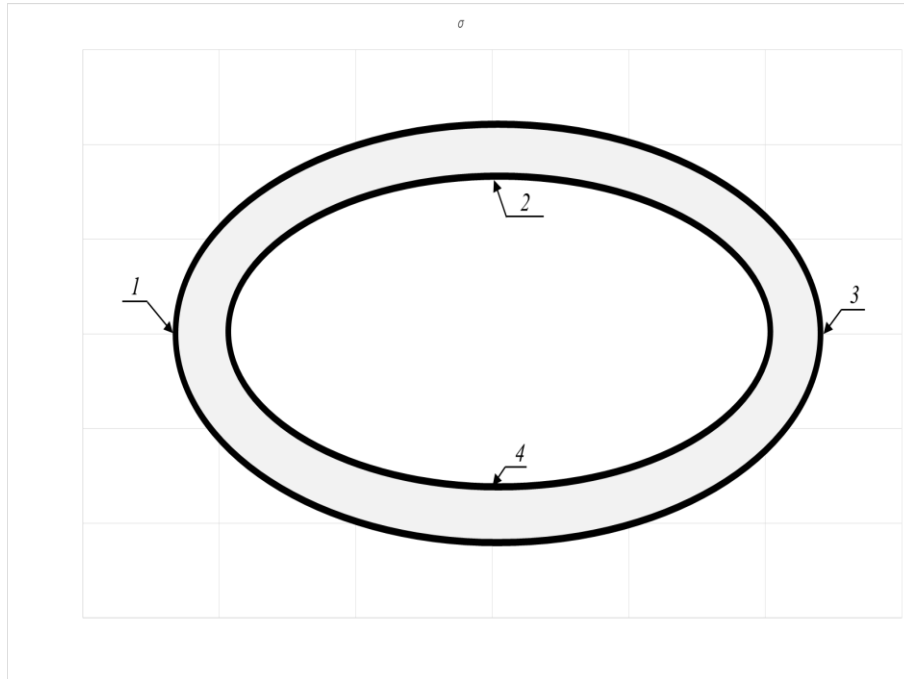


Figure 6

The maximum bending strains in deflected underground pipes occur at the indicated locations. The innermost weep barrier is tensile loaded at the points 2 and 4. The outermost UD ply is tensile loaded at the locations 1 and 3.

The following numerical example expands on figure 6.

Example 4 – Explain the significance of the two ASTM strain-corrosion tests to measure the long-term strengths of underground composite pipes.

The soil overburden deflects the underground pipes, giving rise to hoop bending strains. This condition makes the strain-corrosion failure a fundamental durability issue for underground pipes. There are two ASTM test methods designed to determine the long-term bending strengths of underground composite pipes in sanitation service.

The ASTM D 5365 test method measures the long-term bending strength of pipes deflected in water. The ASTM D 3681 test method does a similar evaluation on pipes deflected in a 5% solution of sulfuric acid.

The prediction of the long-term strengths derives from the regression lines generated from these tests. Extrapolation of the water line gives the long-term bending strength S_b of the pipe in water. Likewise, extrapolation of the acid line give the long-term bending strength S_c of the pipe in sewer service. Both methods measure the extrapolated long-term hoop strengths at room temperature.

- The ASTM D 5365 test evaluates the long-term bending strength S_b of deflected pipe specimens fully immersed in water. The full immersion implies in water contact with the inner weep barrier as well as with the outer UD fibers. Therefore, the ruptures may come from either the outer UD fibers or the inner chopped fibers. The test method does not discriminate between these two obviously different failures. The observed ruptures should most likely come from the outer UD fibers. This, however, is not certain since the test results depend on the laminate construction. The mixing of ruptures from UD and chopped fibers produce useless data. The ASTM D 5365 protocol needs modification to test separately and independently for the outer UD and the inner chopped fibers.

- The ASTM D 3681 test protocol measures the long-term bending strength S_c of pipes deflected in 5% solutions of sulfuric acid in direct contact with the liner. The situation here is similar to the one discussed in the measurement of the HDB, discussed in chapter 14. In both cases, the demands of the test protocol place the pipe specimens under strains significantly above the weep threshold. Such high strains invalidate the test, since they allow fast ingress of the full strength acid solution into the laminate. This eliminates the all-important shielding effect, generating invalid data. The extrapolated S_c

values from the ASTM D 3681 protocol are not representative of the long-term strain-corrosion strength of pipes in sewer service.



Strain-corrosion testing per ASTM D 3681, showing the deflected pipes exposed to 5% solution sulfuric acid.

Table 1 shows some published strain-corrosion parameters for sanitation pipes tested in water and in 5% sulfuric acid. All tests done according to the ASTM protocols. The experimental values of A_{sc} have been adjusted to give strains in % and failure times in hours.

$\log(\epsilon) = A_{sc} - G_{sc} \times \log(\text{hour})$	A_{sc}	G_{sc}	Bending strength at 50 years
5% H_2SO_4 @ 25°C (ASTM D 3681)	0.220	0.071	$S_c = 0.66\%$
Water @ 25°C (ASTM D 5365)	0.334	0.039	$S_b = 1.30\%$

Table 1

Static strain-corrosion parameters and 50 years bending strengths for commercial sanitation pipes. The values of A_{sc} have been adjusted to give strains in % and failure times in hours.

The regression equations corresponding to the parameters in table 1 are:

$$\log \epsilon = 0.220 - 0.071 \times \log(\text{hours}) \quad (\text{Pipes in } 5\% H_2SO_4)$$

$$\log \epsilon = 0.334 - 0.039 \times \log(\text{hours}) \quad (\text{Pipes in water})$$

Figure 7 shows plots of these two equations on log x log space.

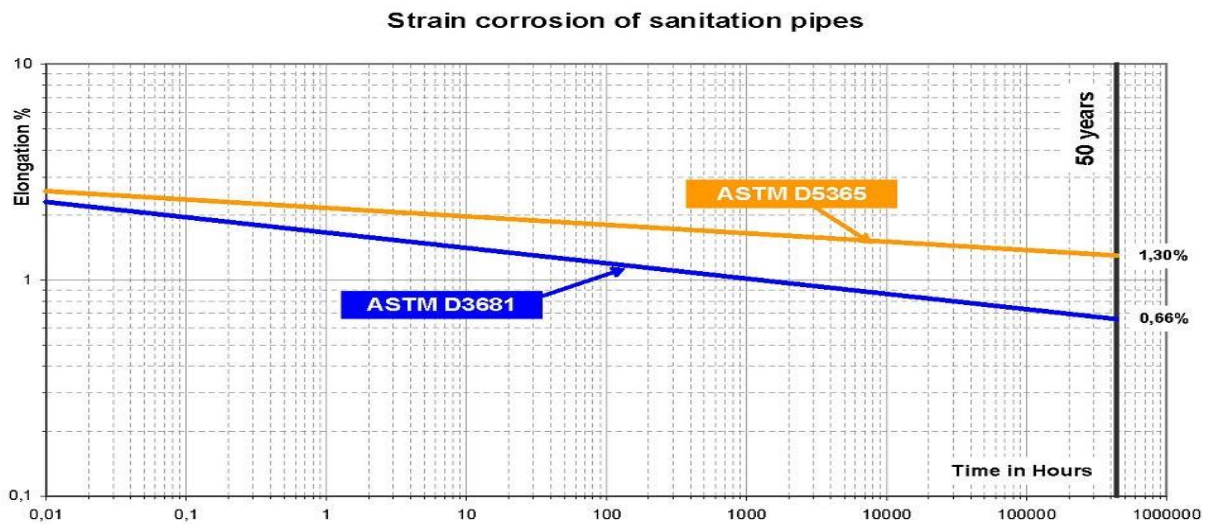


Figure 7
Strain-corrosion lines of sanitation pipes tested in water (ASTM D 5365) and in 5% sulfuric acid (ASTM D 3681). For a failure time of 50 years, the bending strength is $S_b = 1.30\%$ in water and $S_c = 0.66\%$ in acid.

Example 5 – Describe the significance of the long-term strain-corrosion strengths S_c and S_b for sanitation pipes. Establish their relations to the long-term and the short-term strengths of the critical ply.

The S_b strength is the long-term strain-corrosion bending strain of pipes deflected in water. There are two S_b strengths, one for the inner ply of chopped fibers and another for the outer ply of UD fibers. In water service, the outer UD ply should fail first and control the pipe durability. The S_b value should be equal to the long-term S_s strength of UD plies. Therefore, $S_b = S_s$.

The S_c strength has essentially the same meaning as the S_b , except the testing is in 5% solution of sulfuric acid. Since these concepts are applicable to static loadings, we conclude (see example 7) that the long-term S_c strength of the pipe in acid is equal to the weep threshold of the inner ply of chopped fibers. Therefore, $S_c = T_w$.

Both the S_b and the S_c values are pipe features defined by their critical plies. The foregoing discussion allows the computation of the theoretical long-term S_b and S_c strengths of sanitation pipes. The values are in table 2.

	Outer UD90 ply	Inner ply of chopped fibers
S_b in water (50 years)	Equal to the long-term tensile strength of the outer UD ply ($S_b = S_s = 0.92\%$)	Not applicable
S_c in sewer	Not applicable	Equal to the weep threshold of the inner chopped ply ($S_c = T_w = 0.80\%$)

Table 2
Computed values for the long-term strain-corrosion strengths of sanitation pipes in water (S_b) and in urban sewer (S_c). The listed values assume the outer UD plies in the hoop direction, as in hoop-chop pipes.

A close inspection of table 2 reveals

- The long-term bending strength S_b of the outer UD ply in water is the same long-term static strength S_s of UD plies discussed in chapter 15. For boron-free glass, the 50 years strength is $S_s = S_b = 0.92\%$.

- Likewise, the long-term bending strength S_b of the inner chopped fibers in water is the same as the long-term static tensile strength S_s of chopped fibers in water. This S_s value comes from the static rupture equation developed by the British Plastics Federation that gives 50 years $S_b = S_c = 0.85\%$ for brittle polyester resins. The value for the tough polyesters used in sanitation pipes should be higher than 0.92%.
- The S_c strength of the outer UD plies – not in contact with sewage – is not applicable.
- The long-term bending strength S_c of the pipe in acid is equal to the weep threshold of the inner ply of chopped fibers. This assumption is justified by the shielding effect giving low acid presence in the structural plies. For details, see figure 1 of part 3 in chapter 10 and the work of Regester in appendix 1. See also figure 6 in the appendix 1 of chapter 14. Also, see the examples 5 and 6 in chapter 8.

Table 3 compares the theoretical and measured long-term strain-corrosion strengths for sanitation pipes.

	Threshold values	Measured by ASTM test methods
S_b in water (50 years)	0.92%	1.30% Invalid (mixture of data)
S_c in sewer	0.80%	0.66% Invalid (Overstrained specimens)

Table 3

Comparison of the theoretical and measured long-term strain-corrosion strengths of sanitation pipes.

Example 6 – Regester reported a very low solubility of sulfuric acid in bisphenol-A fumarate polyester at 100C. See the appendix 1 of this book. Describe the practical implication of such a low acid solubility.

The low measured solubility implies in a very low acid concentration in the laminate. This is the essence of the shielding effect. The low acid concentration would hardly damage the embedded fibers, suggesting negligible strain-corrosion at strains below the weep threshold.

This situation holds as long as the weep barrier operates below its weep threshold. The high crack densities above the weep threshold allow direct ingress of the full 5.0% acid solution into the underlying structural UD plies. The presence of such a high acid concentration in the structural plies lead to quick strain-corrosion and failure. We have the following situation:

- Above the weep threshold, the full strength acid solution starts a fast strain-corrosion process leading to quick failures.
- Below the weep threshold, the low acid concentration from the shielding effect slows down the strain-corrosion process.
- The strain-corrosion process takes place on the bent locations of the pipe, shown in figure 6. The non-bent locations deteriorate by acid diffusion, as discussed in chapter 10.

The strain-corrosion testing per ASTM D3681 deflects the pipe specimens above the weep threshold, producing regression equations that have no use to estimate the pipe's long-term strength S_c . The static long-term strain-corrosion strength S_c of underground sewage pipes is equal to the weep threshold of the chopped plies. See table 2. See also figure 6 in chapter 14.

Example 7 – Compute the 50 years S_b strength of UD55, UD70 and UD90 underground pipes made of boron-free glass. The long-term static strength of boron-free fibers at 50 years is $S_s = S_b = 0.92\%$. See chapter 15.

The long-term S_b strength defined in pipe standards and recognized in the composites community is the global hoop strain, not the strain in the fiber direction. The recognized S_b value is therefore equal to ϵ_y , not equal to ϵ_1 . Rotating the strains from the global pipe directions to the local fiber direction, gives

$$\varepsilon_1 = \varepsilon_x \cos^2 \alpha + \varepsilon_y \sin^2 \alpha$$

Since the global strain in the axial direction of underground pipes is zero, the above equation reduces to

$$\varepsilon_1 = \varepsilon_y \sin^2 \alpha$$

From which we have

$$S_b = \varepsilon_y = \frac{\varepsilon_1}{\sin^2 \alpha}$$

For UD55

$$S_b = \frac{0.92\%}{\sin^2 55} = 1.37\%$$

For UD70

$$S_b = \frac{0.92\%}{\sin^2 70} = 1.04\%$$

For UD90 (hoop-chop pipes)

$$S_b = \frac{0.92\%}{\sin^2 90} = 0.92\%$$

Example 8 – Prof. Mandell mentions an interesting case of strain-corrosion associated with cyclic fatigue. The application is high voltage UD rods insulators in electrical transmission lines. The rods operate subjected to static tensile strains as well as to wind induced vibrations. The wind vibrations promote crack growth by a fatigue mechanism. The corrosive chemical in this case is nitric acid generated by electric discharges ionizing the atmospheric nitrogen. The stage is thus set for the strain-corrosion process. We have the static tensile loads and the cyclic wind vibrations in the presence of nitric acid. The pultruded rods of UD fibers have no liner or corrosion barrier.

The rods of standard E glass have short lives in this application. This is due to (1) the absence of a protective corrosion barrier to retard the access of the nitric acid to the strained UD fibers and (2) the bending cyclic vibrations. We quote from Prof. Mandell himself:

“Some failures occur at strains which appear to be less than 10% of the short term value (the quality of the UD rods appears to be very good and the strength near the cracks is close to the initial values). The most unusual and baffling aspect of these field failures is the mode of crack growth. The cracks propagate in a planar fashion, perpendicular to the fibers, with no significant splitting or debonding along the fibers. The fracture surfaces are almost perfectly flat over most of the 2 cm rod diameter, with fracture surface features which can be traced back to crack origin, as with many homogeneous materials. Along with the main failure crack, there are often several small cracks which have grown a short distance in from the surface”.

The above is a good description of the strain-corrosion cracks and ruptures. The rupture times are short in this case, since the interaction of cyclic loads with strain-corrosion should be particularly harmful.

Chapter 12

Abrasion life

1 – Introduction: The remarkable performance of composite pipes in aggressive and abrasive service derives mostly from their inherent corrosion resistance. The corrosion resistance of composite materials, unlike that of metal, does not depend on thin protective oxide layers that quickly erodes away in abrasive service. The absence of the dreadful combination erosion-corrosion, together with recent developments in the technology of abrasion resistant barriers, have provided the basis of acceptance for composite pipelines in abrasive service.

Lab tests performed by pipe manufacturers have established the best materials and formulations for enhanced abrasion resistance. Such developments, however, make no mention of durability. The pipe manufacturer know how to combine the best resins and ceramic fillers to last long in abrasive situations. The current state of the technology indicates the best abrasion barriers consisting of hard ceramic fillers embedded in resilient resin matrices. However, knowing the best materials combination is hardly enough to estimate the abrasion life.

There is a striking similarity between the durability of corrosion and abrasion barriers. The composite pipeline is declared unfit for service when its protective abrasion barrier is eroded away, exposing the structural plies. This chapter introduces a quantitative method to predict the durability of abrasion barriers.

2 – The eroded depth Δe : The abrasive slurries carried in pipelines erodes away the protective inner barrier, while leaving the structural plies intact. See figure 1. It takes a long time for the erosive action to advance through the protective barrier and reach the structural plies. The rate of erosion measures the performance of the protective barrier. Furthermore, the residual abrasion life is determined by measuring the remaining barrier thickness after a certain period in service. The analogy with the chemical penetration, discussed in chapter 10, is obvious.

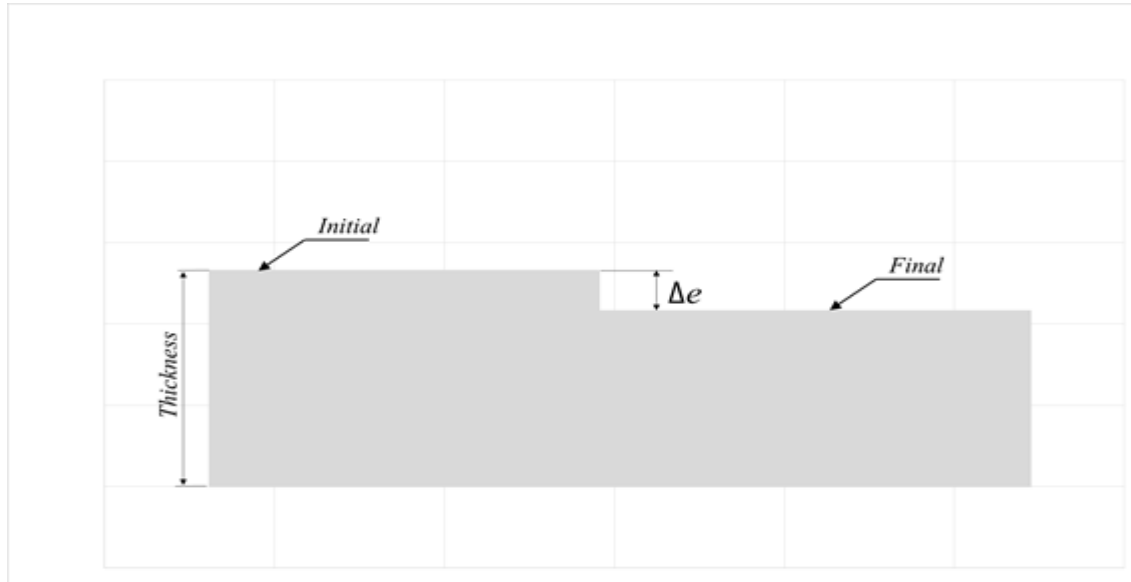


Figure 1 – The worn out depth “ Δe ” measures the extent of erosion. The eroded depth is determined directly by measuring the reduction in wall thickness.

Given enough time, the abrasive slurry will destroy the protective barrier and initiate the rapid erosion of the structural plies. The structural plies are designed for best mechanical properties, not for abrasion resistance.

3 – The wear equation: Having measured the eroded depth, we next derive a mathematical relationship for the wear rate. The derivation takes into consideration the many process and slurry variables known to affect the wear rate. The most important are:

- Pressure
- Temperature
- Particle size
- Hardness
- Slurry concentration
- Flow velocity

In analogy with the chemical case, we estimate the wear rate using a power function of the form

$$\frac{d(\Delta e)}{dt} = K \times P^\alpha \times T^\beta \times D^\gamma \times H^\delta \times C^\epsilon \times V^\theta \quad (1)$$

Where

K is a parameter characteristic of the abrasive barrier

P is the pressure

T is the temperature

D is the particle size

H is the particle hardness

C is the slurry concentration

V is the flow velocity

$\alpha, \beta, \gamma, \delta, \epsilon$ and θ are experimental parameters

The above equation contains many variables and is too complex. The experimental measurement of the exponent parameters in it would involve a series of expensive, laborious and impractical lab tests. The practical approach in such cases consists in dropping the less relevant variables from the equation, capturing their average effects in the constant K . This is the same approach taken in chapter 10 to estimate the chemical durability.

The simplified lab tests to measure the exponent parameters in equation (1) assumes constant deterioration to each of the ignored variables. This simplification reduces the number of variables, having a significant impact on the experimental costs. Looking at equation (1), we can assume constant and typical values for the pressure, temperature, slurry concentration and particle size. With this simplification, the only remaining variables are the particle hardness H and the flow velocity V .

$$\frac{d(\Delta e)}{dt} = K \times H^\alpha \times V^\beta \quad (2)$$

Integrating equation (2) gives the eroded depth

$$\Delta e = K \times H^\alpha \times V^\beta \times t \quad (3)$$

Taking logarithms of both sides of equation (3), we have

$$\log(\Delta e) = \log K + \alpha \times \log(H) + \beta \times \log(V) + \log(t)$$

Or

$$\log(\Delta e) = A + \alpha \times \log(H) + \beta \times \log(V) + \log(t) \quad (4)$$

Equation (4) is similar to the one proposed in chapter 10 to compute the penetrated depth of aggressive chemicals in laminates. As explained there, we make use of multilinear regression analysis technique to determine the wear parameters α , β and A in equation (4). Such parameters, of course, hold for pipelines lined with the same abrasion barrier as the one used in the test.

The constant temperature, pressure, particle size and slurry concentration assumed are those commonly found in practice. The variables that remain are the particle hardness H , the flow velocity V and the time t . The eroded depth Δe is a function of the variables V , H and t . The tested pipeline would ideally include typical fittings to allow the simultaneous determination of their own wear parameters

A simplified test like the one just described may still be too complex and expensive. Even ignoring all variables except three, we still face a sizeable amount of testing. A further simplification would involve the retention of just two variables, flow velocity and time, but the accuracy of equation (4) in this case would perhaps be too low.

4 – Measuring the wear parameters: There is no standardized test method at this time to measure the abrasion parameters. The current abrasion protocols proposed by the pipe manufactures, while excellent to screen and compare formulations and materials are useless to compute the wear parameters. In this section, we suggest a protocol to measure the abrasion parameters of straight pipe sections.

The protocol involves the rotation at several velocities of straight pipe sections partially filled with abrasive slurries of different hardness. The eroded depth “ Δe ” come from direct measurement of the test specimens at several time intervals and velocities. The measured data points, corresponding to the eroded depth “ Δe ”, exposure time “ t ”, abrasive hardness “ H ” and flow velocity “ V ” are annotated and then processed by the usual least squares method to produce a statistical estimation of the wear parameters. This is a typical linear regression problem in three independent variables – velocity, hardness and time. The regression equation (4) obtained in this way predicts the service life of the tested abrasion barrier.

The method just described may give reasonably good values for the wear parameters of straight pipe sections. The determination of the wear parameters for fittings would be more complex and require a more elaborate apparatus. To eliminate this difficulty, we may consider the fittings as expendable items discarded and routinely replaced during the pipeline operation. In this way, the fittings have the same erosive barriers of the straight pipe sections. This simplification greatly facilitates the fabrication and testing, with a negligible impact on the overall performance of the pipeline.

The following examples illustrate the application of the regression equation (4). We assume an abrasion barrier of vinyl ester resin loaded with tungsten carbide. We also assume the following wear parameters

$$A = -2.53, \alpha = 1.03 \text{ and } \beta = 0.80.$$

The above values are valid for “ Δe ” in mm, flow velocity V in m/s, particle hardness H in mhos and exposure time t in months. The wear parameters enter equation (4) to determine (a) the service life of a 2.0 mm abrasion barrier and (b) the required thickness of the abrasion barrier for a projected service life of 25 years.

Example 1 – Estimate the service life of a 2.0 mm abrasion barrier. To increase the service life the pipeline is rotated by 180 degrees when the eroded depth reaches $\Delta e = 2.0$ mm. The service life is estimated for calcite slurry (mhos hardness $H = 3$) at a flow velocity $V = 5$ m/s.

From equation (4)

$$\log(\Delta e) = A + \alpha \times \log(H) + \beta \times \log(V) + \log(t)$$

$$\log 2.0 = -2.53 + 1.03 \times \log 3.0 + 0.80 \times \log 5.0 + \log(t)$$

Solving for the time “ t ” we obtain $t \approx 60$ months (5 years).

The pipeline rotation by 180 degrees after 5 years of service, gives another 5 years. The expected life is, then, 10 years.

Example 2 – Estimate the required thickness of the abrasion barrier for a service life of 20 years. Considering rotation of the pipes, we design the barrier thickness to last 10 years. Per equation (4) the required thickness is

$$\log \Delta e = -2.53 + 1.03 \times \log 3.0 + 0.80 \times \log 5.0 + \log(10 \times 12)$$

$$\Delta e = 4.0 \text{ mm.}$$

Chapter 13

Part 1

Regression equations of plies

1 – Introduction: Figure 1 shows a strain wave with a static and a sinusoidal cyclic component. In the study of durability, we define the following quantities.

ϵ_{min} = static strain component

$\epsilon_{max} - \epsilon_{min}$ = amplitude of the cyclic sinusoidal strain component

$\epsilon_{max} = \epsilon$ = peak strain

$R = \frac{\epsilon_{min}}{\epsilon_{max}}$ = strain ratio

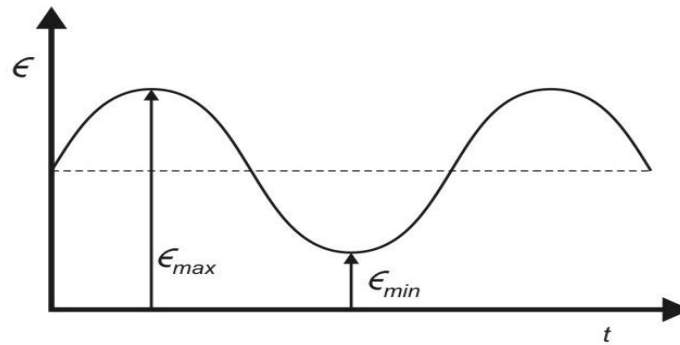


Figure 1

The static strain component is $\epsilon_{min} = R \cdot \epsilon_{max}$.

The sinusoidal cyclic strain component is $\epsilon_{max} - \epsilon_{min} = (1 - R) \cdot \epsilon_{max}$.

The load-dependent durability of composite equipment is a function of the load ratio R and the peak strain ϵ , plus a few parameters controlled by the resin matrix and the reinforcing fibers. There are three methods to estimate the durability of composites equipment.

- The Goodman diagrams
- The regression equations
- The unified equation

The Goodman diagrams are widely covered in the literature, receiving little attention in this book. We will do just a cursory comparison with the unified equation. The regression equations are the topics of the parts 1 and 2 of this chapter. The derivation and application of the unified equation is the topic of chapter 16.

There are two categories of regression lines, one for laminates and another for plies.

2 – Regression equations of laminates. The regression equations of laminates capture the damages from the cyclic and static load components, as well as their interaction. It is customary to express them as a function of the number of cycles to failure, N

$$\log(\epsilon) = A_{lam} - G_c^{lam} \times \log(N)$$

In this equation, the effects of the interaction and the load ratio R are captured by the slope parameter, G_c^{lam} . Therefore, the slope parameter G_c^{lam} is not a laminate property.

Taking $N = 1$ in the above equation, we have

$$\log(\epsilon) = A_{lam} - G_c^{lam} \times \log(1)$$

$$\log(\epsilon) = A_{lam}$$

The intercept A_{lam} is the logarithm of the short-term strength the laminate. However, as discussed in chapter 8, the short-term strengths are just another name for the strain thresholds.

$$A_{lam} = \log(A_i)$$

$$A_{lam} = \log(A_w)$$

$$A_{lam} = \log(A_s)$$

$$A_{lam} = \log(A_r)$$

Where

A_i is the laminate infiltration threshold

A_w is the laminate weep threshold

A_s is the laminate stiffness threshold

A_r is the laminate rupture threshold

In the above, the laminate thresholds refer to the global frame. What we have just done is extend to laminates the ply concepts of failure thresholds. It is as if the laminates were plies with axes oriented in the global directions x and y .

The preceding regression equation gives very accurate predictions for the load-dependent durability of laminates. However, they find no practical use for two reasons:

- *Their development is expensive and time consuming*
- *They are specific to the tested laminate and loading. Any change in the laminate construction or raw materials, or in the loading, requires the development of a new costly and difficult to obtain regression equation.*

The literature lists a few regression equations for laminates, developed for some specific applications like automotive parts, pressure pipes and wind blades. The problems with these equations are their high cost and application specific to the tested laminate. The unified equation has none of these difficulties.

The unified equation can predict the load-dependent durability of any laminate under any loadings. This is possible because the unified equation uses inputs from plies, not laminates. The importance of plies is undeniable. The remainder of this chapter deals with the load-dependent regression equations of plies.

3 – Regression equations of plies: There is no formal distinction between laminate and ply equations. Both have the same format and meaning. The only difference lies in their reference frame, local for plies and global for laminates.

The ply equation for a loading of strain ratio R is

$$\log(\epsilon) = A_c - G_c \times \log(N)$$

Where

ϵ is the peak strain at failure

N is the number of cycles at failure

A_c and G_c are experimental ply parameters.

As in the case of laminates, we have

$$A_c = \log(T_i)$$

$$A_c = \log(T_w)$$

$$A_c = \log(T_s)$$

$$A_c = \log(T_r)$$

Where

T_i is the ply infiltration threshold

T_w is the ply weep threshold

T_s is the ply stiffness threshold

T_r is the ply rupture threshold

In the above, the ply thresholds refer to their local frames.

The slopes G_c vary with the load ratio R. Any given ply has an infinite number of slopes G_c , one for each load ratio R. The full characterization of a ply requires an infinite number of regression equations, each with their own slope G_c . This is the same situation described earlier for laminates. See figure 2

Note: Since plies have well-defined constructions, this difficulty can be alleviated by interpolating between load ratios. This simplification is feasible for plies, not for laminates. The interpolation solution is a basic feature of the Goodman diagrams. For a comparison of the Goodman diagrams with the unified equation, see chapter 16.

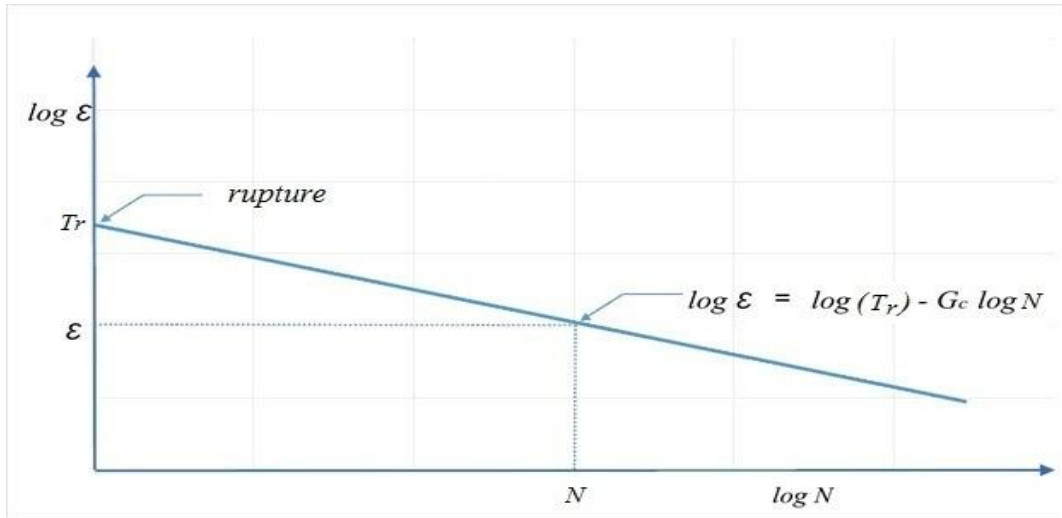


Figure 2a

Rupture line – The cyclic rupture lines of plies initiate at rupture threshold $A_c = \log(T_r)$. The line slopes G_c , however, vary with the load ratio R .

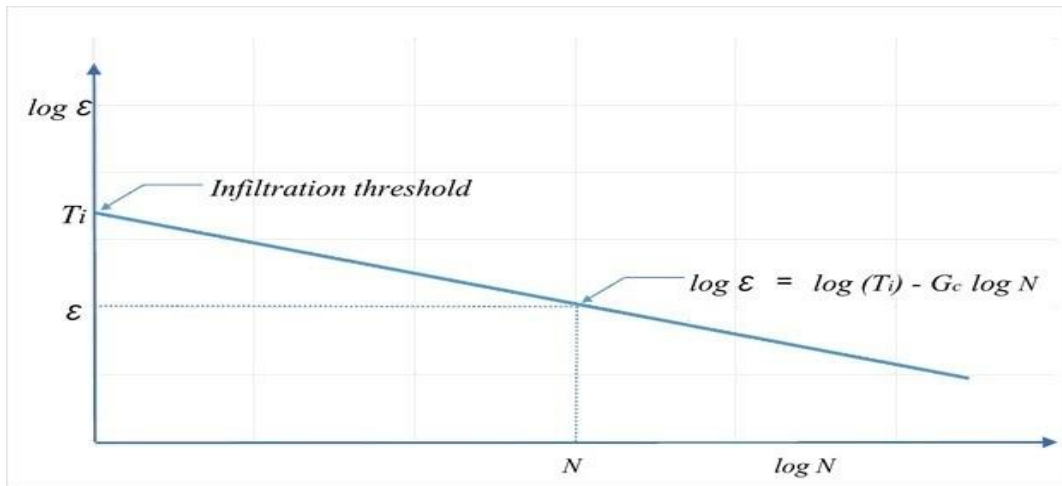


Figure 2b

Infiltration line – The cyclic infiltration lines of plies start at the failure threshold $A_c = \log(T_i)$. The cyclic slope parameters G_c depend on the load ratio R .

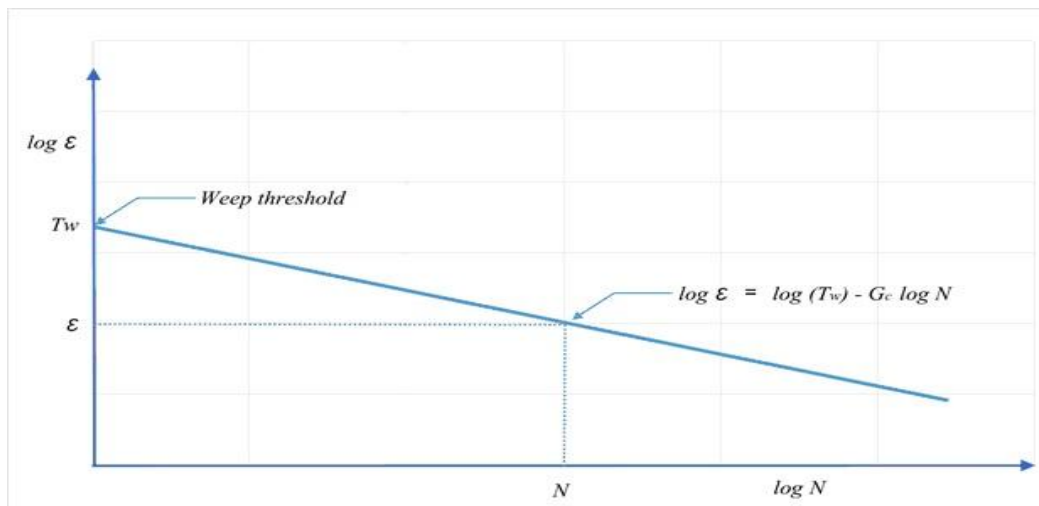


Figure 2c

Weep line – The cyclic weep lines of plies starts at failure threshold $Ac = \log(Tw)$. The slope Gc varies with the load ratio R . As explained in the part 2 of this chapter, the weep equations of plies differ considerably from those of pipes.

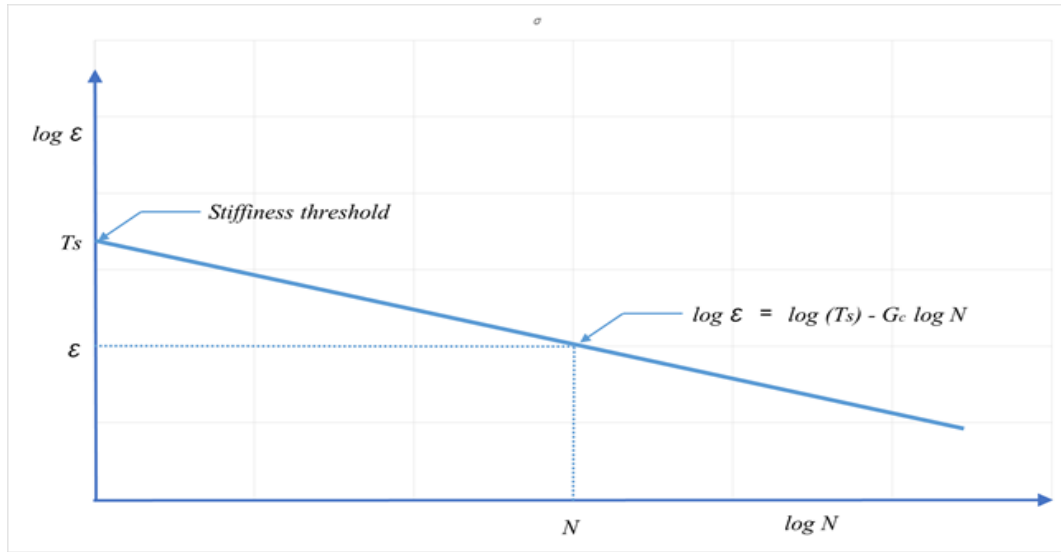


Figure 2d

Stiffness line – The cyclic stiffness lines of plies converge at the failure threshold $Ac = \log(Ts)$. The slopes Gc vary with the load ratio R .

4 – Simplifying the ply equations: We continue our discussion of the ply equations. The difficulty with these equations is their narrow applicability to specific conditions. Changes in resin toughness, load ratio R , temperature and mode of failure require the experimental development of a new and expensive regression line. The discussion that follows simplifies this. Let us see.

- The potentially serious complication posed by the operating conditions is not a problem. As explained in chapter 9, the regression equations for dry plies at room temperature are safe to use at any operating condition.
- All cyclic regression equations turn horizontal on meeting the fatigue threshold $To = 0.05\%$. The number of cycles No varies with the R ratio. See figure 3.
- The fatigue threshold $To = 0.05\%$ is a universal ply constant independent of the resin system and load ratio R . The number of cycles No , however, increases with the load ratio from $No = 10^{20}$ at $R = 0.0$ to infinite at $R = 1.0$.
- Being a ply property, the fatigue threshold $To = 0.05\%$ holds in the local frame.

The above simplifications are helpful, but not enough, since No depends on the R ratio. A further simplification comes from the inputs required in the unified equation, that derive exclusively from pure static ($R = 1.0$) and pure cyclic ($R = 0.0$) equations. Therefore, we need only concern ourselves with pure regression equations. That is momentous simplification.

5 – Pure regression equations of plies: The foregoing discussion involved general ply equations, valid for any R ratio. The pure regression equations of plies, for $R = 1.0$ (pure static) or $R = 0.0$ (pure cyclic), are the topic of this section. From now on, in preparation for the unified equation, we shall focus exclusively on the pure regression equations.

In pure cyclic loadings, the fixed R ratio and absence of interaction produces constant slopes. This is true for both plies and laminates.

Therefore

$$\log(\epsilon) = A_c - G_c^{ply} \times \log(N)$$

$$\log(\epsilon) = A_s - G_s^{ply} \times \log(\text{hour})$$

Where

$A_c = A_s = \log(\text{failure threshold})$
 G_c is the experimental pure cyclic slope
 G_s is the experimental pure static slope
 ϵ is the peak strain
 N is the number of cycles to failure
 Hour is the time to static failure.

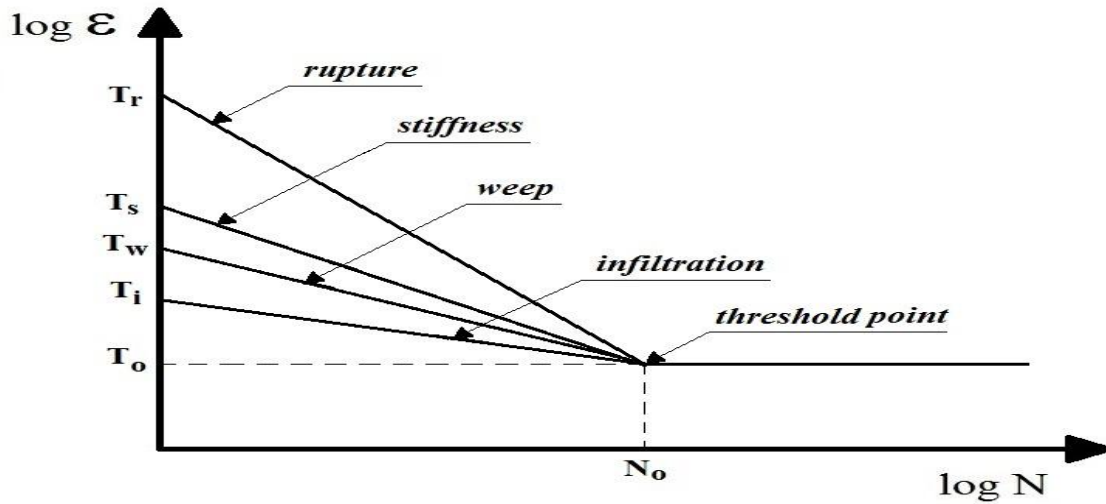


Figure 3

The fatigue threshold point is defined by a pristine strain T_o and a corresponding number of cycles N_o . The pristine strain $T_o = 0.05\%$ is a ply constant independent of the load ratio R and the resin system. The number of cycles N_o increases with the load ratio R . The failure thresholds define the intercepts of the four regression lines. Changes in resin toughness will shift the intercepts up and down, while changes in the load ratio R will move N_o left and right.

Figure 3 shows the four cyclic lines of plies. All regression lines connect the long-term fatigue point (T_o N_o) to the short-term failure thresholds.

- For pure cyclic loadings, $T_o = 0.05\%$ and $N_o = 10^{20}$. For details, see the appendix 2 in this chapter.
- The inputs to the unified equation come from pure regression lines. For details, see chapter 16.

Taking the above into consideration, the pure cyclic regression equations of plies are

$$\log \epsilon = \log(T_r) - \frac{\log(T_r/0.05)}{20} \times \log N$$

$$\log \epsilon = \log(T_s) - \frac{\log(T_s/0.05)}{20} \times \log N$$

$$\log \epsilon = \log(T_w) - \frac{\log(T_w/0.05)}{20} \times \log N$$

$$\log \epsilon = \log(T_i) - \frac{\log(T_i/0.05)}{20} \times \log N$$

As we see, the four pure cyclic equations (one for each mode of failure) derive from known ply quantities.

- The fatigue threshold point for the pure cyclic equations of plies are $T_o = 0.05\%$ and $N_o = 10^{20}$
- The resin-dependent short-term failure thresholds of plies are available in chapter 8

Likewise, the pure static regression equations of plies are available. See tables 2 through 5.

The inputs to the unified equation come from well-known tabulated pure regression equations of plies, requiring no further measurement. This is a tremendous simplification.

Table 1 lists the five pure regression equations of plies available in the literature. They are all rupture lines, since rupture is the only recognized mode of failure at this time. It is amazing that these five rupture equations provide the basis to solve all load-dependent durability problems of composite laminates. This amazing feat results from the unified equation, discussed in chapter 16.

Ply	R ratio	Equation	Developer
UD Fiber direction	$R = 1.0$	$\log(\epsilon) = 0.400 - 0.077 \log(\text{hours})$ Boron-free glass.	Mark Greenwood
	$R = 0.0$	$\log(\epsilon) = 0.347 - 0.130 \log(\text{hours})$ E glass. $\log(\epsilon) = \log(3.00) - 0.089 \log(N)$	Guangxu Wei
UD Transverse	$R = 1.0$	Not applicable	-----
	$R = 0.0$	$\log(\epsilon) = \log(0.25) - 0.034 \log(N)$	Guangxu Wei
Chopped fibers	$R = 1.0$	$\log(\epsilon) = \log(1.50) - 0.045 \log(\text{hours})$	British Plastics Federation
	$R = 0.0$	$\log(\epsilon) = \log(1.50) - 0.075 \log(N)$	

Table 1

These are the only pure rupture regression equations available in the literature. These equations suffice to solve all load-dependent durability problems of composite laminates.

The unified equation holds for both plies and laminates, taking into account the frames of reference, local for plies and global for laminates. That includes the fatigue threshold T_o . We will return to this in the part 2 of this chapter.

Example 1. Discuss the effect of ply thickness on the failure thresholds of transverse loaded UD plies.

We quote from reference 20, taking the liberty of slightly editing the original text to comply with the terminology of this book.

“The transverse cracking process in general cross-ply laminates is influenced by several factors. The first to consider is the so-called constraint effect. As demonstrated by Parzini et al. the thickness of embedded UD plies will influence the transverse failure threshold, with thinner plies having higher values. For ply thicknesses higher than about 0.5 mm, the transverse failure thresholds reach constant values, equal to those of isolated plies. All UD plies embedded in laminates and thinner than 0.5 mm, have higher transverse failure thresholds than isolated plies”

Taking into consideration that most commercial UD plies are thinner than 0.5 mm, we can safely accept the conservative threshold values from isolated plies.

Example 2 – Derive the pure cyclic regression equations for chopped plies of tough polyester resin. The following information is available

$$\begin{aligned} T_o &= 0.05\% \\ N_o &= 10^{20} \text{ cycles} \\ T_i &= 0.40\% \\ T_w &= 0.80\% \\ T_r &= 1.50\% \end{aligned}$$

The infiltration equation is

$$\begin{aligned} \log(\epsilon) &= \log(T_i) - \frac{\log(T_i/T_o)}{\log(N_o)} \times \log N \\ \log(\epsilon) &= \log(0.40) - \frac{\log(0.40/0.05)}{20} \times \log N \\ \log(\epsilon) &= \log(0.40) - 0.045 \log N \end{aligned}$$

The weep equation is

$$\begin{aligned} \log(\epsilon) &= \log(T_w) - \frac{\log(T_w/T_o)}{\log(N_o)} \times \log N \\ \log(\epsilon) &= \log(0.80) - \frac{\log(0.80/0.05)}{20} \times \log N \\ \log(\epsilon) &= \log(0.80) - 0.060 \log N \end{aligned}$$

The rupture equation is

$$\begin{aligned} \log(\epsilon) &= \log(T_r) - \frac{\log(T_r/T_o)}{\log(N_o)} \times \log N \\ \log(\epsilon) &= \log(1.50) - \frac{\log(1.50/0.05)}{20} \times \log N \\ \log(\epsilon) &= \log(1.50) - 0.075 \log N \end{aligned}$$

All ply equations are valid in the local frame.

Example 3 – Derive the pure cyclic regression equations for chopped plies of vinyl ester resin. The following information is available

$$\begin{aligned}
T_o &= 0.05\% \\
N_o &= 10^{20} \text{ cycles} \\
T_i &= 0.50\% \\
T_w &= 1.10\% \\
T_r &= 2.50\%
\end{aligned}$$

The infiltration equation is

$$\log(\epsilon) = \log(T_i) - \frac{\log(T_i/T_o)}{\log(N_o)} \times \log N$$

$$\log(\epsilon) = \log(0.50) - \frac{\log(0.50/0.05)}{20} \times \log N$$

$$\log(\epsilon) = \log(0.50) - 0.050 \log N$$

The weep equation is

$$\log(\epsilon) = \log(T_w) - \frac{\log(T_w/T_o)}{\log(N_o)} \times \log N$$

$$\log(\epsilon) = \log(1.10) - \frac{\log(1.10/0.05)}{20} \times \log N$$

$$\log(\epsilon) = \log(1.10) - 0.067 \log N$$

The rupture equation is

$$\log(\epsilon) = \log(T_r) - \frac{\log(T_r/T_o)}{\log(N_o)} \times \log N$$

$$\log(\epsilon) = \log(2.50) - \frac{\log(2.50/0.05)}{20} \times \log N$$

$$\log(\epsilon) = \log(2.50) - 0.085 \log N$$

All ply equations are valid in the local frame.

Example 4 – Derive the pure cyclic regression equations for transverse loaded vinyl ester UD plies. The following information is available

$$\begin{aligned}
T_o &= 0.05\% \\
N_o &= 10^{20} \text{ cycles} \\
T_i &= 0.50\% \\
T_w &= 0.55\% \\
T_r &= 1.00\%
\end{aligned}$$

The infiltration equation is

$$\log(\epsilon) = \log(T_i) - \frac{\log(T_i/T_o)}{\log(N_o)} \times \log N$$

$$\log(\epsilon) = \log(0.50) - \frac{\log(0.50/0.05)}{20} \times \log N$$

$$\log(\epsilon) = \log(0.50) - 0.050 \log N$$

The weep equation is

$$\log(\epsilon) = \log(Tw) - \frac{\log(Tw/To)}{\log(No)} \times \log N$$

$$\log(\epsilon) = \log(0.55) - \frac{\log(0.55/0.05)}{20} \times \log N$$

$$\log(\epsilon) = \log(0.55) - 0.052 \log N$$

The rupture equation is

$$\log(\epsilon) = \log(Tr) - \frac{\log(Tr/To)}{\log(No)} \times \log N$$

$$\log(\epsilon) = \log(1.00) - \frac{\log(1.00/0.05)}{20} \times \log N$$

$$\log(\epsilon) = \log(1.00) - 0.065 \log N$$

All ply equations are valid in the local frame.

Example 5 – Derive the pure cyclic regression equations for transverse loaded UD plies of tough polyester resins. The following information is available

$$To = 0.05\%$$

$$No = 10^{20} \text{ cycles}$$

$$Ti = 0.40\%$$

$$Tw = 0.45\%$$

$$Tr = 0.70\%$$

The infiltration equation is

$$\log(\epsilon) = \log(Ti) - \frac{\log(Ti/To)}{\log(No)} \times \log N$$

$$\log(\epsilon) = \log(0.40) - \frac{\log(0.40/0.05)}{20} \times \log N$$

$$\log(\epsilon) = \log(0.40) - 0.045 \log N$$

The weep equation is

$$\log(\epsilon) = \log(Tw) - \frac{\log(Tw/To)}{\log(No)} \times \log N$$

$$\log(\epsilon) = \log(0.45) - \frac{\log(0.45/0.05)}{20} \times \log N$$

$$\log(\epsilon) = \log(0.45) - 0.048 \log N$$

The rupture equation is

$$\log(\epsilon) = \log(Tr) - \frac{\log(Tr/To)}{\log(No)} \times \log N$$

$$\log(\epsilon) = \log(0.70) - \frac{\log(0.70/0.05)}{20} \times \log N$$

$$\log(\epsilon) = \log(0.70) - 0.057 \log N$$

All ply equations are valid in the local frame.

6 – Tabulated pure regression equations of plies. There are two methods to compute the load-dependent durability of laminates under simultaneous cyclic and static loads.

- *The experimental method, using regression equations and diagrams measured for specific loadings and laminates. This is unfeasible, given the cost and the time to make the required measurements.*
- *The unified equation, with inputs from available pure regression lines.*

The pure regression equations of plies provide the inputs needed by the unified equation. The fact that previous investigators have developed the pure equations for the two plies of commercial interest is a blessing. We should all be thankful and appreciative of the early works by Mark Greenwood, Guangxu Wei and the British Plastics Federation.

As illustrated in the foregoing numerical examples, knowledge of the ply short-term strengths and the fatigue point ($T_o = 0.05\%$; $N_o = 10^{20}$) trivializes the determination of their pure cyclic regression equations. Tables 2 through 5 list all pure – cyclic and static – regression equations of commercially important plies. These pure equations are useful in two situations:

- *To compute the ply durability under pure cyclic or pure static loadings. Pure loadings are rare in real life. Storage tanks provide examples of pure static loadings. I cannot recall a single application involving a pure cyclic loading. Most engineering applications involve the simultaneous presence of cyclic and static loads.*
- *To provide the inputs to the unified equation, allowing the computation of the load-dependent durability of any laminate for any loading. Herein lies the importance of the pure equations.*

Rupture failure $N_o = 10^{20}$ $T_o = 0.05\%$	
<i>UD plies, fiber direction, static boron-free glass</i>	$\log(\epsilon) = 0.400 - 0.077 \times \log(\text{hours})$
<i>UD plies, fiber direction, static E glass</i>	$\log(\epsilon) = 0.347 - 0.130 \times \log(\text{hours})$
<i>UD plies, fiber direction, cyclic any glass</i>	$\log(\epsilon) = \log(3.00) - 0.089 \times \log(N)$
<i>UD plies, transverse direction, cyclic Tough polyester</i>	$\log(\epsilon) = \log(0.70) - 0.057 \times \log(N)$
<i>UD plies, transverse direction, cyclic vinyl ester</i>	$\log(\epsilon) = \log(1.00) - 0.065 \times \log(N)$
<i>Chopped plies, static Tough polyester</i>	$\log(\epsilon) = \log(1.50) - 0.045 \times \log(\text{hours})$
<i>Chopped plies, static vinyl ester</i>	$\log(\epsilon) = \log(2.50) - 0.045 \times \log(\text{hours})$
<i>Chopped plies, cyclic Tough polyester</i>	$\log(\epsilon) = \log(1.50) - 0.075 \times \log(N)$
<i>Chopped plies, cyclic</i>	$\log(\epsilon) = \log(2.50) - 0.086 \times \log(N)$

vinyl ester	

Table 2

Pure rupture failure equations. The slopes of all static equations are fiber dominated and resin-independent.

Stiffness failure $N_0 = 10^{20}$ $T_0 = 0.05\%$	
UD plies, fiber direction, static boron-free glass	-----
UD plies, fiber direction, static E glass	-----
UD plies, fiber direction, cyclic any glass	-----
UD plies, transverse direction, cyclic Tough polyester	$\log(\epsilon) = \log(0.70) - 0.057 \times \log(N)$
UD plies, transverse direction, cyclic vinyl ester	$\log(\epsilon) = \log(1.00) - 0.065 \times \log(N)$
Chopped plies, static polyester	-----
Chopped plies, static vinyl ester	-----
Chopped plies, cyclic polyester	-----
Chopped plies, cyclic vinyl ester	-----

Table 3

The pure stiffness equations of practical interest are those of transverse loaded UD plies. The UD plies loaded in the fiber direction and the chopped plies are not critical to stiffness failure.

Weep failure $N_0 = 10^{20}$ $T_0 = 0.05\%$	
UD plies, fiber direction, static boron-free glass	-----
UD plies, fiber direction, static E glass	-----
UD plies, fiber direction, cyclic any glass	-----
UD plies, transverse direction Tough polyester	$\log(\epsilon) = \log(0.45) - 0.048 \times \log(N)$
UD plies, transverse direction vinyl ester	$\log(\epsilon) = \log(0.55) - 0.052 \times \log(N)$
Chopped plies, static polyester	-----
Chopped plies, static vinyl ester	-----
Chopped plies, cyclic Tough polyester	$\log(\epsilon) = \log(0.80) - 0.060 \times \log(N)$
Chopped plies, cyclic vinyl ester	$\log(\epsilon) = \log(1.10) - 0.068 \times \log(N)$

Table 4

The pure weep failure equations of interest are those of chopped fibers and transverse loaded UD plies. The UD plies loaded in the fiber direction do not develop resin cracks leading to weep failure. Statically loaded chopped plies do not grow cracks leading to weep failure.

Infiltration failure $N_o = 10^{20}$ $T_o = 0.05\%$	
UD plies, fiber direction, static boron-free glass	-----
UD plies, fiber direction, static E glass	-----
UD plies, fiber direction, cyclic Any glass	-----
UD plies, transverse loaded Tough polyester	$\log(\epsilon) = \log(0.40) - 0.045 \times \log(N)$
UD plies, transverse loaded vinyl ester	$\log(\epsilon) = \log(0.50) - 0.050 \times \log(N)$
Chopped plies, static polyester	-----
Chopped plies, static vinyl ester	-----
Chopped plies, cyclic Tough polyester	$\log(\epsilon) = \log(0.40) - 0.045 \times \log(N)$
Chopped plies, cyclic vinyl ester	$\log(\epsilon) = \log(0.50) - 0.050 \times \log(N)$

Table 5

The pure infiltration failure equations of interest are those of chopped plies. The UD plies never have contact with corrosive chemicals. Statically loaded chopped plies do not grow cracks leading to infiltration failure.

Example 6 – Derive the cyclic regression equations for polyester chopped plies at the load ratio $R = 0.2$.

To compute these equations, we need the resin-dominated cyclic slope at $R = 0.2$. As a rule, slopes for values of R other than zero are not available. However, for illustration purposes, we assume the rupture slope $G_c^r = 0.070$ for $R = 0.2$.

We first determine the number of cycles N_o at $R = 0.2$

$$\frac{\log(T_r/T_o)}{\log(N_o)} = 0.070$$

$$\frac{\log(1.50/0.05)}{\log(N_o)} = 0.070$$

From which we have $N_o = 10^{21}$

Having $N_o = 10^{21}$, we easily derive the desired regression equations.

The infiltration equation at $R = 0.2$ is

$$\log(\epsilon) = \log(T_i) - \frac{\log(T_i/T_o)}{\log(N_o)} \times \log N$$

$$\log(\epsilon) = \log(0.40) - \frac{\log(0.40/0.05)}{21} \times \log N$$

$$\log(\epsilon) = \log(0.40) - 0.043 \log N$$

The weep equation at $R = 0.2$ is

$$\log(\epsilon) = \log(T_w) - \frac{\log(T_w/T_o)}{\log(N_o)} \times \log N$$

$$\log(\epsilon) = \log(0.80) - \frac{\log(0.80/0.05)}{21} \times \log N$$

$$\log(\epsilon) = \log(0.80) - 0.057 \log N$$

The rupture equation at $R = 0.2$ is

$$\log(\epsilon) = \log(T_r) - \frac{\log(T_r/T_o)}{\log(N_o)} \times \log N$$

$$\log(\epsilon) = \log(1.50) - \frac{\log(1.50/0.05)}{21} \times \log N$$

$$\log(\epsilon) = \log(1.50) - 0.070 \log N$$

Different R ratios produce lines of different slopes, all passing through the fixed short-term thresholds and turning horizontal at $T_o = 0.05\%$.

Example 7 – *We wish to compute the number of cycles to infiltrate, weep and rupture a hoop-chop polyester sanitation pipe operating at a load ratio $R = 0.2$ and a peak hoop strain $\epsilon = 0.40\%$.*

The critical ply for weep and infiltration is the weep barrier of chopped fibers. The regression equations for this ply at $R = 0.2$ are available from example 6.

The number of cycles to infiltration is

$$\log(\epsilon) = \log(0.40) - 0.043 \log N$$

$$\log(0.40) = \log(0.40) - 0.043 \log N$$

$$N = 1 \text{ cycle}$$

The pipe infiltrates in the first cycle, since the peak strain (0.40%) is equal to the infiltration threshold. Infiltration, however, is perfectly acceptable for pipes in water service.

The number of cycles to weep is

$$\log(\epsilon) = \log(0.80) - 0.057 \log N$$

$$\log(0.40) = \log(0.80) - 0.057 \log N$$

$$N = 191\,000 \text{ cycles}$$

The number of cycles to weep computed from ply data – instead of pipe data – is overly conservative, since it ignores the travel time of water in traversing the pipe wall. A more accurate number of cycles to weep is determined from pipe data, as discussed in the part 2 of this chapter.

To determine the number of cycles to rupture, we need the rupture regression equation for UD plies loaded in the fiber direction at $R = 0.2$. We do not have this equation.

This numerical example illustrates the enormous power of the regression equations, when available. The problem, as explained, is that such equations are available only for pure loadings.

Example 8 – Let us revisit the numerical example 7, assuming pipes of vinyl ester resin. The infiltration threshold of vinyl ester chopped plies is $T_i = 0.50\%$ and the weep threshold is $T_w = 1.10\%$. The fatigue threshold point is the same for both resins, at $T_0 = 0.05\%$ and $N_0 = 10^{21}$.

The infiltration equation at $R = 0.2$ is

$$\log(\epsilon) = \log(T_i) - \frac{\log(T_i/T_0)}{\log(N_0)} \times \log N$$

$$\log(\epsilon) = \log(0.50) - \frac{\log(0.50/0.05)}{21} \times \log N$$

$$\log(\epsilon) = \log(0.50) - 0.048 \log N$$

The number of cycles to infiltration is

$$\log(0.40) = \log(0.50) - 0.048 \log N$$

$$N = 100 \text{ cycles}$$

The weep equation at $R = 0.2$ is

$$\log(\epsilon) = \log(T_w) - \frac{\log(T_w/T_0)}{\log(N_0)} \times \log N$$

$$\log(\epsilon) = \log(1.10) - \frac{\log(1.10/0.05)}{21} \times \log N$$

$$\log(\epsilon) = \log(1.10) - 0.064 \log N$$

The number of cycles to weep is

$$\log(0.40) = \log(1.10) - 0.064 \log N$$

$$N = 7\,300\,000 \text{ cycles}$$

This numerical example illustrates the superior performance of vinyl esters vis-à-vis polyesters in weep failures.

Example 9 – Explain the difference between the ply general regression lines in figure 1 and the pure equations discussed in the previous section.

The graphs in figure 1 are generic representations valid for plies and laminates subjected to any loading, i.e. any R ratio. The R ratios determine the line slopes. The line slopes decrease from a maximum at $R = 0.0$ (pure cyclic) to zero at $R = 1.0$ (pure static). The intercept parameters in figure 1 are the same as those in the pure regression equations. In fact, the intercepts are short-term strengths independent of the loading. See figure 3.

7 – Hydrostatic pressure tests. The hydrostatic test consists in pressurizing pipes of diameter less than 1400 mm to strains slightly below the infiltration threshold. For example, a pipe rated for 100 PSI may have a weep threshold of 200 PSI and be pressure tested at 175 PSI, just short of the weep threshold. This is safe for pipes operating under static pressure. However, in applications involving cyclic loads, there is a legitimate concern that the large crack density from such a high test-pressure might reduce the durability of the tested pipes.

Pressure excursions above the rated value would enlarge the crack density, thus reducing the cyclic durability of the pipe. The conscious engineer rightfully worries about pressure spikes above rated pressures. The following numerical example provides an elegant answer to this question.

Note: As explained in chapter 18, the pipes with impermeable aluminum foil do not require quality control pressure tests. The impermeable pipes eliminate the need for this expensive and time-consuming quality control test, while assuring weep-free pipes. For details, see chapter 18.

Example 10 – Consider a vinyl ester UD55 oil pipeline with a weep threshold $T_w = 0.55\%$. Assuming $SF = 2$, the allowable peak strain transverse to the UD fibers is $\epsilon = T_w/2 = 0.28\%$. As part of the quality control program, all pipes are factory tested at a transverse strain 0.42% . It is required to quantify the effect of such a pressure test on the expected weep durability. We consider the pipeline operating in two scenarios:

1. Pure static condition.
2. Pure cyclic condition.

Pure static service: The transverse test strain 0.42% is less than the weep threshold 0.55% . Since static loads do not grow cracks, the damages incurred by the pipes in the pressure test have no effect on their static long-term weep failure. The pipeline operates as if it had never been pressure tested. In the specific case of pure static loading, pressure tests below the weep threshold is not a cause of concern.

Pure cyclic service: In the presence of cyclic loads, the peak transverse test strain 0.42% will develop a large crack density that reduces the durability of the pipeline. The complete evaluation of the durability in this case, consisting of a combination of static and cyclic loads, requires the unified equation introduced in chapter 16. However, for illustration purposes, we proceed to offer a solution assuming a simple and more severe case of pure cyclic load.

First, we compute the number of cycles to fail pristine, non-tested pipes. From table 4, the pure cyclic weep equation of transverse loaded vinyl ester UD plies submitted to a peak operating strain of $0.55/2 = 0.275\%$ is

$$\log(0.275) = \log(0.55) - 0.052 \times \log(N_1)$$

From this equation, we obtain $N_1 = 615\,000$ cycles. This is the number of cycles to weep a healthy, pristine, non-tested pipe. The crack density in this case starts to grow from the small damage corresponding to the peak strain 0.275% .

We next compute the number of cycles to grow the damage – crack density – to a level corresponding to the test strain $1.5 \times 0.275 = 0.413\%$. To do this, we assume the pipe weeping at 0.413% under pure cyclic loading. The weep regression equation for this hypothetical case is

$$\log(\epsilon) = \log(0.413) - \frac{\log(0.413/0.05)}{20} \times \log N_2$$

$$\log(0.275) = \log(0.413) - 0.046 \log N_2$$

Solving for N_2 , we have

$$N_2 = 6\,900 \text{ cycles.}$$

This is the number of cycles for a healthy pipe, under a 0.275% peak strain, to attain the damage state equivalent to that of the static test strain 0.413% . The number of cycles required to weep the damaged pipe is the difference between N_1 and N_2 . Therefore

$$N = N_1 - N_2 = 615\,000 - 6\,900 \approx 615\,000 \text{ cycles}$$

As we see, and perhaps to our surprise, the hydrostatic testing of pipes at 1.5 times the rated weep pressure has negligible effect on its cyclic durability. The number of cycles to weep remains practically unchanged after the pressure test.

Example 11 – Determine the spike pressure (strain) required to reduce the weep durability of a vinyl ester UD55 pipe to 1/3 of its original value. Assume the same parameters and operating conditions of example 10.

The weep threshold is $T_w = 0.55\%$ and the peak operating strain is $\varepsilon = T_w/2 = 0.275\%$. Since the pipe operates under pure cyclic loading, the fatigue threshold is $T_o = 0.05\%$ and $N_o = 10^{20}$.

The number of cycles to weep the pristine pipe is, as in example 10

$$\log(0.275) = \log(0.55) - 0.052 \log N_1$$

$$N_1 = 615\,000 \text{ cycles.}$$

We wish to determine the spike strain that would infiltrate the pipe in

$$N_2 = N_1/3 = 205\,000 \text{ cycles}$$

Let ω designate the unknown spike strain. The hypothetical weep regression equation is

$$\log(0.275) = \log(\omega) - \frac{\log(\omega/0.05)}{20} \times \log(205\,000)$$

Solving for ω , we have

$$\omega = 0.51\%$$

This surprising and amazing result shows the required spike strain to be 0.51%, just slightly short of the weep threshold 0.55%. Our analysis seems to indicate that spike pressures up to the weep threshold are not overly detrimental to the durability of pipes.

Appendix 1

Idealized regression lines

The experimental cyclic lines do not capture the failure thresholds. At few cycles, they predict higher strains than the actual values. See figure 4. Failure to capture the short-term thresholds is risky for design purposes.

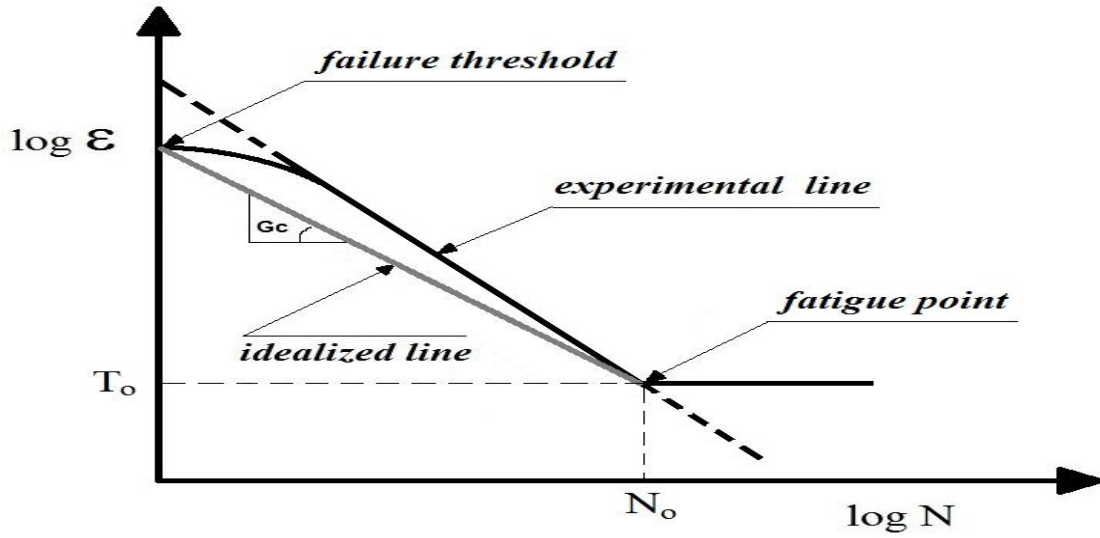


Figure 4

The experimental cyclic regression lines do not capture the actual short-term strengths. The idealized lines capture both the short-term strengths and the fatigue threshold.

To eliminate this difficulty, we modify the cyclic regression lines to capture both the short-term ply strength and the long-term fatigue threshold. The idealized regression equations take the form

$$\log(\epsilon) = A_c - G_c \log(N)$$

Where

$A_c = \log(\text{failure threshold})$

G_c is the slope connecting the short-term failure threshold and the long-term fatigue threshold.

N is the number of cycles to failure

ϵ is the applied peak strain.

The idealized cyclic lines connect the long-term fatigue threshold and the short-term strengths (the failure thresholds). From figure 4, it is apparent that these lines give conservative predictions compared to the actual values.

Appendix 2

Deriving the fatigue point (T_0 , N_0)

In this appendix, we take a closer look at the fundamental concept of fatigue threshold T_0 and the fatigue point (T_0 , N_0). We have an intuitive feeling for the existence of a minimum ply damage, incurred at its creation. It is impossible to create a damage-free ply. We can associate this minimum “creation damage” with an equivalent strain T_0 , that we call the fatigue threshold.

Let us assume a ply failing from growing strains. The damage path of the ply starts at this pristine damage state (the fatigue threshold T_0), growing from there through all intermediate stages until failing at the damage corresponding to the failure threshold. Nothing could be more obvious than that.

The same reasoning applies to decreasing loads. As the loading strain gets smaller, the decreasing crack densities eventually merge with the pristine damage state existing at its creation. No damage can be less than this pristine state of damage. The fatigue threshold T_0 is, therefore, the strain corresponding to the absolute minimum “crack density” at the creation of the ply.

When expressed in the local ply frame, all pristine fatigue thresholds T_0 should have the same value, regardless of ply construction, resin system and R ratios. The only requirement is its reference to the ply local frame.

The scant data available at this time seems to indicate the existence of an isotropic fatigue threshold T_0 independent of the resin matrix and fiber array. The published data indicate the same fatigue strain value $T_0 = 0.05\%$ for all plies, regardless of anything. This situation is particularly puzzling when we consider the different mechanisms of fiber-dominated ruptures and resin-dominated failures. Surprisingly, the experiments seem to indicate the same fatigue threshold $T_0 = 0.05\%$ in all cases. Better measurements are required to confirm this. We will return to this topic later.

In this book, we assume the existence of a fatigue threshold $T_0 = 0.05\%$ for all plies, all resins, all fiber arrays and all modes of failure. The only requirement is its reference to the ply local frame.

Figure 3 shows the four regression lines – one for each mode of failure – connecting the fatigue threshold point, defined by (T_0, N_0) , to the short-term failure thresholds. The four cyclic regression equations come from a simple inspection of figure 2.

$$\begin{aligned} \log \epsilon &= \log(T_r) - \frac{\log(T_r/T_0)}{\log(N_0)} \times \log N \\ \log \epsilon &= \log(T_s) - \frac{\log(T_s/T_0)}{\log(N_0)} \times \log N \\ \log \epsilon &= \log(T_w) - \frac{\log(T_w/T_0)}{\log(N_0)} \times \log N \\ \log \epsilon &= \log(T_i) - \frac{\log(T_i/T_0)}{\log(N_0)} \times \log N \end{aligned}$$

In the above, T_0 and N_0 are respectively the strain and number of cycles at the fatigue point. The fatigue threshold is $T_0 = 0.05\%$ for all loadings and all plies. The number of cycles, N_0 , vary with the load ratio R. For pure cyclic loadings ($R = 0$), $N_0 = 10^{20}$. For non-pure cyclic loadings, $N_0 > 10^{20}$. For pure static equations ($R = 1$), $N_0 = \infty$.

We will pause our discussion to review what we have just done. We have developed the four regression equations for any ply from their known short-term strengths and the fatigue threshold point (T_0, N_0) . The fatigue threshold is $T_0 = 0.05\%$, while the number of cycles N_0 varies with the R ratio. See figure 5.

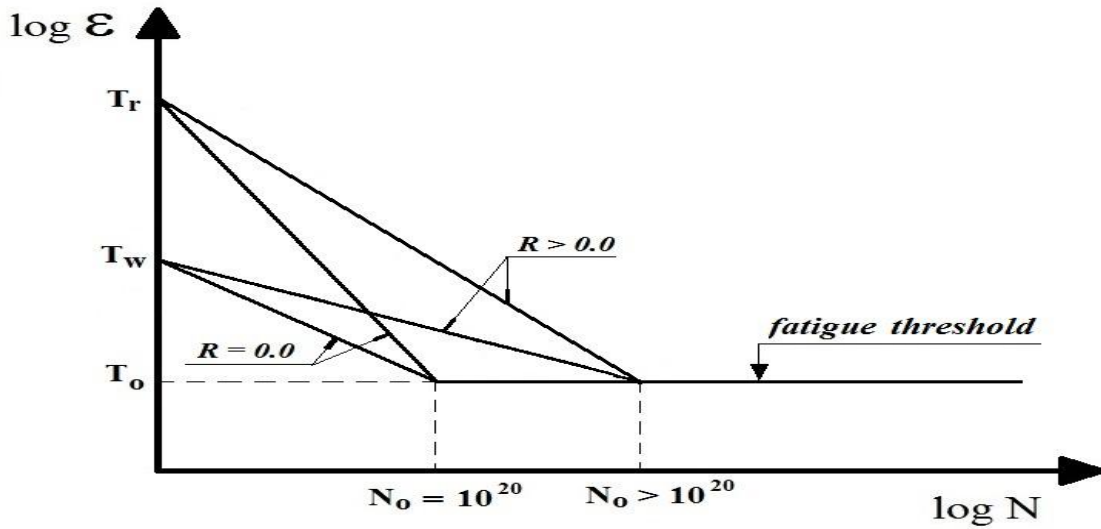


Figure 5

The regression slopes decrease with increasing load ratio R . The fatigue threshold $T_o = 0.05\%$ is a universal constant. The cyclic regression lines connect the fatigue threshold to the short-term failure thresholds. The reader will note the line slopes decreasing with increasing R ratios. The slope is a maximum at $R = 0.0$ (pure cyclic) decreasing to $G_c = 0$ at $R = 1.0$ (pure static).

For pure cyclic loadings, $R = 0.0$, the four regression lines converge at the fatigue point defined by $T_o = 0.05\%$ and $N_o = 10^{20}$. For non-pure loadings ($R > 0.0$) the threshold point moves to the right, resulting in $N_o > 10^{20}$.

It is reasonable to expect different fatigue thresholds T_o for fiber-dominated versus resin-dominated failures. Furthermore, T_o should vary with the fiber array. Yet, experimental evidence seems to indicate the same value for the fatigue threshold strain in all these situations. To confirm this, we consider three pure cyclic regression equations tested at the load ratio $R = 0$

$$\log(\epsilon) = \log(3.00) - 0.089 \log(N)$$

UD ply, fiber-loaded, $R = 0.0$

$$\log(\epsilon) = \log(0.25) - 0.034 \log(N)$$

UD ply, resin-loaded, $R = 0.0$

$$\log(\epsilon) = \log(1.50) - 0.075 \log(N)$$

Chopped fibers, fiber-dominated, $R = 0.0$

The above equations, combining fiber and resin dominated failures, with different fiber arrays and resins, converge on the same point

$T_o = 0.05\%$

$N_o = 10^{20}$ cycles.

This unbelievable result seems to indicate that **all pure cyclic lines**, for both fiber and resin-dominated failures, regardless of resin and fiber array, converge at the same fatigue threshold point $T_o = 0.05\%$ and $N_o = 10^{20}$. The only requirement is the T_o reference to the local frame of the ply.

The fatigue threshold strain $T_o = 0.05\%$ seems to be a universal constant, independent of anything. The number of cycles N_o at the fatigue point, however, increases with the load ratio R . For $R > 0.0$, $N_o > 10^{20}$. This is apparent from the two experimental equations below, developed by Guangxu Wei for UD plies cycled at $R = 0.5$.

$$\log(\epsilon) = \log(3.0) - 0.065 \times \log(N)$$

Fiber loaded UD ply, $R = 0.5$

$$\log(\epsilon) = \log(0.25) - 0.027 \times \log(N)$$

Transverse loaded UD ply, $R = 0.5$

Entering the constant fatigue threshold $T_o = 0.05\%$ in the above equations, we have

$$\begin{aligned} N_o &= 10^{27} && (\text{fiber-dominated failure}) \\ N_o &= 10^{25} && (\text{resin-dominated failure}) \end{aligned}$$

Table 6 summarizes these findings

<i>R ratio</i>	<i>Fatigue threshold point</i>	<i>Regression equations</i>
$R = 0.0$ (pure cyclic)	$T_o = 0.05\% \quad N_o = 10^{20}$	$\log(\epsilon) = \log(T_i) - \frac{\log(T_i/0.05)}{20} \times \log(N)$ $\log(\epsilon) = \log(T_w) - \frac{\log(T_w/0.05)}{20} \times \log(N)$ $\log(\epsilon) = \log(T_s) - \frac{\log(T_s/0.05)}{20} \times \log(N)$ $\log(\epsilon) = \log(T_r) - \frac{\log(T_r/0.05)}{20} \times \log(N)$
$0.0 < R < 1.0$	$T_o = 0.05\% \quad N_o > 10^{20}$	$\log(\epsilon) = \log(T_i) - \frac{\log(T_i/0.05)}{\log(N_o)} \times \log(N)$ $\log(\epsilon) = \log(T_w) - \frac{\log(T_w/0.05)}{\log(N_o)} \times \log(N)$ $\log(\epsilon) = \log(T_s) - \frac{\log(T_s/0.05)}{\log(N_o)} \times \log(N)$ $\log(\epsilon) = \log(T_r) - \frac{\log(T_r/0.05)}{\log(N_o)} \times \log(N)$
$R = 1.0$ (pure static)	$T_o = 0.05\% \quad N_o = \infty$	$\epsilon = T_i$ $\epsilon = T_w$ $\epsilon = T_s$ $\epsilon = T_r$

Table 6

The fatigue threshold T_o corresponds to the ply damage at their formation. $T_o = 0.05\%$ for all plies, regardless of R ratio and fiber array. The number of cycles N_o at the fatigue point, however, increases with the R ratio.

The discussion initiated in chapter 9 and further developed in this chapter, have reached far deeper conclusions than originally expected. My initial goal was simply to present an overview of the regression equations and look for simplifying connections between them. Eventually, this effort led me to the astonishing fatigue threshold concept. I finish this chapter with the amazing realization that knowledge of the fatigue threshold and the short-term failure thresholds suffice to derive all pure regression equations for any ply or laminate, regardless of resin matrix and fiber array.

Example 12 – The pristine cracks developed at the formation of plies have the following causes.

- Resin cure shrinkage.
- Differential resin-fiber thermal and hydric expansions.
- Fiber content and array.
- Peak cure temperature.

The different plies develop different pristine cracks at their formation, giving rise to different T_0 values. The plies of chopped fibers have their own crack densities at formation, depending on the resin, the glass loading and the peak cure temperature. The same goes for UD plies. However, the small differences between these values at the pristine level implies in all plies having the same fatigue threshold $T_0 = 0.05\%$.

Example 13 – The reader may be curious to see at least one laminate equation. There are many such equations in the literature, developed for a variety of applications, like wind blades and others. Figure 6 shows the experimental rupture lines for two laminates, developed by the legendary John Mandel. See reference 31.

The experiments involved the fatigue rupture of epoxy and polyester laminates of identical construction $[\pm 45/0/\pm 45/0/\pm 45]$. The R ratio is $R = 0.1$, which we take as pure cyclic. We have two comments regarding figure 6.

- First, as we have repeatedly warned, the testing of laminates produces specific equations applicable only to the tested specimens and load ratio.
- Second, the equations for the two curves on log-log space are

$$\begin{aligned}\log(\epsilon) &= 0.784 - 0.147 \log N && \text{(Epoxy)} \\ \log(\epsilon) &= 0.456 - 0.128 \log N && \text{(Polyester)}\end{aligned}$$

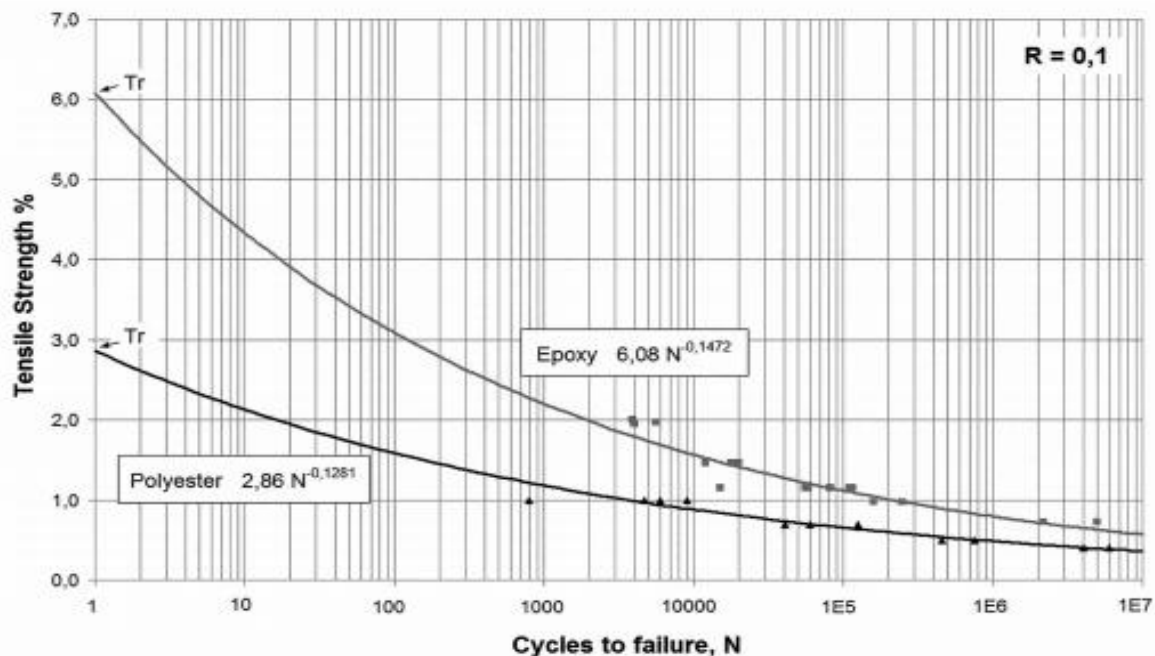


Figure 6

Fatigue regression equations for epoxy and polyester laminates of equal construction. On log-log space, these equations plot as straight lines.

By definition, the fatigue threshold coordinates (T_0, N_0) for the tested laminates are obtained by solving the system

$$\log(T_0) = \log(6.08) - 0.147 \times \log N_0$$

$$\log(T_0) = \log(2.86) - 0.128 \times \log N_0$$

$$\log(T_0) = 0.784 - 0.147 \log N_0$$

$$\log(T_0) = 0.456 - 0.128 \log N_0$$

Which gives

$$N_0 = 10^{20}$$

$$T_0 = 0.01\%$$

This is the rupture fatigue point for the tested laminates. The reader will note the discrepancy between the fatigue threshold $T_0 = 0.05\%$ and the computed value 0.01% . This discrepancy comes from the wide scatter in the rupture failure of complex laminates.

Chapter 13

Part 2

Weep equations of pipes

1 – Introduction: The first part of this chapter dealt with the pure static and the pure cyclic regression equations of plies. The high point of those discussions was the introduction of the fatigue point, essential to derive the cyclic equations of any ply. The fatigue point concept applies to both plies and laminates, since laminates are a combination of plies.

The second part of this chapter deals with the laminate regression equations. We start by recognizing that the laminate equations should be equal to those of their critical plies, since both have the same failure thresholds and fatigue points. In other words, failure of the critical ply implies in failure of the laminate. This is true for the infiltration, stiffness and rupture failure of laminates, whose thresholds are identical to those of their plies. The exception is the weep failure of pipes, where the critical plies always fail first.

The weep delay in pipes results from the time taken by the leaking water to traverse the cracked wall. To take into account the delay time, we define a weep failure threshold for pipes, A_w , which is substantially higher than the weep threshold of plies, T_w .

Failure of the critical ply indicates the onset of the weep process, not the actual weep. The actual weep occurs later, after the leaking water traverses the pipe wall.

2 – Weep equation of pipes: From this introduction, we take three conclusions.

- *The infiltration, stiffness and rupture equations of plies are unconditionally applicable to any laminate, including pipes.*
- *The weep equation of plies does not account for the travel time observed in pipes*
- *The actual weep equation should use the pipe weep threshold, A_w , in place of the ply weep threshold, T_w .*

Table 1 summarizes the failure thresholds for pipes and plies

<i>Failure threshold</i>	<i>Plies</i>	<i>Pipes</i>
<i>Infiltration</i>	T_i	T_i
<i>Weep</i>	T_w	A_w

<i>Stiffness</i>	T_s	T_s
<i>Rupture</i>	T_r	T_r
<i>Fatigue</i>	T_0	T_0

Table 1.

The pipes and plies share the same failure thresholds for infiltration, stiffness and rupture. The exception is the pipe weep threshold, A_w , which is substantially higher than T_w .

The cyclic weep equation of plies, derived in the part 1 of this chapter, is

$$\log(\epsilon) = \log(T_w) - \frac{\log(T_w/T_0)}{\log(N_0)} \times \log(N)$$

The cyclic weep equation of pipes is similar, substituting A_w for T_w

$$\log(\epsilon) = \log(A_w) - \frac{\log(A_w/T_0)}{\log(N_0)} \times \log(N)$$

It is customary to express the pipe strains in the global frame x, y. Therefore, the weep threshold, A_w , is the hoop strain that weeps the pipe in the short-term – in other words, the weep strength of the pipe. For consistency, the fatigue threshold T_0 should enter the pipe equation rotated to the hoop direction.

The unified equation requires two inputs, the long-term static and the long-term cyclic weep strengths of the pipe.

To compute the long-term cyclic weep strength, we need the pure cyclic weep equation of the pipe, at $N_0 = 10^{20}$.

$$\log(\epsilon) = \log(A_w) - \frac{\log(A_w/T_0)}{20} \times \log(N)$$

With T_0 expressed in hoop direction.

The long-term cyclic weep strength S_c of the pipe is

$$\log(S_c) = \log(A_w) - \frac{\log(A_w/T_0)}{20} \times \log(Y)$$

Where Y is the number of cycles expected in the lifetime of the pipe.

The unified equation also requires the pure static weep strength of the pipe. As explained in the next section, this static strength is equal to the weep threshold of the critical ply.

$$S_s = T_w$$

For consistency, all strains should enter the pipe equations in the same reference frame, usually the global frame. Therefore, we rotate T_0 and T_w from the local to the global frame.

Table 2 summarizes the preceding arguments.

<i>Long-term strengths</i>	<i>Pipes</i>	<i>Plies</i>
<i>Pure cyclic</i>	$\log(S_c) = \log(A_w) - \frac{\log(A_w/T_0)}{20} \times \log(Y)$	$\log(S_c) = \log(T_w) - \frac{\log(T_w/T_0)}{20} \times \log(Y)$
<i>Pure static</i>	$S_s = T_w$	$S_s = T_w$
<i>Remarks</i>	<i>All strains in the global frame, including Tw and To</i>	<i>All strains in the local frame</i>

Table 2

Computation of the pure cyclic and pure static long-term weep strengths for pipes and plies.

Example 1 – Explain the difference between the weep equations for pipes and plies.

The pipe equation predicts the actual weep time. The ply equation predicts the onset of the weeping process.

Example 2. Compare the travel times (weep times) in pure cyclic and pure static failures.

The travel times, or weep times, depend on the crack densities and the crack openings. The following situations apply.

$$\begin{bmatrix} \text{travel time} \\ \text{static load} \end{bmatrix} = \begin{bmatrix} \text{large strains (above } T_w) \\ \text{cracks fully open all the time} \\ \text{small density of large cracks} \end{bmatrix}$$

$$\begin{bmatrix} \text{travel time} \\ \text{cyclic load} \end{bmatrix} = \begin{bmatrix} \text{small strains (below } T_w) \\ \text{cracks partially open during the cycle} \\ \text{large density of small cracks} \end{bmatrix}$$

For equal peak strains, the weep times are longer in cyclic than in static loadings. .

3 – Reconciling A_w , T_o and T_w in UD55 pipes: The absence of weep barriers in UD55 oil pipes requires the global hoop strains of the pipes, A_w , and local transverse strain of the UD55 ply, T_w , be expressed in the same reference frame. In our case, we choose the global frame. The critical plies of isotropic chopped fibers present in UD70 sanitation pipes require no such reconciliation.

The reconciliation of A_w , T_o and T_w for UD55 pipes is necessary to give the same results in the global and the local frames. The following numerical example shows this

Example 3. Derive the expression to convert local into global strains in UD55 pipes.

We illustrate the computation for UD55 pipes under 2:1 loading. As discussed in chapter 5, the ply strains in the global frame of UD55 pipes under 2:1 loading are

$$\begin{bmatrix} N \\ 2N \\ 0 \end{bmatrix} = \begin{bmatrix} 132470 & 96220 & 0 \\ 96220 & 235070 & 0 \\ 0 & 0 & 101220 \end{bmatrix} \times \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix}$$

Where N is the external force per unit length with the strains in the global system. Expanding the above, we have

$$\epsilon_y = 7.71 \times 10^{-6} \times N$$

$$\epsilon_x = 1.95 \times 10^{-6} \times N$$

These are the global hoop and axial strains. Dividing through, we obtain

$$\epsilon_y = 3.96 \times \epsilon_x$$

The local strains are

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \end{bmatrix} = \begin{bmatrix} \cos^2 55 & \sin^2 55 \\ \sin^2 55 & \cos^2 55 \end{bmatrix} \times \begin{bmatrix} \epsilon_x \\ \epsilon_y \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \end{bmatrix} = \begin{bmatrix} 0.33 & 0.67 \\ 0.67 & 0.33 \end{bmatrix} \times \begin{bmatrix} \epsilon_x \\ \epsilon_y \end{bmatrix}$$

From this, the local transverse strain in the UD55 ply is

$$\epsilon_2 = 0.67 \times \epsilon_x + 0.33 \times \epsilon_y$$

$$\epsilon_2 = 0.67 \times \frac{\epsilon_y}{3.96} + 0.33 \times \epsilon_y$$

$$\epsilon_2 = 0.50 \times \epsilon_y$$

For UD55 pipes under 2:1 loadings, the hoop (global) strains are twice as large as those transverse to the ply.

From this, we have

$$\left[\begin{array}{l} \text{Weep threshold} \\ \text{in the global frame} \end{array} \right] = 2 \times T_w$$

Likewise, the fatigue threshold T_0 in the global frame is

$$\left[\begin{array}{l} \text{Fatigue threshold} \\ \text{in the global frame} \end{array} \right] = 2 \times T_0$$

It is easier and more intuitive to discuss the durability of pipes in the global system. This, however, is a matter of preference, not of necessity. We could certainly carry our discussion in the local frame as well.

Example 4 – Compute the weep pure regression equations for UD55 pipes under 2:1 loading. Assume pipes of vinyl ester Derakane DK 411.

The UD55 pipes have no weep barrier of chopped fibers, meaning all strains referred to the global system.

The pipe weep threshold is $A_w = 2.5\%$ in the global system

The ply weep threshold is $T_w = 0.55\%$ in the local system

The fatigue threshold is $T_0 = 0.05\%$ in the local frame

Pure static line. The pure static weep equation for the pipes in the global frame is

$$\epsilon = 2 \times T_w = 2 \times 0.55\% = 1.10\%$$

Pure cyclic line. The pure cyclic weep regression equation for the pipes in the global system is

$$\log(\epsilon) = \log(A_w) - \frac{\log(A_w/T_0)}{\log(N_0)} \times \log(N)$$

The measured hoop weep strength of the pipe is $A_w = 2.5\%$.

Therefore, the pure cyclic weep equation for the UD55 pipe is

$$\log(\epsilon) = \log(2.5) - \frac{\log(2.5/2 \times 0.05)}{\log(10^{20})} \times \log(N)$$

$$\log(\epsilon) = \log(2.5) - 0.070 \times \log(N)$$

This global pure cyclic weep equation is applicable to all UD55 pipes of wall thickness equal to that of the tested specimen. The above equation, derived from a simple static pressure testing, replaces the expensive and extremely hard to obtain experimental ASTM D2992A cyclic line.

As a comparison, the ply pure cyclic weep equation in the hoop direction is

$$\log(\epsilon) = \log(2 \times 0.55) - \frac{\log(2 \times 0.55/2 \times 0.05)}{\log(10^{20})} \times \log(N)$$

$$\log(\epsilon) = \log(1.10) - 0.052 \times \log(N)$$

The global ply equation marks the onset of weeping, i.e., the start of water movement that eventually leads to weeping. By contrast, the global pipe equation marks the start of the actual weeping. You can actually see the pipe weep, but not the onset of the weeping process.

Example 5. Compare the cyclic global pipe and ply lines derived in examples 3 and 4.

The global pipe cyclic weep equation is

$$\log(\epsilon_{pipe}) = \log(2.50) - 0.070 \times \log(N_{pipe})$$

The global ply cyclic equation at the weep onset is

$$\log(\epsilon_{ply}) = \log(1.10) - 0.052 \times \log(N_{ply})$$

Subtracting the two equations, we have

$$\log(\epsilon_{pipe}/\epsilon_{ply}) = \log(2.50/1.10) + 0.052 \times \log(N_{ply}) - 0.070 \times \log(N_{pipe})$$

$$\log(\epsilon_{pipe}/\epsilon_{ply}) = 0.357 + 0.052 \times \log(N_{ply}) - 0.070 \times \log(N_{pipe})$$

Since the hoop strains on the pipe are the same as those on the ply, we have

$$\log(1.00) = 0.357 + 0.052 \times \log(N_{ply}) - 0.070 \times \log(N_{pipe})$$

$$0.070 \times \log(N_{pipe}) = 0.357 + 0.052 \times \log(N_{ply})$$

$$\log(N_{pipe}) = 5.100 + 0.743 \times \log(N_{ply})$$

This equation relates the number of cycles to weep pipes to number of cycles to the onset of weep. Taking, as an example, $N_{ply} = 1$ cycle, we have

$$\log(N_{pipe}) = 5.1$$

$$N_{pipe} = 125\,000 \text{ cycles}$$

Table 3 shows a few select numbers derived from the above equation. The number of cycles to weep pipes is so large as to render the ply equations meaningless. The travel times control the weep failure of pipes.

<i>Number of cycles to the onset of weeping</i>	<i>Number of cycles to actual weeping</i>
$N_{ply} = 1$	$N_{pipe} = 125\,000$
$N_{ply} = 1\,000$	$N_{pipe} = 21,3 \times 10^6$
$N_{ply} = 100\,000$	$N_{pipe} = 653.1 \times 10^6$
$N_{ply} = 1\,000\,000$	$N_{pipe} = 3.6 \times 10^9$

Table 3

The number of cycles to weep pipes is several orders of magnitude higher than that for the weep onset of plies. This implies in weep failures controlled by the travel times in actual pipes. This table compares pure cyclic loadings. A complete analysis, including static and cyclic load components, requires the unified equation

Example 6 – *A central topic in the computation of the load-dependent weep durability of pipes is the complexity introduced by the simultaneous action of cyclic and static load components.*

The classical solution to this problem consists in developing experimental regression equations for the pipes at the R ratio of interest. The experimental regression equations are very accurate. However, their specific application to the tested pipes and loading implies that changes in laminate construction, materials, wall thickness, and R ratio requires a new expensive equation.

A second possibility is to use the unified equation with tabulated ply data. The tabulated ply data require no further testing. This solution, however, does not capture the delay time, giving inaccurate results.

A third solution, introduced in this chapter, is to use the unified equation with pipe data. The pipe data require a simple pressure test measurement. This solution includes the travel time, giving good results.

We have, therefore, three competing solutions to the weep durability of pipes

- *The very accurate experimental equations, which are expensive and specific to the tested pipes and loadings*
- *The inaccurate unified equation with ply inputs, at zero cost.*
- *The accurate unified equation with pipe inputs, at low cost and specific to the tested pipes*

The best solution is, no doubt, the unified equation with pipe data. The only downside in this solution is the cost to pressure test pipe specimens to weep.

Example 7 – In this numerical example, we compute the pure cyclic and pure static weep equations for sanitation underground UD70 polyester pipes. The reader knows that all sanitation pipes have a weep barrier of chopped fibers.

The critical weep barrier of chopped fibers is isotropic, meaning T_o and T_w having the same values in all frames.

The following data is available:

$\epsilon_x = 0$ (Underground pipes constrained by surrounding soil)
 $D = 500$ mm is the pipe diameter
 $T_w = 0.80\%$ is the ply weep threshold in the hoop direction
 $T_o = 0.05\%$ is the fatigue threshold in the hoop direction
 $t = 7.0$ mm is the wall thickness
 $E_y = 250\,000$ kg/cm² is the hoop modulus.
 $P_w = 120$ kg/cm² is the short-term weep pressure, determined at $\epsilon_x = 0$

The pipe weep threshold A_w in the hoop direction is

$$A_w = \frac{P_w \times D}{2 \times E_y \times t}$$

$$A_w = \frac{120 \times 500}{2 \times 250000 \times 7.0} = 1.70\%$$

The pipe's weep pure cyclic regression equation is

$$\log(\epsilon) = \log(A_w) - \frac{\log(A_w/T_o)}{\log N_0} \times \log(N)$$

Entering the fatigue threshold point $N_0 = 10^{20}$ and $T_o = 0.05\%$ in the above equation, we have.

$$\log(\epsilon) = \log(1.70) - \frac{\log(1.70/0.05)}{20} \times \log(N)$$

$$\log(\epsilon) = \log(1.70) - 0.077 \times \log(N)$$

This is the pure cyclic weep equation for the pipe.

In the present case, all strains are in the hoop direction. Since the critical ply is isotropic (chopped fibers) the values of T_o and T_w need no rotation. Likewise, the pure static weep equation of the pipe in the hoop direction is

$$\epsilon = T_w = 0.80\%$$

We have just derived the pure cyclic and the pure static weep equations for the underground UD70 sanitation pipe. These equations provide the required inputs to the unified equation, completely solving the weep durability for any load ratio R . The downside of this solution is the measurement of A_w .

This numerical example illustrates the derivation of the pipe's pure cyclic weep equation from a simple, quick and inexpensive short-term pressure test. The pure cyclic weep equation we have

just derived is not the same as the ASTM D 2992A equation. The attractiveness of this pure cyclic pipe weep equation is its extremely low cost and quick derivation.

Example 8 – *Compute the weep durability of the pipe discussed in example 7.*

This computation requires knowledge of the unified equation, discussed in chapter 16. For the solution, see the numerical example 11 in the part 1 of chapter 16

Example 9 – *This next numerical example is an afterthought, included in the hope that corny repetition clarifies new concepts.*

We propose to illustrate the different methods to compute the weep durability of pipes with an analogy to aircraft crashes

- *The unified equation with ply data predicts the time for the aircraft to run out of fuel*
- *The unified equation with pipe data predicts the time for the aircraft to hit the ground, disregarding the air resistance.*
- *The experimental regression equation predicts the time for the aircraft to hit the ground, taking into account the air resistance.*

These are the differences between the classical experimental method and our proposed new methods.

Appendix 1

Static weep equation of pipes

The static weep equation of pipes is

$$\log(\epsilon) = \log(A_w) - G_s^{pipe} \times \log(hour)$$

Where

A_w is the hoop weep threshold of the pipe

G_s^{pipe} is the regression slope

(hour) is the time to weep

(ϵ) is the static hoop strain

In the above, (hour) indicates the time to actual weeping, i. e, the time for the pressurized water to traverse the strained wall

$$[\text{hour}] = \left[\begin{array}{c} \text{static} \\ \text{weep time} \end{array} \right] = \left[\begin{array}{c} \text{travel time through} \\ \text{the pipe wall} \end{array} \right]$$

Figure 1 shows three static weep lines designated as pipe 1, pipe 2 and pipe 3, bound by two horizontal lines. The lower horizontal bound is the weep threshold, T_w . The upper is the rupture threshold, T_r . No static weeping occurs in pipes below T_w .

Increments in the pipe thickness cause the pure static lines to move up and rotate counterclockwise, as shown in figure 1. All static weep lines turn horizontal on meeting the weep threshold T_w . The intercepts A_w increases with the wall thickness. Thicker pipes mean higher A_w and longer weep times. We have

- $A_w = T_r$ (Extremely thick pipes weep at rupture)
- $A_w = A_3$ (Thicker pipes take longer times to weep)
- $A_w = A_1$ (Thinner pipes take shorter times to weep)
- $A_w = T_w$ (Extremely thin pipes weep instantaneously, like plies)

For consistency, all strains enter figure 1 in the global system, including P_w .

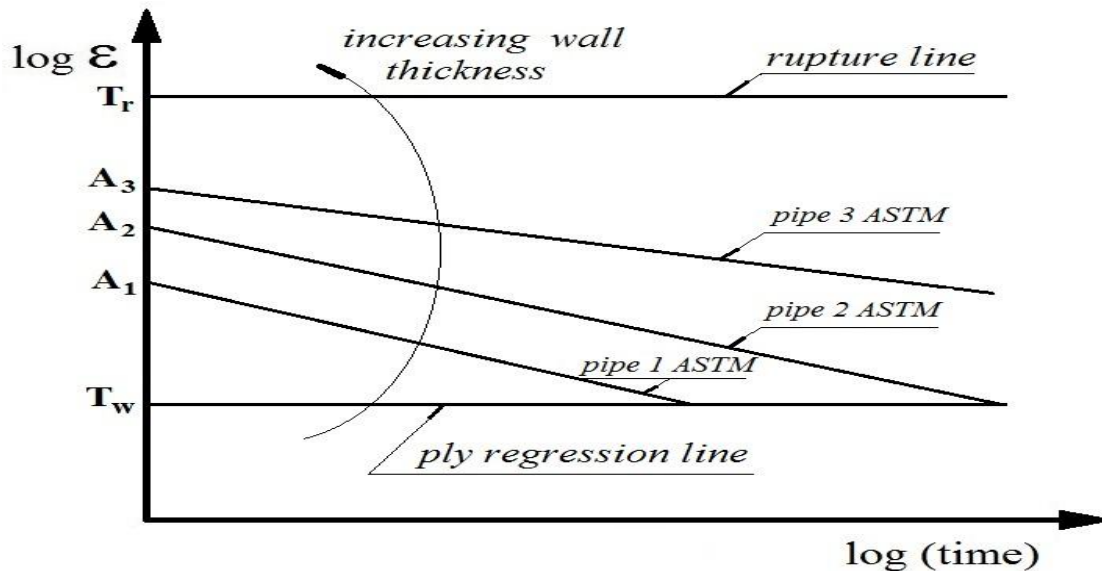


Figure 1

The pure static weep lines of pipes have two horizontal bounds, namely the pipe rupture, T_r , and the ply weep threshold T_w . The static lines – experimentally measured per ASTM D2992B – move up and rotate counterclockwise with increasing wall thickness, as indicated in the figure. All static weep lines turn horizontal on meeting T_w . All strains expressed in the global system.

Appendix 2

Cyclic weep equations of pipes

The lower left corner of figure 2 shows the pure cyclic regression lines of a pipe and a ply. Since these are pure cyclic lines, they connect the fatigue point ($T_0 = 0.05\%$ and $N_0 = 10^{20}$) to their respective weep thresholds, T_w for the pipe, and A_w for the ply.

Figure 2 also show three additional cyclic regression lines, marked as pipe1, pipe 2 and pipe 3. A simple observation of figure 2 leads to the conclusions:

- The regression lines move up with larger thickness
- The regression lines rotate counterclockwise with larger R ratios.
- There is a regression line for every pipe and every loading

For consistency, all strains enter figure 2 in the global system, including T_w and T_o .

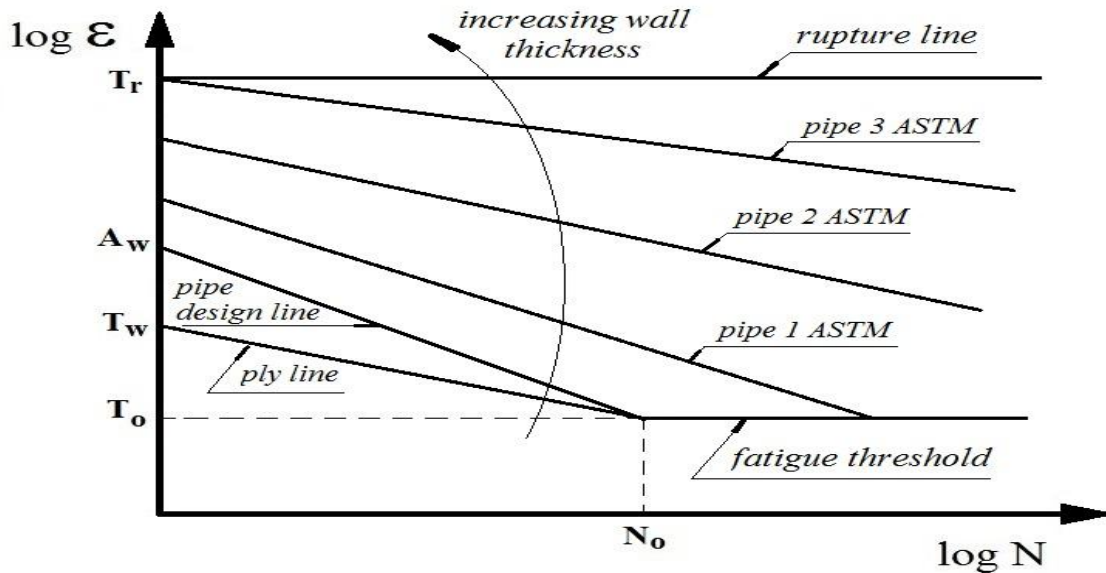


Figure 2

Shows cyclic regression lines bound by the rupture threshold, T_r , and the fatigue threshold, T_o . The weep lines move up with increasing wall thickness. The weep lines and rotate counterclockwise with increasing load ratio R . The pipe pure cyclic equation connects the weep strength, A_w , to the pure cyclic fatigue threshold point $T_o = 0.05\%$ and $N_o = 10^{20}$.

Appendix 3

Measuring the pipe weep threshold A_w

The pipe weep threshold A_w is the hoop strain that weeps pressurized water-filled pipes in less than 10 minutes. The protocol for its measurement is in table 4.

<i>Hoop strain (%)</i>	<i>Test pressure (Kg/cm²)</i>	<i>Hold time (minutes)</i>	<i>Weep</i>	<i>Comments</i>
1.50	$P = 3\,810 \times t/D$	10	No	
1.60	$P = 4\,064 \times t/D$	10	No	
1.70	$P = 4\,318 \times t/D$	10	No	
1.75	$P = 4\,445 \times t/D$	10	No	
1.80	$P = 4\,572 \times t/D$	10	No	
1.85	$P = 4\,699 \times t/D$	10	Yes	$A_w = 1.85\%$

Table 4

Measuring the weep threshold A_w of pipes. The test pressures indicated are for UD55 pipes and 2:1 loadings.

The test pressures should produce the hoop strains indicated. Assuming UD55 pipes under 2:1 loadings, the test pressures are

$$P = \frac{\epsilon \times t}{D(1/2E_y + \nu_{xy}/4E_x)}$$

For UD55 pipes, we have

$$\begin{aligned} E_y &= 170\,800 \text{ kg/cm}^2 \\ E_x &= 96\,300 \text{ kg/cm}^2 \\ \nu_{xy} &= 0.39 \end{aligned}$$

Therefore

$$P = 2540 \times \epsilon \times \frac{t}{D}$$

As an option, we can obtain the hoop strains from strain gauges.

Chapter 14

Part 1

Weep failure and the HDB

1 – Introduction: The tough glass fibers prevent the growth of cracks in composite laminates. As a result, the load-dependent failures of composites comes from the accumulation of many micro-cracks, instead of the growth of a single crack. The rare large cracks observed in composites grow in the resin matrix, parallel to the fibers. As examples, we mention the delamination between plies and the cracks that develop alongside the fibers in UD plies. These resin cracks, together with those from local strain-corrosion, are the only large cracks observed in composite laminates.

The four load-dependent modes of failure of laminates – infiltration, weeping, stiffness and rupture – correspond to four critical crack densities. This chapter discusses the relation between the classical HDB and the critical crack density at the weep threshold.

2 – The classical HDB: The ASTM D 2992B regression equation computes the time to weep of water-filled pipes under constant pressure. The equation itself comes from testing a minimum of 18 water-filled pipe specimens at various constant pressure and annotating the times to weep. To assure failure, the constant pressures – this is a static test – are set above the weep threshold. The weep times depend on the wall thickness and the intensity of the hoop strain. The hoop strains, combined with the corresponding times to weep, generate a regression line of the form

$$\log(\epsilon) = A_w + B_w \times \log(t) - G_w \times \log(\text{hour}) \quad (1)$$

Where

ϵ is the static hoop strain that weeps the pipe.

t is the pipe wall thickness

(hour) is the time to weep

A_w , G_w and B_w are experimental parameters

The ASTM D 2992B weep equation is

$$\log(\epsilon) = A_w - G_s^{pipe} \times \log(\text{hour}) \quad (1A)$$

This is a pipe, not a ply, equation. The intercept A_w is the hoop short-term weep strength of the pipe, introduced in chapter 9. Later in this book, we will make extensive use of this pipe

parameter A_w . The regression slope G_s^{pipe} is a pipe parameter that decreases with increments in the wall thickness. For details, see the part 2 of chapter 13.

Contrary to widespread belief, the weep times of pressurized pipes do not reflect any crack growth or progressive pipe damage. Rather, they represent the times taken by the pressurized water to traverse the cracked pipe wall. The weep times from equation (1A) are water travel times, not related to any progressive pipe deterioration.

From the ASTM protocol, the static hoop strain that weeps the pressurized pipe in the long-term comes from extrapolating the regression equation (1A) to 20 years (oil pipes) or 50 years (sanitation pipes). The extrapolated static hoop strain is the classical HDB – Hydrostatic Design Basis – of the tested pipe. The allowable hoop strain is

$$[\text{allowable hoop strain}] = \frac{HDB}{1.8}$$

Where $SF = 1.8$ is the short-term safety factor

The classical HDB is the static, constant, hoop strain that weeps the tested pipes in the long-term. Later in this chapter, we will argue against the HDB and propose its replacement by the weep threshold T_w of the critical ply. For now, however, we proceed with our discussion of the classical HDB.

Example 1. *Explain the difference between the HDB and the pipe weep threshold A_w .*

Both A_w and the HDB are pipe (not ply) strains measured in the hoop direction.

A_w is the static hoop strain that weeps the pipe in the short-term. A_w varies with the pipe thickness and construction.

The HDB is the extrapolated static hoop strain that weeps the pipe in the long-term. The HDB varies with the pipe thickness and construction. The HDB is conceptually incorrect, to be eventually replaced by the ply threshold T_w .

Figure 1 shows typical experimental static weep lines of oil and sanitation pipes, with the classical HDB extrapolated to 50 years.

HDB - Hydrostatic Design Basis

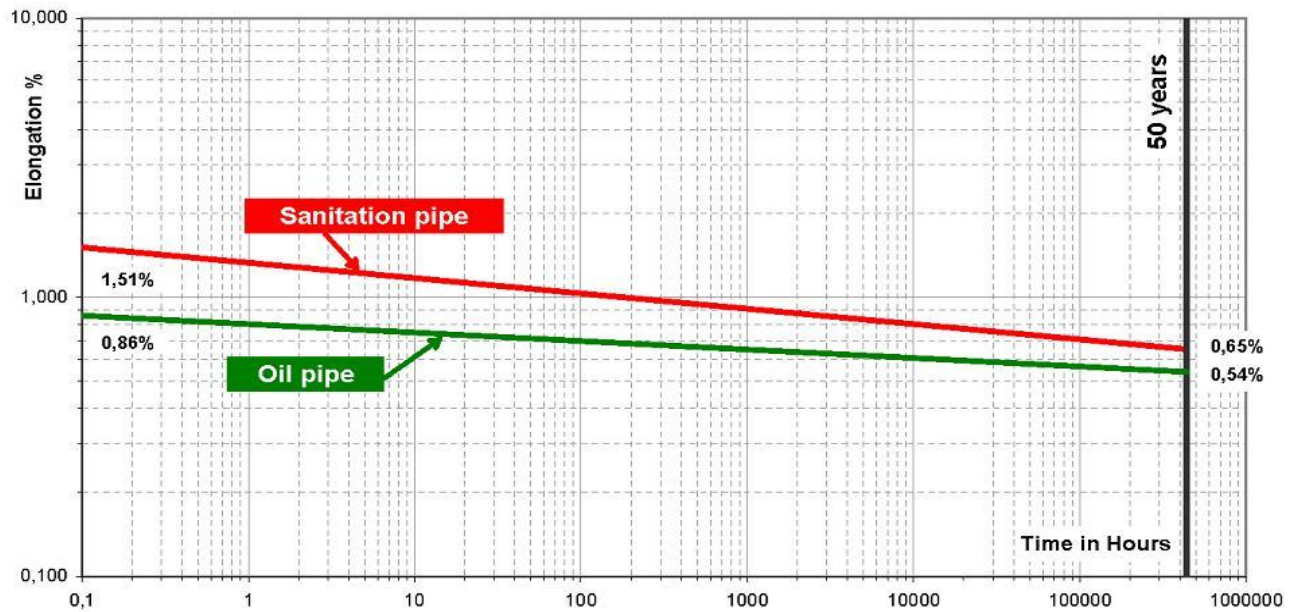


Figure 1

Typical static weep lines of sanitation and UD55 oil pipes. The gradients of these lines depend on the pipe wall thickness and the load ratio R .

Example 2 – Compare the weep and rupture durability of pipes under (a) monotonically increasing pressure, (b) constant pressure, (c) constant peak cyclic pressure and (d) a combination of cyclic and static pressure components.

(a) Suppose a water-filled pipe under a monotonically increasing pressure. The crack density grows in response to the increasing pressure. At first, the low pressures are not large enough to link up the cracks. As the pressures increase, the growing crack densities link up to start the weep process. The weep time is not immediate, as the water should first traverse the cracked wall. Increasing the applied pressure will shorten the weep time. Further pressure increments will eventually burst the pipe. The weep failure usually occurs before the pipe bursts. However, if the rate of pressurization is too fast, or the wall thickness too large, the pipe may burst before weeping.

(b) Consider now water-filled pipes subjected to different levels of non-increasing static pressures. The static pressures do not cause the crack density to grow. If the strains are less than the weep threshold, the pressurized pipes never weep. In this scenario, the pipes eventually rupture from long-term fiber strain-corrosion, without weeping.

(c) Cyclic pressures, unlike static, are capable of growing crack densities regardless of the weep threshold. Cyclic pressures eventually weep pipes before rupture.

(d) The discussion of failure combining cyclic and static pressure components requires the unified equation, introduced in chapter 16.

It is instructive to ponder this numerical example. The arguments presented are indeed powerful and show beyond a reasonable doubt that:

- 1 – Pure static strains above the weep threshold eventually weep the pipes.
- 2 – Pure static strains below the weep threshold never weep the pipes.
- 3 – Pure cyclic strains always weep the pipes.

In the presence of combined static and cyclic loads, no such sweeping generalizations are possible. The solution of durability problems in such cases requires the unified equation.

Example 3 – The weep static regression line for sanitation pipes in figure 1 indicates an extrapolated long-term (50 years) hoop strain of 0.65%. What is the 50 years HDB? What is the allowable hoop strain for a durability of 50 years?

The 50 years HDB is, by definition, 0.65%. The allowable 50 years hoop strain is

$$[\text{allowable hoop strain}] = \frac{0.65\%}{1.8} = 0.36\%$$

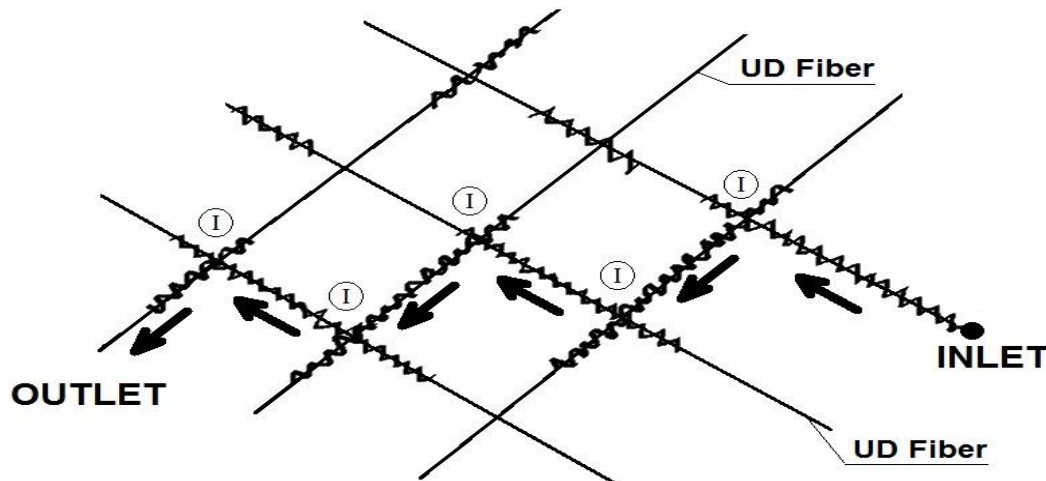


Figure 2

The arrowheads indicate the pathway followed by the weeping water in UD55 oil pipes. The advancing water moves from ply to ply at the crack intersection points “I”.

3 – The weep thresholds. So far, we have been discussing the classical HDB. We move next to the new concept of the weep threshold.

We open the discussion with a description of the mechanism of water percolation in UD pipes. The arrowheads in figure 2 shows the pathway followed by water in traversing the pipe wall. The water moves from the inner to the outer surface of the pipe at the intersection of the large resin cracks running along the fibers in each ply. The weep threshold of UD plies is the transverse strain that allows the formation of crack densities large enough to form these pathways. Since UD plies develop long cracks in the resin along the fibers, the weep threshold should occur immediately after the formation of the first cracks. In other words, the weep threshold of UD plies should not be much higher than their infiltration threshold.

We discuss next the same situation for plies of chopped fibers. Figure 3a shows no cracks under low strains. Figure 3b shows the formation of the first micro-cracks in response to higher strains. The subsequent figures show the formation of pathways in response to increasing crack densities.



Figure 3a

Small strains, below the infiltration threshold, produce negligible crack densities that do not coalesce into pathways.

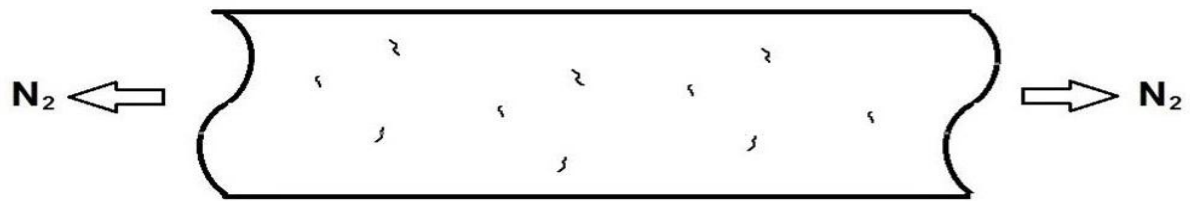


Figure 3b
At the infiltration threshold, the crack density is high enough to coalesce into incipient pathways.

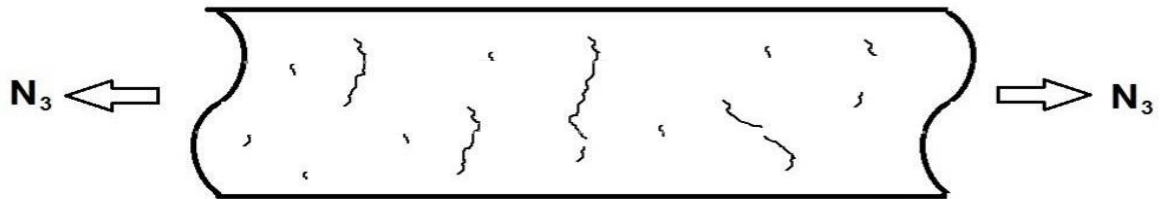


Figure 3c
Larger strains increase the crack density, promoting more coalescence. The reader will recall that the chopped fibers keep the cracks small in the entire process. The larger cracks shown in the figure come from the coalescence of smaller cracks.

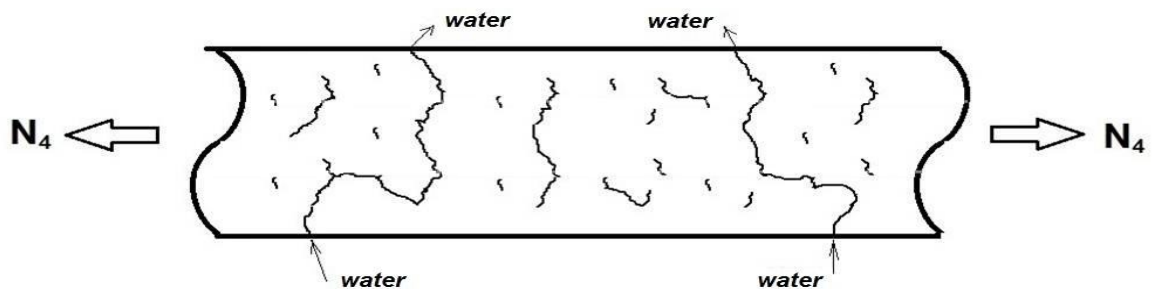


Figure 3d
At still higher strains, the crack densities form pathways for the passage of water. Further strain increments will increase the number and opening of the already coalesced cracks. The critical crack density – expressed as strain – that initiates the formation of such pathways is the weep threshold of the ply. If the strain is less than the weep threshold, the pathways do not form, and no weeping occurs. If the strain exceeds the weep threshold, the pathways will form and weeping will certainly occur. The pathways formed at the weep threshold are too narrow and few, resulting in long times for the water to traverse the pipe wall.

The small thickness of plies imply in their weeping immediately upon reaching the weep threshold. However, in pipes, the times to weep will depend on the wall thickness, the opening and the length of the coalesced cracks. Higher pressures will open the cracks and shorten the time to weep. Larger wall thicknesses will increase the pathway and produce long weep times.

The discussions pertaining to figure 3 leave no doubt that pipes under constant pressures never weep below the weep threshold. However, at pressures above that threshold, all pipes eventually weep. At the weep threshold, the density and openings of the cracks are small, resulting in extremely long weep times. This, of course, depends on the wall thickness. Higher strains will increase the crack densities and their openings, reducing the weep times. The crack density that weeps pipes in the short-term, designated as A_w , are extremely high.

We define two weep thresholds:

- *The ply weep threshold, T_w , is the strain at the onset of weeping. The attainment of T_w marks the first movement of water toward weeping.*
- *The pipe weep threshold, A_w , is the hoop strain causing the immediate weeping of pipes.*

For consistency and proper comparison, these weep thresholds should be in the same reference frame, usually the global pipe frame. From the above definitions, it is obvious that $A_w \gg T_w$.

4 – Liner effect on the weep threshold: This is the moment to correct a widespread misconception about liner toughness. Common sense indicates that increased liner resiliency should improve the weep resistance, A_w , of pipes. This appealing argument is false, however, since thin liners are simply not tough enough to arrest the growth of large coalesced cracks in the underlying laminate.

Therefore, contrary to common sense and widespread belief, liner resiliency is not critical in the weeping process. The large coalesced cracks emerging from the substrate quickly fail the thin liners of neat resin.

Many authors have observed and reported that in-situ liner cracks originate in the substrate, not in the liner itself. For a quick review, see references 5 and 9. The “in-situ rupture strains” of embedded liners are significantly lower than the values measured in isolated sheets. An interesting illustration of this is the rupture of the tough aluminum foils used to make impermeable pipes. A quick and uncritical consideration would lead us to expect the pipes with resilient aluminum foils to fail by rupture, without weeping. Not true. When pressurized above the weep threshold, the pipes with impermeable aluminum foil weep just like any other. The explanation for this “unexpected” result is straightforward and simple. The substrate plies govern the in-situ foil rupture, not the foil itself. The impermeable pipes discussed in chapter 18 have many interesting features, but weep prevention is not one of them.

5 – Inadequacy of the classical HDB. The static lines are excellent to predict the weep times of pipes strained above the weep threshold, that is, of those pipes that have already failed. The commercial pipes operate below the weep threshold and never weep. So, why bother with the development of expensive regression lines applicable to overstrained and unrealistic pipe applications? Furthermore, the static weep lines are specific to the tested pipe. Pipes of different construction and thickness require the development of their own regression lines.

The idea of extrapolating static regression lines comes from the false assumption that stationary cracks grow with time. This false assumption is in direct contradiction with the theory of fracture mechanics. As already mentioned, the weep delay observed in the static testing of pipes are in fact the travel times of water in traversing the pipe wall. The static weep times are wrongfully associated with non-existent stationary crack growth.

The first author to reject the classical HDB was Frank Pickering (reference 8) who, in 1983, proposed to replace the pipe hoop strain at the onset of nonlinear response for the expensive and meaningless HDB. He even named this failure strain as PEL – Proportional Elastic Limit. That idea was a good start, still alive today under the more difficult name of UEWS, for Ultimate Elastic Wall Strain. The PEL, however, is not a replacement for the HDB, as they apply to different situations. The PEL describes the hoop strain of damaged pipes at the stiffness threshold, and very close to A_w . The extrapolated HDB, on the other hand, is the hoop strain of undamaged pipes, far from A_w .

In this book, we are proposing the weep threshold as a proper replacement for the classical HDB. The weep threshold is a better indicator of performance than the onset of pipe nonlinearity. We will have more to say about this in the following section. For now, it suffices to say

that the classical HDB is a flawed predictor of the long-term weep performance of pipes under static pressures.

The substitution of the weep threshold for the classical HDB brings the following benefits to the study of pipes under static loads.

- *The weep threshold is conceptually consistent*
- *The weep threshold of the critical ply is a fundamental material property, valid for all pipes.*
- *The tabulated weep thresholds require no testing or measurement when designing new pipes*

Figure 4 shows the horizontal line corresponding to the weep threshold $T_w = 0.80\%$, superimposed on a classical static weep line of polyester sanitation pipes. The reader will note the solid short-term sloping part of the classical line, valid for strains above the weep threshold. The extrapolated long-term HDB predicted by the classical line (0.65%) falls below the weep threshold $T_w = 0.80\%$.

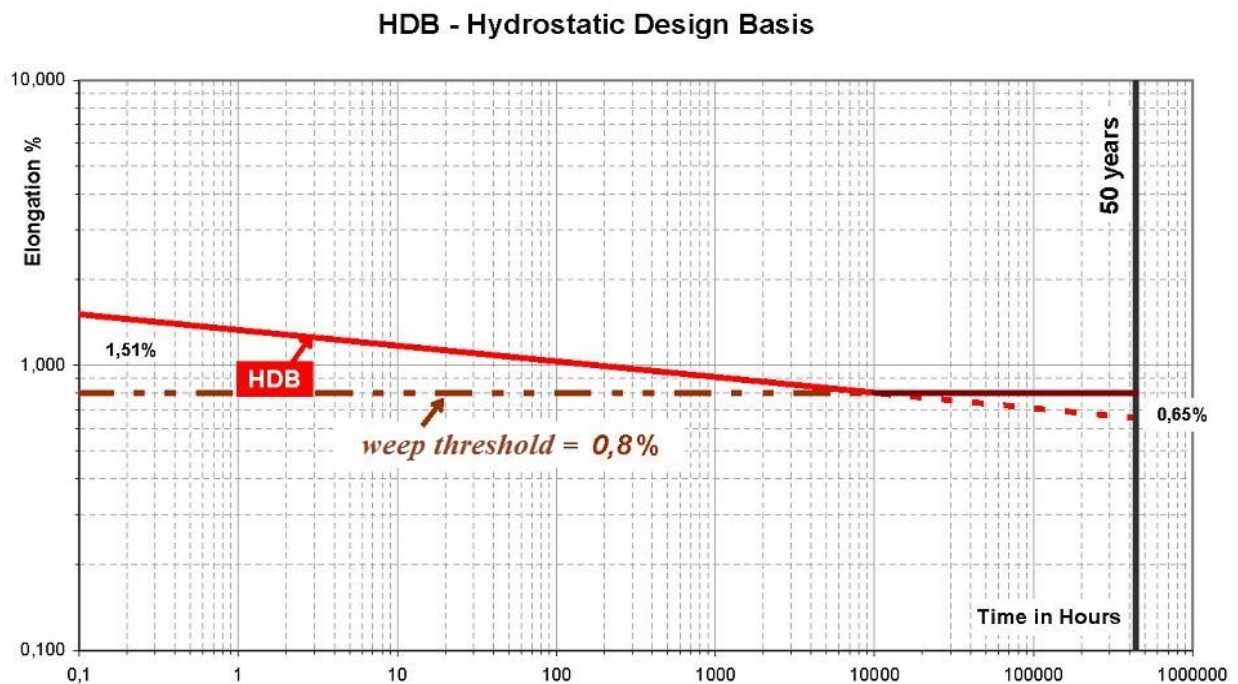


Figure 4
Static weep line of polyester sanitation pipes compared to the weep threshold line. The weep threshold $T_w = 0.80\%$ is higher than the 50 years classical HDB = 0.65% . Looking at the figure, we see a weep line composed of two parts. The sloping part computes the weep times of pipes strained above the weep threshold. The horizontal part corresponds to strains less than the weep threshold. Pipes operating at static strains less than the weep threshold never weep.

6 – Measuring the failure thresholds: This book is about the durability of composites in industrial service, not the measurement of ply properties. However, we believe the reader may be curious to know how to measure the still unfamiliar infiltration, stiffness and weep thresholds. We start by recalling the correlation between ply damage and applied strains. This correlation allows the use of simple to measure ply strains as indicators of the impossible to measure crack densities.

The failure thresholds correspond to special transitions observed in the ply response under load. The indicators of such transitions are:

- *Sudden and sharp increase in the acoustic emissions*
- *Sudden and sharp drop in the light transmission*
- *Sudden and sharp change from linear to non-linear behavior*

- *Sudden and sharp onset of weeping*

The above transitions indicate the strains corresponding to the failure thresholds. Tables 1 and 2, imported from chapter 8, list the threshold values of the most important commercial plies.

	<i>Failure thresholds for transverse-loaded UD plies</i>	
	<i>Tough polyester</i>	<i>Vinyl ester</i>
<i>Infiltration</i>	$T_i = 0.40\%$	$T_i = 0.50\%$
<i>Weep</i>	$T_w = 0.45\%$	$T_w = 0.55\%$
<i>Stiffness</i>	$T_s = 0.70\%$	$T_s = 0.90\%$
<i>Rupture</i>	$T_r = 0.70\%$	$T_r = 0.90\%$

Table 1

This is the same table 4 of chapter 8. We always express the ply thresholds in the local frame 1, 2. For proper comparison, the ply strains need rotation to the global frame x, y.

	<i>Failure thresholds for chopped fiber plies</i>	
	<i>Tough polyester</i>	<i>Vinyl ester</i>
<i>Infiltration</i>	$T_i = 0.40\%$	$T_i = 0.50\%$
<i>Weep</i>	$T_w = 0.80\%$	$T_w = 1.10\%$
<i>Stiffness</i>	-----	-----
<i>Rupture</i>	$T_r = 1.50\%$	$T_r = 2.50\%$

Table 2

This is identical to table 5 of chapter 8. The failure thresholds of isotropic plies do not require rotation.

The weep threshold of pipes, A_w , is a laminate property dependent on the wall thickness, the composition and the R ratio. Every pipe has its own weep threshold A_w , measured by pressure testing water-filled specimens to short-term weeping.

Let us pause for a moment and examine the failure thresholds listed in tables 1 and 2. The following points need consideration.

- *The isotropic chopped plies have the same thresholds in any direction.*
- *The weep threshold of chopped polyester plies is 0.80%. The classical 50 years hoop HDB of polyester sanitation pipes is 0.65%. Assuming a safety factor $SF = 1.8$, the allowable weep hoop strain of sanitation pipes would be $0.8/1.8 = 0.45\%$ and $0.65/1.8 = 0.35\%$ respectively from the weep threshold and the classical HDB.*
- *The vinyl ester resins are the best choice for weep-controlled pipes.*

Note: The long-term safety factor SF for all load-dependent failures (infiltration, weep, stiffness and rupture) derives from the unified equation. In chapter 16, we discuss the unified equation and the long-term SF . The above $SF = 1.8$ is a short-term value.

The failure thresholds in table 1 are valid in the local frames 1, 2 of UD plies. For consistent comparisons with the pipe global strains, they need rotation to the global frame. Equation (2) shows this rotation.

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \frac{1}{2}\gamma_{12} \end{bmatrix} = \begin{bmatrix} \cos^2 \alpha & \sin^2 \alpha & 2(\cos \alpha)(\sin \alpha) \\ \sin^2 \alpha & \cos^2 \alpha & -2(\cos \alpha)(\sin \alpha) \\ -(\cos \alpha)(\sin \alpha) & (\cos \alpha)(\sin \alpha) & \cos^2 \alpha - \sin^2 \alpha \end{bmatrix} \times \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ 0 \end{bmatrix} \quad (2)$$

Where:

α is the angle formed by the UD fibers and the longitudinal (axial) direction.

ε_1 is the strain in the fiber direction

ε_2 is the strain transverse to the fiber

ε_x is the global strain in the axial direction

ε_y is the global strain in the hoop direction.

γ_{12} is the local shear strain

Expanding equation (2), we obtain three strain components on the local frame 1, 2 of the UD plies.

$$\varepsilon_1 = \varepsilon_x \cos^2 \alpha + \varepsilon_y \sin^2 \alpha \quad (2A)$$

$$\varepsilon_2 = \varepsilon_x \sin^2 \alpha + \varepsilon_y \cos^2 \alpha \quad (2B)$$

$$\gamma_{12} = 2 \sin \alpha \cos \alpha (\varepsilon_y - \varepsilon_x) \quad (2C)$$

Equation (2A) gives the strain in the fiber direction. This strain controls the rupture durability of UD plies, as discussed in chapter 15.

Equation (2B) gives the strain transverse to the fiber direction. This strain controls the weep and stiffness failures of oil pipes and wind blades.

Equation (2C) captures the shear strain, relevant in the estimation of the long-term laminate stiffness and weep performance.

The chopped plies are isotropic and do not require strain rotation.

Example 4 – Compute the number of cycles to infiltrate UD55 pipes under pure cyclic loadings. Assume pipes of vinyl ester resin with peak strains $\varepsilon_y = 0.60\%$ and $\varepsilon_x = 0.20\%$

To compute the number of cycles to infiltration failure, we need the transverse strain in the local frame of the UD55 plies

$$\varepsilon_2 = \varepsilon_x \sin^2 \alpha + \varepsilon_y \cos^2 \alpha$$

$$\varepsilon_2 = 0.20 \times 0.67 + 0.60 \times 0.33 = 0.33\%$$

Since these are pure cyclic loadings, the pure cyclic infiltration equations apply, with no need of the unified equation.

The pure cyclic infiltration equation for the UD55 plies in the local frame is

$$\log(\epsilon) = \log(0.50) - \frac{\log(0.50/0.05)}{20} \times \log(N)$$

$$\log(\epsilon) = \log(0.50) - 0.05 \times \log(N)$$

Entering the transverse strain in this equation, we have

$$\log(0.33) = \log(0.50) - 0.050 \times \log(N)$$

$$N = 4100 \text{ cycles}$$

This numerical example emphasizes the importance of always working with strains in the same reference frame.

Example 5 – *There are two approaches to predict the long-term weep failure of composite pipes under static pressures.*

- *The classical ASTM D 2992B assumes pipe deterioration from the steady growth of initially small cracks.*
- *The weep threshold proposes a failure strain below which static weeping never occurs.*

A conclusive experiment to validate either approach requires long-term testing. In 2009, Hogni Johnson reported some test results from pipes deflected in 5.0% sulfuric acid for a period of 30 years. Figure 5 shows the flattened regression line reported for the deflected pipes, exactly as the weep threshold proposed in this chapter. This is the best evidence available at this time in support of the weep threshold concept. For a brief discussion of this interesting experiment, see chapter 11.

Some may argue that strain-corrosion tests on deflected pipes are not valid indicators of weep failure. For a rebuttal of this argument, we show in chapter 11 that the strain-corrosion of deflected pipes are essentially identical to weep failures. The results reported in figure 5 are valid proof of the weep threshold concept.

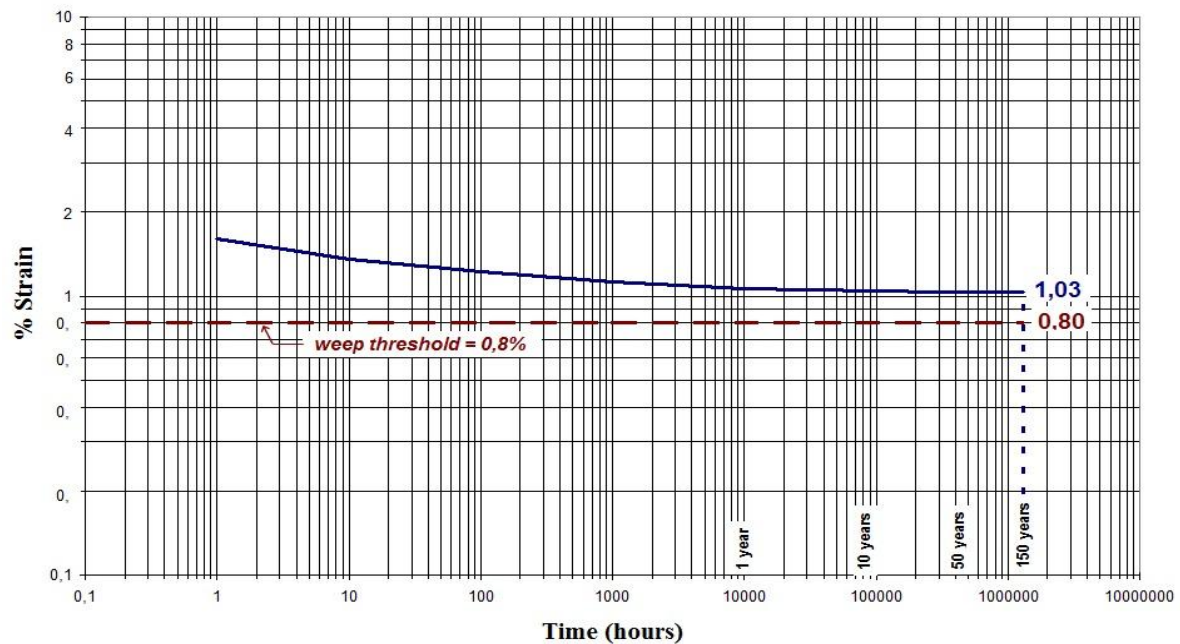


Figure 5

The long-term strain-corrosion tests of composite pipes deflected in 5.0% sulfuric acid shows a regression line levelling off at a hoop strain of 1.03%. The graph also shows the proposed weep threshold value $T_w = 0.80\%$ for the polyester chopped plies in the weep barrier of the tested pipes.

This completes our discussion of the classical HDB versus the weep threshold..

7 – Cyclic loadings: The cyclic and static weep failure of pipes differ in one important point. Static loads do not grow cracks and never fail pipes below the failure thresholds. Cyclic loads, by contrast, always grow cracks, weeping pipes regardless of the weep threshold.

Cyclic loads are the real game changers in the analysis of load-dependent pipe durability. The unified equation is the only tool available to solve the weep durability of pipes.

Chapter 14

Part 2

Weep failure envelopes

1 – Introduction: The weep threshold T_w of the critical ply marks the onset of the weep process in pressurized pipes. The weep times at strains higher than T_w are not immediate, due to the delay from the water traversing the pipe wall. The immediate, short-term leakage of pressurized pipes occurs at a much higher strain A_w , the pipe weep threshold.

At the weep threshold A_w , the pipe displays significant laminate damage, above the stiffness threshold T_s . This is in agreement with experiments that indicate the onset of weeping at pressures slightly above the loss of pipe stiffness.

Table 1 lists the pipe and ply strains of interest in the weep process.

<i>Pipe strain (global frame)</i>	<i>Ply strain (local frame)</i>	<i>Remarks</i>
<i>HDB</i>	<i>T_w</i>	<i>These strains indicate the onset of the weeping process. The pipe damage is negligibly small at these low strains, producing long times to weep. Pipes never weep at static strains below T_w. For consistent comparison, both the HDB and T_w should be in the same reference frame.</i>
<i>UEWS</i>	<i>T_s</i>	<i>These strains indicate the onset of loss of stiffness. At these strains, the pipe damage is significant. Pipes strained at the UEWS, or at the stiffness threshold T_s, are on the verge of weeping. For consistent comparison, both strains should be in the same reference frame.</i>
<i>A_w</i>	<i>A_2</i>	<i>A_w is the hoop strain that weeps the pipe in the short-term. A_2 is the transverse strain in the UD ply corresponding to A_w. A_2 is the hoop strain A_w rotated to the transverse direction of the UD ply</i>
<i>-----</i>	<i>γ_w</i>	<i>Shear strains by themselves do not open cracks and do not cause weeping. Their effect comes from the strong interaction with the transverse strains near the weep threshold.</i>
<i>-----</i>	<i>γ_s</i>	<i>The strong interaction between transverse and shear strains near the stiffness threshold is a concern.</i>

Table 1
Corresponding pipe and ply strains in the weep process.

The UEWS (ultimate elastic wall strain) or PEL (proportional elastic limit), are pipe hoop strains corresponding to T_s , the stiffness threshold of the critical ply. These are high strains, corresponding to significant laminate damage and loss of stiffness. The pipe strain UEWS is close to the pipe weep threshold A_w . Likewise, T_s is not far from A_2 .

Of the three pipe strains in table 1, only A_w has a direct relation to the weeping of pressurized pipes.

- *The high strain UEWS indicates impending weeping.*
- *The low strain HDB indicates the onset of the weeping process.*
- *The high strain A_w indicates the actual short-term weeping*

The pure cyclic weep equation of pipes, discussed in the part 2 of chapter 13, connect A_w and the fatigue threshold T_o rotated to the global frame. This was a central topic in the part 2 of chapter 13.

2 – Weep envelopes. The weep envelopes proposed by ISO 14692 are expensive, inaccurate and difficult to obtain. Furthermore, their validity is limited to pipes of wall equal wall thickness and construction as those tested. This implies in the development of different envelopes for every pipe construction and wall thickness. In this chapter, we propose an analytical derivation of the weep envelopes, bypassing the cost and difficulties involved in their experimental development.

The weep envelopes are obvious when presented in the local ply frame. Figure 1 shows the pipe and ply weep envelopes referred to the frame of the critical ply. The boundaries of the ply envelope are $\epsilon_2 = T_w$ and $\gamma_{12} = \gamma_w$. For the pipe envelope, the boundaries are $\epsilon_2 = A_2$ (A_w rotated to the ply frame) and $\gamma_{12} = \gamma_s$. For consistent comparisons, all strains are in the same frame of reference, in this case the ply frame.

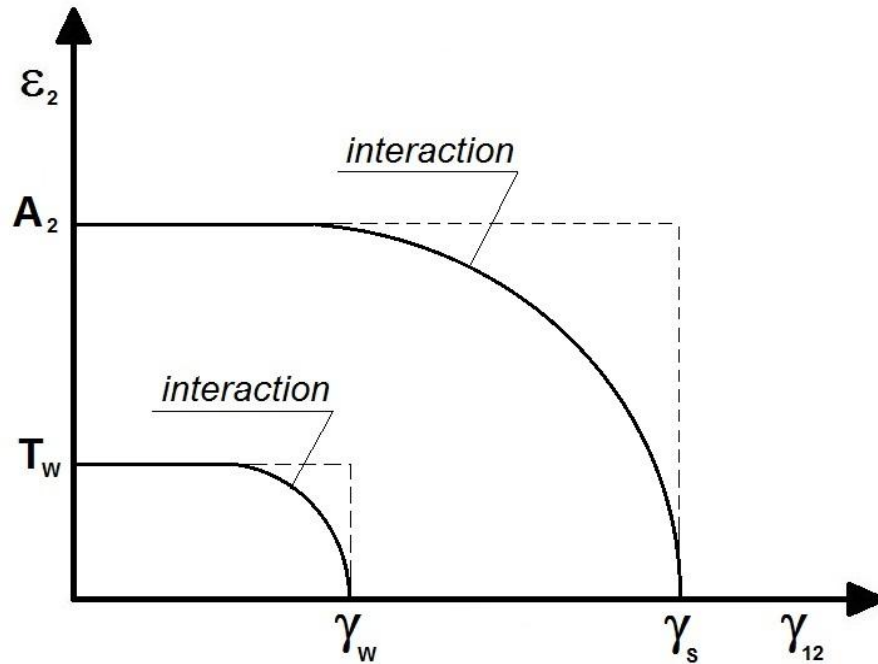


Figure 1

Showing the pipe and ply weep envelopes. The transverse strains open the shear cracks that would otherwise remain closed and have no effect on weeping. The shear strain at the onset of crack coalescence in the ply is $\gamma_w = 0.015$. The shear strain at the onset of loss of stiffness in the pipe is $\gamma_s = 0.020$.

The pipe and ply transverse failure strains taking into account the shear interaction are

$$\epsilon_2 = T_w \times \sqrt{1 - \left(\frac{\gamma_{12}}{\gamma_w}\right)^2}$$

$$\epsilon_2 = A_2 \times \sqrt{1 - \left(\frac{\gamma_{12}}{\gamma_s}\right)^2}$$

The transverse weep threshold, T_w , of commercial plies is available in chapter 8. The transverse weep threshold of the pipe, A_2 , is the pipe hoop strain A_w rotated to the ply frame. The shear weep thresholds for pipes and plies are unknown. For discussion purposes, we are assuming $\gamma_w = 0.015$ and $\gamma_s = 0.020$

These are the ply and pipe weep failure envelopes, expressed in their natural frame of reference, the ply frame.

However, this is not the solution envisaged by ISO 14 692, that chooses to express the failure envelopes in terms of stresses (not strains) in the global frame of the pipe. The justification for this extra complication is that engineers do not like to think in terms of strains, mostly when expressed in the ply frame of reference. I assure you this is not my case.

Be it as it may, compliance with ISO 14 692 requires rotation of the simple weep envelopes of figure 1 to the pipe frame of reference, expressed in stresses, not in strains.

We start with the constitutive stress-strain relations in the global pipe system

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \frac{1}{t} \times [A] \times \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix}$$

Inverting the above equation, we obtain the global strains in terms of the global stresses

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} = t \times [A]^{-1} \times \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}$$

The local strains are

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ 1/2 \gamma_{12} \end{bmatrix} = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix} \times \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ 1/2 \gamma_{xy} \end{bmatrix}$$

In the absence of external torque, the local strains simplify to

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ 1/2 \gamma_{12} \end{bmatrix} = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix} \times \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ 0 \end{bmatrix}$$

The local strains in terms of the global stresses are

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ 1/2 \gamma_{12} \end{bmatrix} = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix} \times t \times [A]^{-1} \times \begin{bmatrix} \sigma_x \\ \sigma_y \\ 0 \end{bmatrix}$$

Where

t is the pipe thickness
 $[A]$ is the known stiffness matrix of the pipe
 m and n are the cosine and sine of the ply angle

The unknowns in the above three equations are

ϵ_1, ϵ_2 and γ_{12}

σ_x and σ_y

To compute the failure envelopes, we set the transverse strain equal to the transverse weep threshold, taking into account the interaction of the transverse and shear strain.

Therefore, to compute the ply envelope, we take the transverse strain as

$$\epsilon_2 = T_w \times \sqrt{1 - \left(\frac{\gamma_{12}}{\gamma_w}\right)^2}$$

For the UWES envelope, we have

$$\epsilon_2 = T_s \times \sqrt{1 - \left(\frac{\gamma_{12}}{\gamma_s}\right)^2}$$

Finally, for the pipe weep envelope, we have

$$\epsilon_2 = A_2 \times \sqrt{1 - \left(\frac{\gamma_{12}}{\gamma_s}\right)^2}$$

Furthermore, we also set $\sigma_y = k \times \sigma_x$, where k is the ratio of the hoop and axial stresses.

After doing all this, we end up with three equations in three unknowns, from which we can readily compute the failure stresses in the global frame.

Example 2. Develop the protocol to compute the pipe weep envelope of UD55 pipes.

The stiffness matrix $[A]$ of UD55 pipes is

$$[A] = \begin{bmatrix} 132470 & 96220 & 0 \\ 96220 & 235070 & 0 \\ 0 & 0 & 101220 \end{bmatrix} \times t$$

The rotation matrix for UD55 pipes is

$$\begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix} = \begin{bmatrix} 0.33 & 0.67 & 0.94 \\ 0.67 & 0.33 & -0.94 \\ -0.47 & 0.47 & -0.34 \end{bmatrix}$$

The local strains are

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ 1/2 \gamma_{12} \end{bmatrix} = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix} \times t \times [A]^{-1} \times \begin{bmatrix} \sigma_x \\ \sigma_y \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ 1/2 \gamma_{12} \end{bmatrix} = \begin{bmatrix} 0.33 & 0.67 & 0.94 \\ 0.67 & 0.33 & -0.94 \\ -0.47 & 0.47 & -0.34 \end{bmatrix} \times \begin{bmatrix} 132470 & 96220 & 0 \\ 96220 & 235070 & 0 \\ 0 & 0 & 101220 \end{bmatrix}^{-1} \times \begin{bmatrix} \sigma_x \\ \sigma_y \\ 0 \end{bmatrix}$$

To compute the pipe weep envelope, we take the pipe weep threshold A_2 equal to the transverse strain

$$\epsilon_2 = A_2 \times \sqrt{1 - \left(\frac{\gamma_{12}}{\gamma_s}\right)^2}$$

Furthermore, we also take

$$\sigma_y = k \times \sigma_x$$

Where k is the loading ratio.

Solving the above three equations in three unknowns (ϵ_1 γ_{12} σ_x), we obtain the failure stresses σ_x and σ_y that form the failure envelope.

The above approach involves the inversion of the laminate stiffness matrix $[A]$. This is not to the liking of practical engineers working with pocket calculators.

Example 3. Compute the pipe weep envelope for UD55 vinyl ester pipes

We start with the constitutive equation

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ 0 \end{bmatrix} = \frac{1}{t} \times [A] \times \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix}$$

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ 0 \end{bmatrix} = \frac{1}{t} \times \begin{bmatrix} 132470 & 96220 & 0 \\ 96220 & 235070 & 0 \\ 0 & 0 & 101220 \end{bmatrix} \times t \times \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix}$$

Cancelling out the pipe thickness and taking $\sigma_y = k \times \sigma_x$

$$\begin{bmatrix} \sigma_x \\ k\sigma_x \\ 0 \end{bmatrix} = \begin{bmatrix} 132470 & 96220 & 0 \\ 96220 & 235070 & 0 \\ 0 & 0 & 101220 \end{bmatrix} \times \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix}$$

Expanding and solving for the global strains, we have

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} (10.75 - 4.43k) \times 10^{-6} \times \sigma_x \\ (6.07k - 4.40) \times 10^{-6} \times \sigma_x \\ 0 \end{bmatrix}$$

Next, we rotate the global strains into the local frame

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ 1/2 \gamma_{12} \end{bmatrix} = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix} \times \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ 1/2 \gamma_{xy} \end{bmatrix}$$

Where $m = \cos 55 = 0.57$ and $n = \sin 55 = 0.82$

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ 1/2 \gamma_{12} \end{bmatrix} = \begin{bmatrix} 0.33 & 0.67 & 0.94 \\ 0.67 & 0.33 & -0.94 \\ -0.47 & 0.47 & -0.34 \end{bmatrix} \times \begin{bmatrix} (10.75 - 4.43k) \times 10^{-6} \times \sigma_x \\ (6.07k - 4.40) \times 10^{-6} \times \sigma_x \\ 0 \end{bmatrix}$$

Expanding the above and collecting terms, we have

$$\epsilon_1 = (0.60 + 2.61k) \times 10^{-6} \times \sigma_x$$

$$\epsilon_2 = (5.75 - 0.97k) \times 10^{-6} \times \sigma_x$$

$$\gamma_{12} = (14.24 - 9.87k) \times 10^{-6} \times \sigma_x$$

These are the strains in the local frame of the UD55 plies. They depend, as they should, on the loading ratio k .

The strain in the fiber direction controls the pipe rupture, with no effect on the weep failure.

The transverse strain controls the weep failure. In computing the failure envelopes, we set the transverse strain equal to the pipe or ply weep threshold. In doing this, we recall the interaction between the shear and transverse strains.

To compute the axial and hoop stresses of the weep envelope of UD55 pipes, we set

$$\epsilon_2 = A_2 \times \sqrt{1 - \left(\frac{\gamma_{12}}{\gamma_s}\right)^2}$$

$$(5.75 - 0.97k) \times 10^{-6} \times \sigma_x = A_2 \times \sqrt{1 - \left(\frac{(14.24 - 9.87k) \times 10^{-6} \times \sigma_x}{\gamma_s}\right)^2}$$

From which we have

$$\sigma_x = \frac{10^6 \times A_2}{\sqrt{(5.75 - 0.97k)^2 + \left(\frac{A_2}{\gamma_s}\right)^2 \times (14.24 - 9.87k)^2}}$$

This equation computes the axial stress of the pipe weep envelope for any loading k . To do the computation, we enter $\gamma_s = 0.020$ and the measured value of A_2 and in the above equation.

To determine the value of the pipe parameter A_2 , we first measure the pipe weep threshold A_w at the loading ratio k . Let us perform this measurement at the loading ratio $k = 2$. Suppose the measured hoop stress at short-term weep is $A_w = 4\,200 \text{ kg/cm}^2$. From this hoop stress, and taking into account that $k = 2$, we compute the axial stress as $\sigma_x = 2\,100 \text{ kg/cm}^2$. Entering this value, together with $k = 2$ and $\gamma_s = 0.020$ in the above equation, we have

$$2100 = \frac{10^6 \times A_2}{\sqrt{(5.75 - 0.97 \times 2)^2 + \left(\frac{A_2}{0.020}\right)^2 \times (14.24 - 9.87 \times 2)^2}}$$

From which we compute

$$A_2 = 0.010$$

$A_2 = 0.010$ is the transverse pipe strain corresponding to A_w . A_2 captures the travel time, giving very accurate weep envelopes.

The equation to compute the axial stress at the short-term weep envelope becomes.

$$\sigma_x = \frac{10\,000}{\sqrt{(5.75 - 0.97k)^2 + 0.25 \times (14.24 - 9.87k)^2}}$$

This equation breaks down for pure hoop loadings, at $k = \text{infinite}$. In this case, we resort to the basic constitutive equation

$$\begin{bmatrix} 0 \\ \sigma_y \\ 0 \end{bmatrix} = \begin{bmatrix} 132470 & 96220 & 0 \\ 96220 & 235070 & 0 \\ 0 & 0 & 101220 \end{bmatrix} \times \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} -5.63 \times 10^{-6} \times \sigma_y \\ 6.05 \times 10^{-6} \times \sigma_y \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ 1/2 \gamma_{12} \end{bmatrix} = \begin{bmatrix} 0.33 & 0.67 & 0.94 \\ 0.67 & 0.33 & -0.94 \\ -0.47 & 0.47 & -0.34 \end{bmatrix} \times \begin{bmatrix} -5.63 \times 10^{-6} \times \sigma_y \\ 6.05 \times 10^{-6} \times \sigma_y \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 2.20 \times 10^{-6} \times \sigma_y \\ -1.78 \times 10^{-6} \times \sigma_y \\ 10.98 \times 10^{-6} \times \sigma_y \end{bmatrix}$$

The compressive transverse strain for pure hoop loadings indicates shear failure. Furthermore, compressive transverse strains do not interact with shear strains.

$$\gamma_{12} = \gamma_s$$

$$10.98 \times 10^{-6} \times \sigma_y = 0.020$$

$$\sigma_y = 1820 \text{ kg/cm}^2$$

$$\sigma_x = 0$$

The computed failure stresses at several values of k are in table 2

Stresses (kg/cm ²)	$k = 0.0$	$k = 1.0$	$k = 2.0$	$k = 4.0$	$k = \infty$
σ_x	1100	1900	2100	790	0
σ_y	0	1900	4200	3160	1820

Table 2

Stress points of the weep envelope for the tested UD55 pipe. The measured envelope varies with the pipe thickness.

Example 4. Compute the ply weep envelope for UD55 vinyl ester pipes

The protocol is the same as in example 3, using the interaction equation

$$\epsilon_2 = T_w \times \sqrt{1 - \left(\frac{\gamma_{12}}{\gamma_w}\right)^2}$$

$$\sigma_x = \frac{10^6 \times T_w}{\sqrt{(5.75 - 0.97k)^2 + \left(\frac{T_w}{\gamma_w}\right)^2 \times (14.24 - 9.87k)^2}}$$

To compute the failure envelope at the onset of coalescence we enter $T_w = 0.0055$ and $\gamma_w = 0.015$ in the above equation.

$$\sigma_x = \frac{5500}{\sqrt{(5.75 - 0.97k)^2 + 0.14 \times (14.24 - 9.87k)^2}}$$

Likewise for the UWES envelope, where we make use of the transverse stiffness threshold for vinyl ester UD plies, $T_s = 0.009$. The stiffness shear threshold is $\gamma_s = 0.020$

$$\epsilon_2 = T_s \times \sqrt{1 - \left(\frac{\gamma_{12}}{\gamma_s}\right)^2}$$

$$\sigma_x = \frac{10^6 \times T_s}{\sqrt{(5.75 - 0.97k)^2 + \left(\frac{T_s}{\gamma_s}\right)^2 \times (14.24 - 9.87k)^2}}$$

Entering $T_s = 0.009$ (see chapter 8) and $\gamma_s = 0.020$, we have

$$\sigma_x = \frac{9\,000}{\sqrt{(5.75 - 0.97k)^2 + 0.20 \times (14.24 - 9.87k)^2}}$$

The computed failure stresses are in tables 3 and 4.

Stresses (kg/cm ²)	$k = 0.0$	$k = 1.0$	$k = 2.0$	$k = 4.0$	$k = \infty$
σ_x	780	1100	1300	750	0
σ_y	0	1100	2600	2250	1350

Table 3

Stress points at the onset of weeping

Stresses (kg/cm ²)	$k = 0.0$	$k = 1.0$	$k = 2.0$	$k = 4.0$	$k = \infty$
σ_x	1050	1750	1980	785	0
σ_y	0	1750	3960	3140	1820

Table 4

Stress points at the onset of loss of stiffness (UWES)

The computed weep envelopes are in figure 2.

FAILURE ENVELOPES UD55° VER PIPES

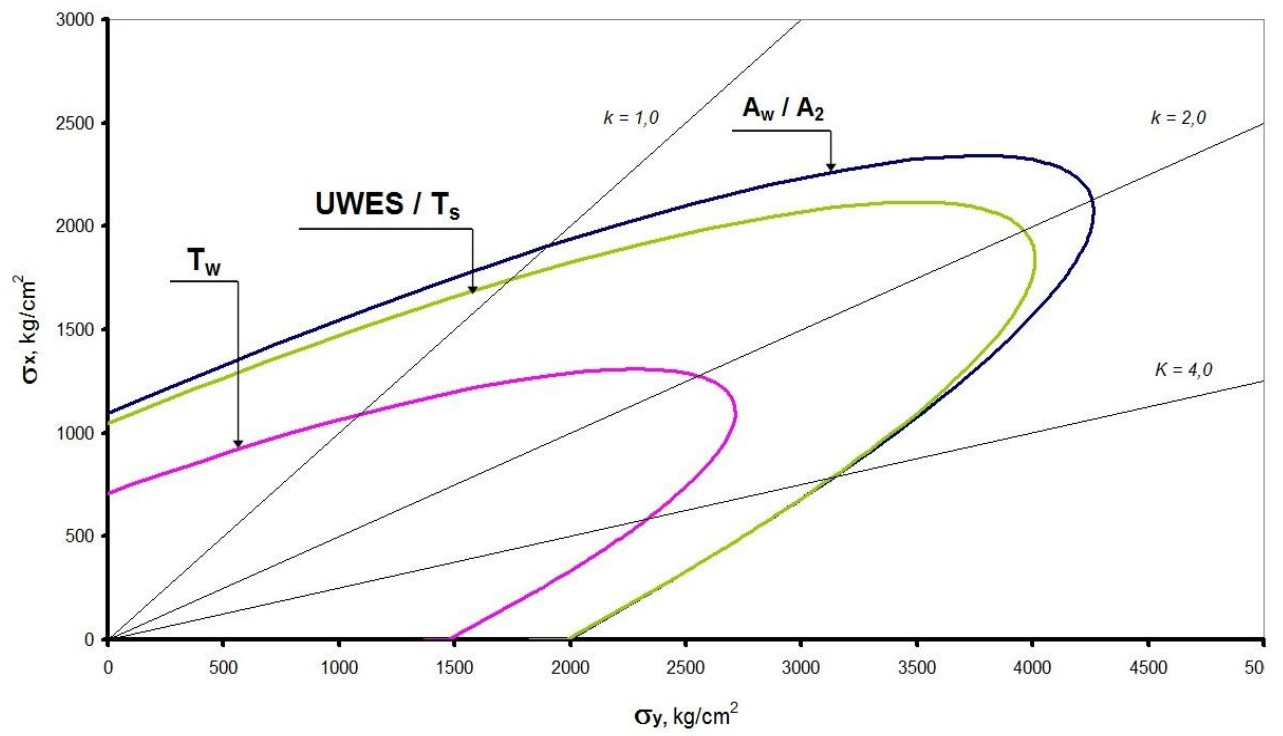


Figure 2
Typical weep failure envelopes. The envelope defined by A_w governs the weep performance of pressurized pipes.

Chapter 15

Rupture failure

1 – Introduction: As discussed in chapter 8, the four load-dependent modes of failure of industrial composite equipment are

- *Infiltration failure, from crack densities large enough to allow the fast ingress of fluids.*
- *Weep failure, from crack densities large enough to allow the leakage of fluids.*
- *Stiffness failure, from damage densities large enough to affect the laminate stiffness.*
- *Rupture failure, from accumulated fiber damages large enough to break the laminate.*

The above failures usually occur in sequence, first infiltration, followed by weeping, then loss of stiffness and finally rupture. There may be exceptions to this sequence, however. As examples, we mention fast loading rates that rupture water-filled pipes before they weep. Likewise, static loaded pipes below the weep threshold rupture before weeping.

The chemical and weep durability were the topics of chapters 10 and 13. In this chapter, we deal with the rupture durability. There are two types of long-term rupture of composite laminates:

- *Static rupture, combining pure static loads with global strain-corrosion from water*
- *Cyclic or fatigue rupture from cyclic loads*

In the present chapter, we derive the static and cyclic regression equations to compute the durability of laminates. We begin with a discussion of static durability. The cyclic case will follow.

2 – Global strain-corrosion of glass fibers: There are two types of static failures of composite laminates. The first type comes from the global attack of water on all fibers. The second type involves the local attack on a few fibers by aggressive chemicals. This chapter deals with the global strain-corrosion by water. For a comprehensive discussion of the local strain-corrosion by chemicals, see chapter 11.

Example 1. *The strain-corrosion of fibers requires the presence of chemicals at the fiber surface. To clarify this, consider the following thought experiments.*

In the first experiment, we imagine three sets of single bare (not resin-coated) glass filaments under equal tensile forces F (a) in air, (b) immersed in water and (c) immersed in acid solution. The times to rupture of the bare glass filaments under the static forces F are:

- *In air, the small presence of water at the fiber surface gives long rupture times.*
- *In acid solution, the high concentration of H^+ ions at the fiber surface gives short times to rupture.*
- *In water, the times to rupture fall in between.*

In the second experiment, we imagine the same three sets of single glass filaments resin-coated before testing. The rupture times of the resin-coated fibers are

- *In air, the times to rupture are the same for the coated and the uncoated fibers. This is so, because the water presence at the fiber surfaces is the same in both cases.*
- *In water, the times to rupture are also the same for the coated and the uncoated fibers. The resin coat gives no protection against water.*
- *In acid solution, however, the coated filaments have significantly longer lives than the bare fibers. The resin-coat substantially depresses the acid concentration at the fiber surface.*

The above thought experiments explain the two mechanisms of fiber protection by resin matrices. First, the resin delays the chemical penetration and second – more important – it forms a protective shield that depresses the chemical presence at the surface of the embedded fibers. The “shield effect” plays a decisive role in the creep-rupture tests of laminates, as explained in the appendix 1 of chapter 10.

As discussed in chapter 11, the depleted presence of sulfuric acid at the fibers in sewage pipes, establishes the existence of a “putative sewer strain-corrosion threshold” equal to the weep threshold of the critical ply.

The discussions that follow center on the global strain-corrosion of fibers from water attack. Water is the only known chemical capable of reaching all fibers in the entire laminate. Inert solvents, that also reach all fibers, do not cause strain-corrosion.

As explained in the numerical example 1, these processes are identical for laminates in air or in water. The ubiquitous water molecules are present in the entire laminate at all times. Water is the only known chemical species capable of causing global strain-corrosion in laminates. Furthermore, the high mobility of the small water molecules assures that all plies reach hydric equilibrium with the environment (atmosphere, soil, or chemical solutions) in a short time. The presence of water at the surface of embedded fibers is unavoidable.

For details on the mechanism of water attack on strained glass fibers, see references 27, 28 and 29.

3 – Static load ruptures – In this section, we discuss the rupture of plies under static loads. We start with the strain-corrosion of single fibers.

To understand the process of crack growth in strain-corrosion, consider a single fiber strained by the static force F , as in figure 1. The crack growth from the simultaneous action of a sustained static strain and chemical attack (in this case, by water) characterizes the strain-corrosion process. The combined action of chemical attack with static loads can grow cracks. This is the essence of the strain-corrosion process. We have a conclusion:

Strain-corrosion is the only known process capable of growing cracks under static loads.

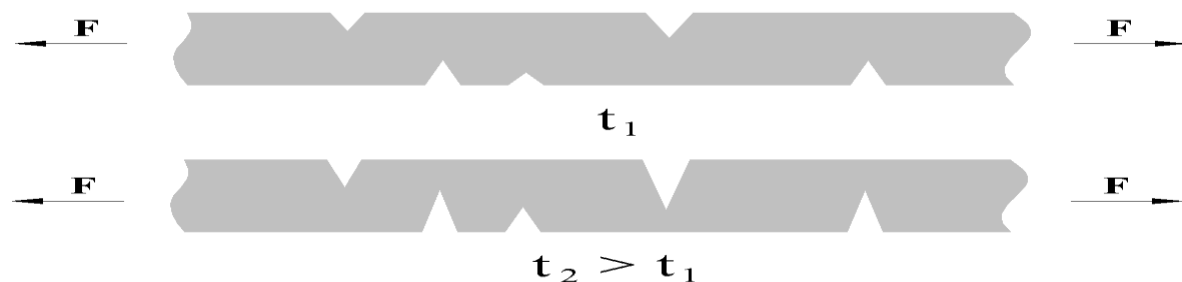


Figure 1

Water attack on pre-existing surface flaws produces many small cracks randomly distributed along the fiber. The static force F opens these cracks exposing fresh material to further attack. Strain-corrosion is the only known process that grows cracks under static loads.

The failure of statically loaded plies involves the rupture of an enormous number of fibers. From the theory of fracture mechanics, we know that all cracks larger than a certain critical size will fracture immediately on application of the static force. This creates an enormous number of ruptured fiber points. The cracks smaller than the critical size do not rupture immediately on application of the tensile load. Instead, they slowly grow in response to a steady strain-corrosion process, until failure. Ply failure occurs when the growing density of ruptured fiber points reaches a critical level, determined by the applied external force. The density of ruptured fibers points controls the time to ply failure.

The regression equation to predict the rupture of statically loaded plies is

$$\log(\epsilon) = A_s - G_s \times \log(\text{hour}) \quad (1)$$

Where “ ϵ ” is the total (residual plus mechanical) static strain on the ply and “hour” is the time to failure. The regression parameters A_s and G_s (the subscripts “s” denote static) are determined experimentally. Equation (1) also appears in chapter 13, where we have identified the regression parameter A_s with the short-term ply rupture strength

$$\log(T_r) = A_s$$

Where T_r is the rupture threshold, or short-term rupture strength, of the ply.

4 – Pure cyclic load ruptures: In the preceding section, we described the pure static strain-corrosion rupture of plies. In this section, we will briefly discuss the rupture of plies under pure cyclic loadings. As before, we start with the analysis of a single fiber.

The cyclic crack growth in fibers comes from fatigue, not strain-corrosion. The rate of fatigue crack growth proceeds in response to the kinetic energy delivered by the cycling load. In analogy with the static case, the regression equation to compute the number of cycles to ply rupture is

$$\log(\epsilon) = A_c - G_c \times \log(N) \quad (2)$$

Where “ ϵ ” is the peak cyclic strain and N is the number of cycles to rupture. As in the static case, we have

$$\log(T_r) = A_c$$

Example 2 – In this numerical example, we define the strain ratio R , as well as the peak and the static strain components of complex loadings. Figure 3 illustrates the static and cyclic strain components.

$$(\epsilon)_{\text{static}} = \epsilon_{\min} \quad (\text{Static strain component})$$

$$(\epsilon)_{\text{cyclic}} = \epsilon_{\max} - \epsilon_{\min} \quad (\text{Cyclic strain component})$$

In the above, $\epsilon_{\max}$ and $\epsilon_{\min}$ are the extreme strain values. The R ratio is

$$R = \frac{\epsilon_{\min}}{\epsilon_{\max}}$$

The static and cyclic strain components expressed in terms of the R ratio and the peak strain $\epsilon_{\max}$ are

$$\epsilon_s = R\epsilon_{\max} = R\epsilon \quad (\text{Static strain component})$$

$$\epsilon_c = (1 - R)\epsilon_{\max} = (1 - R)\epsilon \quad (\text{Cyclic strain component})$$

To facilitate the notation, we drop the “max” subscript from the peak strain. The cyclic and static strain components are direct inputs to the unified equation.

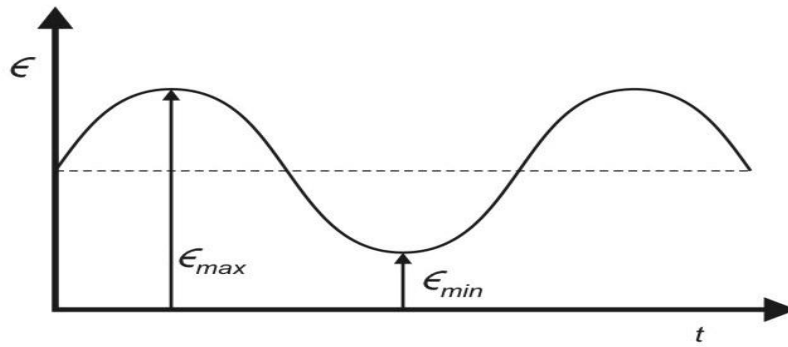


Figure 3

The static strain component is the minimum strain $\epsilon_{min} = R \times \epsilon_{max}$. The cyclic strain component is $\epsilon_{max} - \epsilon_{min} = (1 - R) \times \epsilon_{max}$

5 – The work of Mark Greenwood: Mark Greenwood creep-tested pultruded UD rods immersed in water. The tested rods had the following specification:

- Diameter 6.0 mm
- Glass loading 75%
- High reactivity polyester resin
- Rods made with two glass compositions, E glass and boron-free glass

The glass loading, rod diameter and type of resin are not relevant. The important variable in this study is the glass composition, which reflects the strain-corrosion stability of the fiber in the presence of water. The test consisted in tensioning pultruded rods immersed in water at room temperature. See figure 4. The different tensile loads was constant throughout the test. This is a static test.



Figure 4

Tensile strained rod segment immersed in water. Static tests performed by Mark Greenwood. (Courtesy Owens Corning)

As expected, the rod segments under higher strains ruptured first. A statistical analysis of the failure data produced two straight lines, one for each glass composition. The slopes of the regression lines measure the glass resistance to water attack. Figure 5 shows the boron-free fibers to be less susceptible to water attack than the traditional E glass. The failure strains extrapolated to 50

years indicate a long-term static strength $S_s = 0.92\%$ for the boron-free glass versus $S_s = 0.41\%$ for the traditional E glass. The interpretation is as follows.

1. Water attack lowers the long-term static strength of both rods.
2. The 50 years static strength of boron-free glass fibers is $S_s = 0.92\%$
3. The 50 years static strength of E glass fibers is $S_s = 0.41\%$

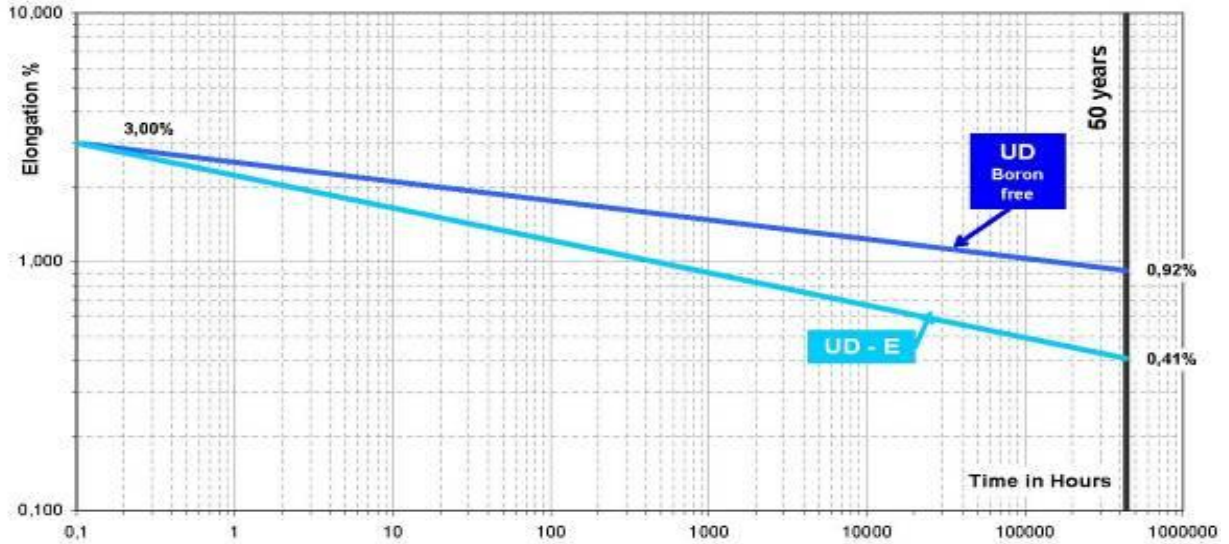


Figure 5
Static regression lines of UD plies in water. Adapted from the creep-rupture tests conducted by Mark Greenwood

A back extrapolation of figure 5 indicates essentially the same short-term strength for the two glass compositions. The difference between E glass and boron-free glass is in the long-term, not in the short-term. Mark Greenwood never published the regression equations for his data. However, inspection of the published data indicates the following

$$\text{For boron-free glass:} \quad \log(\varepsilon) = 0.400 - 0.077 \log(\text{hours}) \quad (3A)$$

$$\text{For E glass:} \quad \log(\varepsilon) = 0.347 - 0.130 \log(\text{hours}) \quad (3B)$$

In the above, the strains enter in %.

Example 3: Compute the long-term and the short-term static strengths of UD plies of boron-free and E glass.

We take 50 years (438 000 hours) as long-term and 0.1 hour as short-term.

Equation (3A) gives the short-term strength of boron-free glass UD plies in 0.1 hour as

$$\log(\varepsilon) = 0.400 - 0.077 \log(\text{hours})$$

$$\log(\varepsilon) = 0.400 - 0.077 \times -1 = 0.477$$

$$S_s = 3.0\%$$

Equation (3B) gives the short-term strength of standard E glass UD plies in 0.1 hour as

$$\log(\varepsilon) = 0.347 - 0.130 \log(\text{hours})$$

$$\log(\varepsilon) = 0.347 - 0.130 \times -1 = 0.477$$

$$S_s = 3.0\%$$

The two glass compositions have the same short-term strength, as expected.

The long-term tensile strength of UD plies of boron-free glass in 50 years is

$$\log S_s = 0.400 - 0.077 \log(438000)$$

$$\log S_s = 0.400 - 0.077 \times 5.641 = -0.034$$

$$S_s = 0.92\%$$

For E glass, the 50 years strength is

$$\log S_s = 0.347 - 0.130 \log(438000)$$

$$\log S_s = 0.347 - 0.130 \times 5.641 = -0.386$$

$$S_s = 0.41\%$$

These strengths are the sustained static strains that fail the UD rods in the long-term. The long-term strength is a metric of the accumulated water damage.

6 – The work of Guangxu Wei: As we have just seen, Mark Greenwood determined the pure static ($R = 1.0$) regression lines of UD plies loaded in the fiber direction.

Guangxu Wei did the same thing for pure cyclic loads ($R = 0.0$) in the fiber and transverse directions of brittle polyester UD plies. See ref. 25. The pure cyclic ($R = 0.0$) UD equations reported by Guangxu Wei are:

$$\log(\epsilon) = -0.602 - 0.040 \log(N) \quad (\text{Transverse direction}) \quad (4A)$$

$$\log(\epsilon) = 0.519 - 0.089 \log(N) \quad (\text{Fiber direction}) \quad (4B)$$

Equation (4A) is the pure cyclic ($R = 0.0$) transverse infiltration equation for UD plies of brittle polyester. This is so, because the failure point defined in the tests by Guangxu Wei was the first crack appearing in the transverse direction of the loaded UD plies. As we know, the first crack in transverse loaded UD plies marks their infiltration threshold. The pure cyclic equations of UD plies listed in the part 1 of chapter 13 derive from equation (4A).

Equation (4B) is the pure cyclic ($R = 0.0$) longitudinal rupture equation for UD plies. Equation (4B) holds for both E and boron-free glasses.

The reader perhaps has noted that the strains mentioned so far acted either *in the fiber direction or in the transverse direction to the fiber*. Both Mark Greenwood and Guangxu Wei developed their data in those directions. Mark's data were developed on UD rods tensile tested by pure static loads in the fiber direction. Guangxu Wei tested UD plies under pure cyclic loadings in the fiber and transverse directions.

7 – The work of the British Plastics Federation: The British Plastics Federation completed the set of pure regression lines by testing chopped plies of polyester resins. Their results follow:

The rupture pure static regression equation for chopped plies of brittle polyester is

$$\log \epsilon = \log(1.50) - 0.045 \times \log(\text{hours}) \quad (5)$$

Where 1.50% is the short-term rupture strength, or rupture threshold, of the ply.

The rupture pure cyclic equation for chopped plies of brittle polyester is

$$\log \epsilon = \log(1.50) - 0.075 \times \log(N) \quad (6)$$

Where ϵ is the peak strain and 1.50% is, again, the ply short-term rupture strength.

The pure regression equations presented in this chapter generate the inputs needed in the unified equation. We are indeed lucky to have these unplanned and unsolicited regression equations. As discussed in the remainder of this book, these equations suffice to solve all load-dependent durability problems of composite laminates.

8 – The unified equation: Equations (1) and (2) are applicable for pure static or pure cyclic loads. They are not valid for complex loadings, with both components acting simultaneously. The unified equation computes the durability in such cases, when the static and cyclic load components act together. Equations (7A) and (7B) show the full and the simplified versions of the unified equation, borrowed from chapter 16.

$$\left(\frac{R \times \epsilon \times SF}{S_s} \right)^{\frac{1}{G_s}} + \left(\frac{(1-R) \times \epsilon \times SF}{S_c} \right)^{\frac{1}{G_c}} + \left(\frac{R \times (1-R) \times \epsilon^2 \times SF^2}{S_s \times S_c} \right)^{\frac{1}{G_{sc}}} = 1.00 \quad (7A)$$

$$\left(\frac{(1-R) \times \epsilon \times SF}{S_c} \right)^{\frac{1}{G_c}} + \left(\frac{R \times (1-R) \times \epsilon^2 \times SF^2}{S_s \times S_c} \right)^{\frac{1}{G_{sc}}} = 1.00 \quad (7B)$$

In the above equations, G_s and G_c are the pure static and the pure cyclic regression slopes available from equations (1) and (2). The long-term static and cyclic strengths S_s and S_c also derive directly from their pure regression equations. With the exception of G_{sc} , all inputs in the unified equation come from pure regression lines.

The full equation (7A) predicts the time-dependent long-term rupture of laminates involving fiber global strain-corrosion. The simplified version, equation (7B), predicts the time-independent infiltration, weep and stiffness failures. For details on the unified equation, see chapter 16.

Example 4: Suppose a UD55 oil pipe carrying pressurized water under a complex loading comprising both static and cyclic components. Which failure occurs first, loss of stiffness, infiltration, weep or rupture?

Infiltration will probably occur ahead of the others, but is not a failure in this case, since the ingress of water is not harmful to laminates. Our attention is therefore focused on the loss of stiffness, weep and rupture failures.

- Equation (7A) computes the long-term pipe rupture from fiber degradation. The long-term safety factor SF , expressing the rupture durability, depends on the load ratio R , the peak strain, the number of cycles and the time duration.
- Equation (7B) computes the long-term weeping and loss of stiffness from transverse loads. The safety factor SF in this case is fully cycle-dependent and not affected by the time duration.

The answer to which failure occurs first will depend on the load ratio R , the number of cycles and the load duration. In most cases, weeping occurs first. In others, rupture may come first. Problems like this are the purview of the unified equation.

Appendix 1

Spontaneous helical cracking of glass fibers

In the fiber forming process, the molten glass near the outer surface of the filaments cools off and hardens sooner than the inner core. The pulling force from the winder puts this hardened outer material in tension. As the formed fiber cools down to room temperature, the pulling force congeals as residual tensile strains on the fiber surface. We have a conclusion:

The high pull speed in the forming process introduces longitudinal residual tensile strains on the outer surface of the fibers.

In the presence of water, these residual tensile strains develop and grow spontaneous tiny strain-corrosion cracks on the fiber surface. This spontaneous cracking forms from the residual strains, not from external loads.

In the presence of aggressive chemicals, this cracking is even worse. Figure 2 shows the spontaneous helical cracks formed on the surface of glass fibers immersed in a 5.0% HCl solution at 60C for 2 hours. The shallow spontaneous cracks cease to grow as soon as the residual surface strains dissipate. The helical pattern shown in figure 2 changes with the pulling speed at the formation of the fiber.

Under applied external forces, these shallow surface cracks resume their growth until fiber rupture.

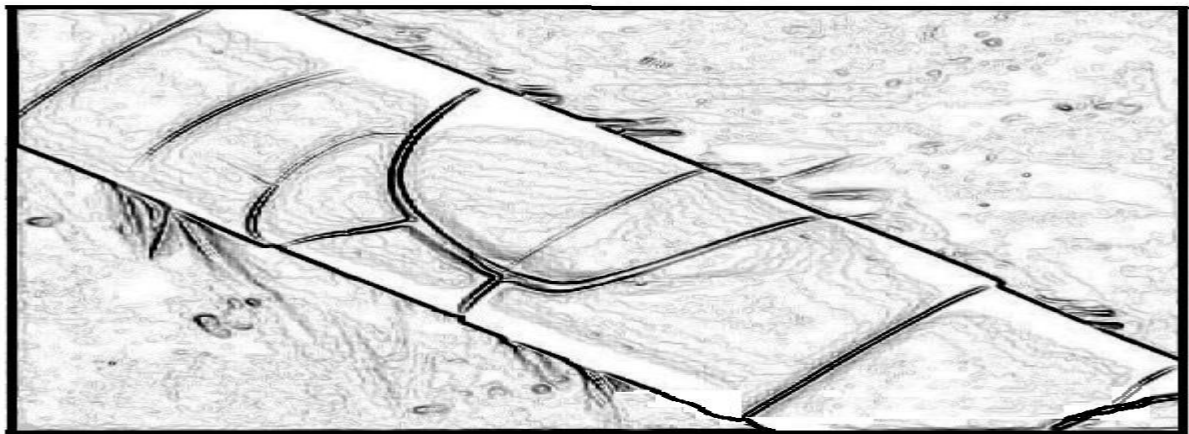


Figure 2

The pulling force in the forming process introduces an enormous number of spontaneous strain-corrosion surface cracks on the glass fibers. The figure shows the spontaneous surface cracking of glass fibers after 2 hours immersion in 5% HCl @ 60C. The open cracks suggest the presence of residual surface strains. This spontaneous surface fiber cracking is similar to the environmental stress cracking observed in resin-rich liners. (Drawing by Gabriel Gonzalez)

Chapter 16

Part 1

The unified equation

1 – Introduction: In this chapter, we introduce and explain the use of the unified equation in solving load-dependent durability problems of composites. To facilitate the presentation, we initially neglect the shear strains.

The unified equation is a logical extension of the regression equations discussed in chapter 13. The validity of the regression equations implies in the unquestionable acceptance of the unified equation. Both methods give exactly the same result.

2 – Pure regression lines: The unified equation uses easily available inputs derived from known pure static and pure cyclic regression equations. This eliminates the enormous experimental effort that would otherwise be required. Let us proceed to derive the unified equation from the pure static and the pure cyclic regression lines.

The following equations predict the laminate/ply failures under pure static or pure cyclic loads.

$$\log \varepsilon_s = A_s - G_s \log(t) \quad (1)$$

$$\log \varepsilon_c = A_c - G_c \log(N) \quad (2)$$

These equations compute the pure static and the pure cyclic long-term ply/laminate residual strengths at any given time “t” or number of cycles “N”. The experimental regression parameters “A” and “G” take different subscripts “s” or “c” to denote the pure static or the pure cyclic cases. The above equations are pure, meaning the static and cyclic loads act separately. The unified equation applies when these loads act simultaneously.

Note: The unified equation holds for both plies and laminates, which is obvious, since laminates are complex plies. However, to facilitate the presentation, the discussion that follows refers only to plies. The application to laminates is implied.

To derive the unified equation, we rearrange the pure regression equations (1) and (2) in a suitable form. Suppose a ply under the separate action of pure static and pure cyclic load components. Let us say its lifetime is X years under the pure static load. Furthermore, let us say its life is Y cycles under the pure cyclic load. Our task is to compute the durability of this ply under the simultaneous action of the cyclic and the static loads.

To do so, we first consider the static load component. For a lifetime of “X” years, the long-term static strength of the ply is

$$\log S_s = A_s - G_s \log(X) \quad (1A)$$

Where S_s is the long-term static strength of the ply, corresponding to failure at X years. Combining equations (1) and (1A), we have

$$\frac{X}{t} = \left(\frac{\epsilon_s}{S_s} \right)^{\frac{1}{G_s}} \quad (1B)$$

In equation (1B), the static strain “ ϵ_s ” and the time “t” describe pairs of failure points. For any given time “t” there is a corresponding static strain “ ϵ_s ” that fails the ply. In a like manner, the long-term static strength “ S_s ” corresponds to failure at the time X.

The analog equation for pure cyclic loads is

$$\frac{Y}{N} = \left(\frac{\epsilon_c}{S_c} \right)^{\frac{1}{G_c}} \quad (2B)$$

Where S_c is the long-term pure cyclic strength of the ply, corresponding to Y cycles to failure. The value of S_c comes from equation (2) by taking $N = Y$.

$$\log S_c = A_c - G_c \log(Y)$$

Equations (1B) and (2B) are applicable to pure static and pure cyclic loads acting alone. Our objective is to combine them into a unified equation that handles both loads simultaneously. This combination requires that the number of cycles to failure Y correspond to the static lifetime X years. For example, if the lifetime is $X = 50$ years, then Y must be the number of cycles in 50 years.

The following definitions apply

S_s is the static strength of the ply in X years

S_c is the cyclic strength of the ply in Y cycles

X is the time lifetime under the combined loading

Y is the cyclic lifetime under the combined loading. Y and X must be compatible.

ϵ_s is the static strain component.

ϵ_c is the cyclic strain component.

N is the number of cycles to fail the ply under ϵ_c

t is the time to fail the ply under ϵ_s

3 – Deriving the unified equation: The left-hand sides of equations (1B) and (2B) give the ratios of the ply lifetimes under the combined loading (X and Y) to the pure static or the pure cyclic lifetimes (t and N). These ratios measure the pure static and the pure cyclic “contributions” to failure. Better yet, they measure the relative “damage” contributed by each load component. At failure, these ratios add up to 1.00.

$$\left(\frac{X}{t}\right)_{\text{STATIC}} + \left(\frac{Y}{N}\right)_{\text{CYCLIC}} = 1.00$$

This, in view of equations (1B) and (2B), is the same as

$$\left(\frac{\epsilon_s}{S_s}\right)^{\frac{1}{G_s}} + \left(\frac{\epsilon_c}{S_c}\right)^{\frac{1}{G_c}} = 1.00 \quad (3)$$

In equation (3) G_s and G_c are the known slopes of the pure regression lines, S_s is the long-term pure static strength of the ply at X years, and S_c is the long-term pure cyclic strength of the ply at Y cycles. In real life, the applied strains ϵ_s and ϵ_c (both known) are too small to fail the ply in X years and Y cycles. To cause failure, they require multiplication by a factor SF . Entering this SF factor in (3) we obtain

$$\left(\frac{\epsilon_s \times SF}{S_s}\right)^{\frac{1}{G_s}} + \left(\frac{\epsilon_c \times SF}{S_c}\right)^{\frac{1}{G_c}} = 1.00 \quad (3A)$$

Equation (3A) computes the factor SF that multiplies the applied load components to fail the ply in X years and Y cycles. SF is the long-term safety factor.

The derivation of equation (3A) ignored the interaction between the cyclic and the static load components. The complete unified equation, including this interaction, is

$$\underbrace{\left(\frac{\epsilon_s \times SF}{S_s}\right)^{\frac{1}{G_s}}}_{\text{static damage}} + \underbrace{\left(\frac{\epsilon_c \times SF}{S_c}\right)^{\frac{1}{G_c}}}_{\text{cyclic damage}} + \underbrace{\left(\frac{\epsilon_s \times \epsilon_c \times SF^2}{S_s \times S_c}\right)^{\frac{1}{G_{sc}}}}_{\text{interaction damage}} = 1.00 \quad (4)$$

Equation (4) is the final form of the unified equation. The interaction parameter G_{sc} is determined experimentally, as shown in the part 2 of this chapter. The only unknown in equation (4) is the long-term safety factor SF . The unified equation computes the long-term safety factor SF for any given combination of static and cyclic load components acting simultaneously for any period of time X .

Note: The number of cycles Y must be compatible with the time X . For failure at $X = 50$ years, Y must be the number of cycles at 50 years.

Example 1 – Suppose failure at $X = 50$ years and $Y = 300\,000$ cycles.

From the pure regression equations (1) and (2), we compute the number of cycles N and the time t that would fail the ply if the pure static and the pure cyclic load components acted alone.

Suppose we compute $N = 1\,000\,000$ cycles to fail the ply if the cyclic component ϵ_c acted alone. The actual number of cycles at failure under the combined loading in $X = 50$ years is $Y = 300\,000$, which is less than $N = 1\,000\,000$ cycles. The actual life of $X = 50$ years accounts for only $300/1000 = 0.30 = 30\%$ of the pure cyclic lifetime.

Suppose also we obtain $t = 200$ years to fail the ply if the static load component ϵ_s acted alone. By the same reasoning, the actual lifetime $X = 50$ years accounts for only $50/200 = 0.25 = 25\%$ of the pure static lifetime.

The immediate question is.... Would the above load components fail the ply in $X = 50$ years and $Y = 300\,000$ cycles if added together? The two load components together would account for just $40\% + 30\% = 70\%$ of the ply lifetime. This is not enough to fail the ply. After 50 years of uninterrupted simultaneous loading, the ply apparently retains $100\% - 70\% = 30\%$ of its life.

The interaction between the static and the cyclic load components accounts for this missing 30%.

The unified equation computes the long-term safety factor SF for a sustained loading over the lifetime of X years and Y cycles. The complete unified equation should account also for the shear strains, which have not been included here. The discussion of the complete equation, including the shear strains, is in the part 5 of this chapter.

The unified equation is a neat, symmetrical and elegant mathematical expression applicable to all load-dependent long-term modes of failure. When applied to the fiber direction of UD plies, it predicts the long-term SF to rupture. When applied to the transverse direction of UD plies, it predicts the long-term SF to weeping, stiffness or infiltration failures, as the case may be. Similar arguments apply to chopped fiber plies.

4 – Applying the unified equation: The unified equation solves any load-dependent durability problems for any laminate. Its application involves the following steps:

- *Determination of the critical ply.*
- *Computation of the total strains (mechanical and residual) on the critical ply.*
- *Decomposition of the actual loading in its pure static and pure cyclic components.*
- *Computation of the partial damages from each pure loading component acting alone.*
- *Computation of the interaction damage.*
- *The total damage determines the ply durability*

These are the steps to compute the load-dependent durability of plies under any loading. The focus of the unified equation is on the critical ply, not on laminates. However, with the exception of weep failure, laminate durability is equal to that of the critical ply.

The inputs required by the unified equation are.

- *The total static and cyclic strain components referred to the ply frame*
- *The slope G_s of the pure static regression line of the critical ply*
- *The slope G_c of the pure cyclic regression line of the critical ply*
- *The interaction parameter G_{sc} at the load ratio R .*
- *The long-term pure static strength S_s of the critical ply at the lifetime of X years*
- *The long-term pure cyclic strength S_c of the critical ply at Y cycles and X years.*

The requirements for laminates would be the same as for plies, except that all strains should be in the global system. The problem with laminates is the cost to develop their pure regression equations, which are readily available and tabulated for plies. See chapter 13.

The reader will keep in mind that, although derived for plies, the unified equation holds for laminates as well. As an example, we mention the weep failure of pressurized pipes, that is best computed using pipe, instead of ply equations. For details, see the part 2 of chapter 13.

The total static and cyclic strain components entering the unified equation include the mechanical and residual contributions computed for the critical ply, as shown in the part 1 of this book. The pure slopes G_s and G_c come from the ply pure regression equations listed in chapter 13. The long-

term pure strengths S_s and S_c derive from the pure equations and the desired lifetimes. The remaining unknowns in equation (4) are the long-term safety factor SF and the interaction parameter G_{sc} . The next two chapters will discuss the determination of the interaction parameter G_{sc} .

The critical ply depends on the mode of failure. For long-term ruptures, the UD plies strained in the fiber direction are critical. For infiltration failures, the plies of chopped fibers are critical. For weep failures of pressure oil pipes, the UD55 plies are critical. The engineer should have no problem in identifying the mode of failure and the critical ply in each case.

The classical idea of durability is commonly associated with the computation of the time or the number of cycles to failure at a given safety factor SF . The unified equation works in reverse, computing the long-term safety factor SF from pre-defined lifetimes X and number of cycles Y . The pre-defined lifetime X and the corresponding number of cycles Y are not explicit in the unified equation. Rather, they are implicit in the long-term ply strengths S_s and S_c .

As a closing note to this section, we recognize that it is easier to work with the unified equation expressed in terms of the R ratio and the operating peak strain ε , instead of the static and cyclic strain components. Introducing this new notation, the unified equation (4) becomes

$$\left(\frac{R \times \varepsilon_{\max} \times SF}{S_s} \right)^{\frac{1}{G_s}} + \left(\frac{(1-R) \times \varepsilon_{\max} \times SF}{S_c} \right)^{\frac{1}{G_c}} + \left(\frac{R \times (1-R) \times \varepsilon_{\max}^2 \times SF^2}{S_s \times S_c} \right)^{\frac{1}{G_{sc}}} = 1.00$$

Dropping the subscript “max”, we have

$$\left(\frac{R \times \varepsilon \times SF}{S_s} \right)^{\frac{1}{G_s}} + \left(\frac{(1-R) \times \varepsilon \times SF}{S_c} \right)^{\frac{1}{G_c}} + \left(\frac{R \times (1-R) \times \varepsilon^2 \times SF^2}{S_s \times S_c} \right)^{\frac{1}{G_{sc}}} = 1.00 \quad (5)$$

Where R is the load ratio and ε is the peak strain. This notation is in full agreement with the classical authors.

The unified equation covers all modes of load-dependent failures, whether infiltration, loss of stiffness, weeping or rupture. Its application starts with the definition of the loading and the desired lifetime. It goes like this:

- *The user informs the operating conditions.*
- *The user informs the expected number of cycles Y in the lifetime X .*
- *The designer identifies the mode of failure and the critical ply.*
- *The designer computes the peak total strain and the R ratio for the critical ply.*
- *The informed durability X and Y define the load path. See next chapter*
- *The designer picks the pure regression lines for the critical ply and the mode of failure. These lines are in the part I of chapter 13.*
- *The slope parameters G_s and G_c come from the pure regression lines.*
- *The long-term strengths S_s and S_c come from the pure regression lines and the informed durability X and Y .*
- *The interaction parameter G_{sc} come from tables in chapters 16.*
- *The only unknown in the unified equation is the long-term safety factor SF .*

The unified equation does not compute the lifetime. Rather, it computes the long-term safety factor SF for the user-specified lifetime. The numerical examples that follow will help illustrate and consolidate the concepts behind the unified equation. As indicated earlier, to simplify the presentation, the shear strains have not been included.

Example 2 – This numerical example computes the rupture durability of underground hoop-chop pipes used in water transmission. The user-defined operating conditions are:

Hoop static tensile strain component	$\epsilon_s = 0.25\%$
Hoop static bending strain component	$\epsilon_b = 0.15\%$
Hoop cyclic tensile strain component	$\epsilon_c = 0.10\%$
Target lifetime	$X = 50 \text{ years} = 438\,000 \text{ hours}$
Number of cycles in 50 years	$Y = 158\,000\,000 \text{ cycles}$

The UD90 ply of hoop-chop pipes are critical for rupture failure. We will solve the problem for two scenarios:

- Pipes of E glass
- Pipes of boron-free glass.

Since the UD90 fibers align in the hoop direction, strain rotation is not necessary. The minimum hoop strain comes from adding the known tensile and bending static components.

$$\epsilon_{\min} = 0.25 + 0.15 = 0.40\% .$$

The maximum hoop strain comes in the same way, including the cyclic component.

$$\epsilon_{\max} = 0.25 + 0.15 + 0.10 = 0.50\%$$

The R ratio is

$$R = \frac{0.40}{0.50} = 0.80$$

The load path (loading frequency) is

$$\text{frequency} = \frac{158\,000\,000}{50 \times 365 \times 24 \times 60 \times 60} = 0.10 \text{ Hertz}$$

Parameter	Value	Source
Gs	0.077	Mark Greenwood for boron-free glass (chapter15)
	0.130	Mark Greenwood for E glass (chapter 15)
Gc	0.089	Guangxu Wei (fiber direction chapter 15)
	0.040	Guangxu Wei (transverse direction chapter 13)
Gsc	NA	Gsc varies with the lifetime, the number of cycles and the strain ratio R.
Ss	0.92%	Mark Greenwood for boron-free glass and a target lifetime of X = 50 years (chapter 15)
	0.41%	Mark Greenwood for E glass and a target lifetime of X = 50 years (chapter 15)
Sc	0.62%	Guangxu Wei for Y = 158 000 000 cycles in the fiber direction (chapter 13)

Table 1

Inputs to the unified equation, for a target lifetime of $X = 50$ years and $Y = 158\,000\,000$ cycles in 50 years.

First, as a curiosity, we compute the short-term safety factor against rupture. The short-term SF is the ratio of the rupture threshold (Tr) of the UD ply and the peak hoop strain.

$$SF = \frac{3.0\%}{0.50\%} = 6$$

The short-term safety factor is the same for E glass and boron-free glass.

We next compute the long-term safety factor. The input data needed in the unified equation for a target lifetime of $X = 50$ years and $Y = 158\,000\,000$ cycles are in table 1. The long-term static strengths S_s of the UD plies come from the pure static equations developed in chapter 15.

For boron-free glass in $X = 50$ years:

$$\log \varepsilon_s = 0.400 - 0.077 \log(\text{hours})$$

$$\log S_s = 0.400 - 0.077 \times \log(438000)$$

$$\log S_s = 0.400 - 0.077 \times 5.641 = -0.034$$

$$S_s = 0.92\%$$

For E glass in $X = 50$ years:

$$\log \varepsilon_s = 0.347 - 0.130 \log(\text{hours})$$

$$\log S_s = 0.347 - 0.130 \times 5.641 = -0.386$$

$$S_s = 0.41\%$$

The long-term cyclic strength S_c of the UD plies in the fiber direction come from the pure cyclic equation.

$$\log S_c = 0.519 - 0.089 \log Y \quad (\text{Cyclic rupture in the fiber direction})$$

Where $Y = 158\,000\,000$ is the number of cycles in the lifetime $X = 50$ years. Entering $Y = 158\,000\,000$ cycles in this equation, we have

$$\log S_c = 0.519 - 0.089 \log 158 \times 10^6$$

$$S_c = 0.62\%.$$

We now proceed to apply the unified equation. We address the boron-free pipes first, by entering the appropriate inputs in the unified equation (5).

$$\left(\frac{R \times \varepsilon \times SF}{S_s} \right)^{\frac{1}{G_s}} + \left(\frac{(1-R) \times \varepsilon \times SF}{S_c} \right)^{\frac{1}{G_c}} + \left(\frac{R(1-R) \times \varepsilon^2 \times SF^2}{S_s \times S_c} \right)^{\frac{1}{G_{sc}}} = 1.00$$

$$\left(\frac{0.80 \times 0.50 \times SF}{0.92} \right)^{\frac{1}{0.077}} + \left(\frac{(1-0.80) \times 0.50 \times SF}{0.62} \right)^{\frac{1}{0.089}} + \left(\frac{0.80(1-0.80) \times 0.50^2 \times SF^2}{0.92 \times 0.62} \right)^{\frac{1}{G_{sc}}} = 1.00$$

The interaction parameter G_{sc} is not known for the load path 0.10 Hz and $R = 0.80$. However, for illustration purposes, we assume $G_{sc} = 2$. Hence,

$$\left(\frac{0.40 \times SF}{0.92}\right)^{\frac{1}{0.077}} + \left(\frac{0.10 \times SF}{0.62}\right)^{\frac{1}{0.089}} + \left(\frac{0.40 \times 0.10 \times SF^2}{0.92 \times 0.62}\right)^{\frac{1}{2}} = 1.00$$

Solving this equation, we obtain $SF = 1.8$. The meaning of this safety factor is like follows. The applied loading by itself is not enough to fail the pipe in 50 years. For rupture to occur in 50 years, the applied loading should be 1.8 times higher. Therefore, the long-term safety factor is $SF = 1.8$.

Setting $SF = 1.8$ in the above equation we have an idea of the damages from each loading.

$$\left(\frac{0.40 \times 1.8}{0.92}\right)^{\frac{1}{0.077}} + \left(\frac{0.15 \times 1.8}{0.62}\right)^{\frac{1}{0.089}} + \left(\frac{0.40 \times 0.15 \times 1.8^2}{0.92 \times 0.62}\right)^{\frac{1}{2}} = 1.00$$

$$4.14\% + 0.01\% + 95.91\% = 100.0\%$$

The above computations indicate the pure static and the pure cyclic loading components to have little effect on the pipe durability. The real damage comes from their interaction. This conclusion, however, is purely illustrative, since the correct value of G_{sc} is not available for this loading.

We next do a similar analysis for the pipes of E glass. The long-term pure cyclic strength S_c for E glass is 0.62%, the same as for boron-free glass. The long-term static strength for E glass is (table 1) $S_s = 0.41\%$. Entering these in equation (5)

$$\left(\frac{0.40 \times SF}{0.41}\right)^{\frac{1}{0.130}} + \left(\frac{0.10 \times SF}{0.62}\right)^{\frac{1}{0.089}} + \left(\frac{0.40 \times 0.10 \times SF^2}{0.41 \times 0.62}\right)^{\frac{1}{2}} = 1.00$$

Which gives $SF = 0.73$, meaning pipes of E glass under the stated loading rupture before completing 50 years in service.

This example illustrates the superiority of boron-free glass vis-à-vis the traditional E formulation. The higher hydrolytic stability of boron-free glass produces pipes of higher resistance to long-term rupture.

Example 3 – Compute the 50 years rupture safety factor SF for hoop-chop pipes operating in the following conditions.

- The HDT of the vinyl ester resin matrix is $HDT = 105C$
- The peak post-cure temperature is $PCT = 100C$.
- The pipes carry water at room temperature $OT = 25C$
- The operating pressure is $P = 10$ bar under a 2:1 loading.
- The pipe diameter is $D = 500$ mm
- The water pickup of vinyl ester resins is $\Delta m = 1.0\%$
- There is no cyclic loads

The operating conditions allow the computation of all strains on the critical ply of the pipe. The mode of failure is rupture. The UD ply in the hoop direction is critical. The total strain in the fiber direction, as computed in the part 1 of this book, is $\epsilon_1 = 0.0012 = 0.12\%$. There is no cyclic load component.

First, as a curiosity, we determine the short-term safety factor against rupture.

$$SF = \frac{3.0\%}{0.12\%} = 25$$

This healthy short-term safety factor is not dependent on the glass formulation. We proceed to the computation of the long-term safety factor. We assume pipes made of boron-free glass with the following static regression equation.

$$\log \epsilon_s = 0.400 - 0.077 \log(\text{hours})$$

$$\log S_s = 0.400 - 0.077 \log(438000) \quad (\text{Pure static strength at 50 years})$$

$$S_s = 0.92\%$$

The unified equation is

$$\left(\frac{R \times \varepsilon \times SF}{S_s} \right)^{\frac{1}{G_s}} + \left(\frac{(1-R) \times \varepsilon \times SF}{S_c} \right)^{\frac{1}{G_c}} + \left(\frac{R(1-R) \times \varepsilon^2 \times SF^2}{S_s \times S_c} \right)^{\frac{1}{G_{sc}}} = 1.00$$

In the absence of cyclic loads, $R = 1.0$, reducing the unified equation to

$$\left(\frac{\varepsilon \times SF}{S_s} \right)^{\frac{1}{G_s}} = 1.00$$

$$\left(\frac{0.12 \times SF}{0.92} \right)^{\frac{1}{0.077}} = 1.00$$

Which gives $SF = 7.6$.

This example indicates a 50-year long-term rupture safety factor $SF = 7.6$, against a short-term safety factor $SF = 25$.

Example 4 – We continue the analysis of the pipe in example 3, assuming the presence of a hoop cyclic load component of amplitude $\varepsilon_c = 0.10\%$. The expected number of cycles at 50 years is $Y = 10\,000\,000$. The analysis is for long-term rupture and the focus is on the fiber direction.

The long-term pure cyclic strength in the fiber direction computed from equation (6) for $Y = 10\,000\,000$ cycles is

$$\log S_c = 0.519 - 0.089 \log Y$$

$$\log S_c = 0.519 - 0.089 \log 10^7$$

$$S_c = 0.79\%$$

The minimum strain in the fiber direction is $\varepsilon_{\min} = 0.12\%$. The maximum strain in the fiber direction is $\varepsilon_{\max} = 0.12 + 0.10 = 0.22\%$. The R ratio in the fiber direction is

$$R = \frac{0.12}{0.12 + 0.10} = 0.55$$

The loading frequency is

$$\text{frequency} = \frac{10\,000\,000}{50 \times 365 \times 24 \times 60 \times 60} = 0.01 \text{ cycles/sec}$$

Again, we do not have the G_{sc} parameter for the load path 0.01 Hertz and load ratio $R = 0.55$. Assuming, for illustration purposes, $G_{sc} = 5$, we obtain

$$\left(\frac{R \times \varepsilon \times SF}{S_s} \right)^{\frac{1}{G_s}} + \left(\frac{(1-R) \times \varepsilon \times SF}{S_c} \right)^{\frac{1}{G_c}} + \left(\frac{R(1-R) \times \varepsilon^2 \times SF^2}{S_s \times S_c} \right)^{\frac{1}{G_{sc}}} = 1.00$$

$$\left(\frac{0.55 \times 0.22 \times SF}{0.92} \right)^{\frac{1}{0.077}} + \left(\frac{(1-0.55) \times 0.22 \times SF}{0.79} \right)^{\frac{1}{0.089}} + \left(\frac{0.55(1-0.55) \times 0.22^2 \times SF^2}{0.92 \times 0.79} \right)^{\frac{1}{5}} = 1.00$$

Which gives $SF = 5.7$

The reader will note the arbitrarily chosen $G_{sc} = 5$ in this example, different from the $G_{sc} = 2$ chosen in example 1. The actual values of G_{sc} have not yet been measured, being unknown at this time. The protocol to measure the interaction parameter G_{sc} is discussed in the next two chapters.

5 – Time-independent failures: So far, we have been dealing with long-term rupture failures from strains in the fiber direction of UD plies. The fiber-dominated rupture failures involve the time variable and the complete form of the unified equation. The resin-dominated failures do not involve the time variable, requiring a simplified version of the unified equation. Let us derive this simplified version. There are two simplifications.

- The time (static) contribution is set equal to zero, since resin-dominated failures are time-independent.
- For the same reason, the long-term pure static strength S_s is set equal to the ply failure threshold.

The simplified, time-independent, version of the unified equation is

$$\left(\frac{(1-R) \times \epsilon \times SF}{S_c} \right)^{\frac{1}{G_c}} + \left(\frac{R(1-R) \times \epsilon^2 \times SF^2}{S_s \times S_c} \right)^{\frac{1}{G_{sc}}} = 1.00 \quad (6)$$

Where the long-term pure static strength S_s of the critical ply is set equal to the applicable failure threshold.

$$S_s = T_i \quad (\text{Infiltration failure})$$

$$S_s = T_w \quad (\text{Weep failure})$$

$$S_s = T_s \quad (\text{Stiffness failure})$$

The long-term pure cyclic strength, S_c , derives from the applicable pure cyclic regression equation and the user-informed number of cycles Y . The simplified equation (6) applies to both transverse loaded UD plies and to chopped fiber plies.

Example 5 – Compute the safety factor at the onset of weep failure for a UD55 vinyl ester pipe.

The onset of weep failure of oil pipes occurs when the transverse strain equals the weep threshold T_w of the UD ply. The focus is now on weep failure, meaning a shift from the fiber direction to the transverse direction of the critical UD55 plies. UD55 pipes have no weep barrier of chopped fibers.

The following data is available

$\epsilon_c = 0.065\%$ is the cyclic transverse strain component

$\epsilon_s = 0.15\%$ is the static transverse strain component

$0.22\% = 0.065\% + 0.15\%$ is the peak transverse strain

$R = 0.15/0.22 = 0.68$

$X = 50$ years is the user-informed expected durability

$Y = 10\,000\,000$ is the user-informed number of cycles in $X = 50$ years

0.01 Hz is the loading frequency.

The pure weep regression equation of transverse loaded vinyl ester UD plies is in table 4 of chapter 13.

$$\log(\epsilon) = \log(0.55) - 0.052 \times \log(N) \quad (7)$$

The long-term pure cyclic weep strength of the ply is

$$\log (S_c) = \log (0.55) - 0.052 \times \log (Y)$$

Where $Y = 10\,000\,000$ is the user-informed number of cycles at failure.

$$S_c = 0.24\%$$

The long-term transverse static strength of UD55 vinyl ester plies is the weep threshold $S_s = T_w = 0.55\%$.

The interaction parameter G_{sc} is not available. For illustration purposes, we assume $G_{sc} = 2500$.

This is a time-independent problem requiring the simplified equation (6)

$$\left(\frac{(1-R) \times \epsilon \times SF}{S_c} \right)^{\frac{1}{G_c}} + \left(\frac{R(1-R) \times \epsilon^2 \times SF^2}{S_s \times S_c} \right)^{\frac{1}{G_{sc}}} = 1.00$$

Entering the data in the above, we have

$$\left(\frac{(1 - 0.68) \times 0.22 \times SF}{0.24} \right)^{1/0.052} + \left(\frac{0.68 \times (1 - 0.68) \times 0.22^2 \times SF^2}{0.55 \times 0.24} \right)^{1/2500} = 1.00$$

From which we obtain the long-term safety factor $SF = 2.3$. The unified equation predicts a long-term safety factor $SF = 2.3$ at the onset of weeping after 50 years of continuous service.

The short-term safety factor in this case, as a matter of curiosity, is

$$SF = \frac{0.55\%}{0.22\%} = 2.5$$

The reader will noticed we did all computations on the local frame of the critical ply. Therefore, the computed long-term safety factor SF , refers to the onset of weeping, not to the actual weeping. To compute the actual weep SF , we need the pipe weep pure cyclic equation.

Example 6 – Repeat the analysis of example 5 for polyester pipes.

The pure weep regression equation for transverse loaded polyester UD plies is available in table 4 of chapter 13.

$$\log (\epsilon) = \log (0.45) - 0.048 \times \log (N) \quad (7A)$$

The long-term pure cyclic weep strength of the ply is

$$\log (S_c) = \log (0.45) - 0.048 \times \log (Y)$$

Where $Y = 10\,000\,000$ is the user-informed number of cycles.

$$\log (S_c) = \log (0.45) - 0.048 \times \log (10^7)$$

$$S_c = 0.21\%$$

The long-term transverse static strength of polyester UD ply is equal to the weep threshold $S_s = T_w = 0.45\%$.

The interaction parameter G_{sc} is not available. For illustration purposes, we assume $G_{sc} = 2500$.

This is a time-independent problem requiring the simplified equation (6)

$$\left(\frac{(1-R) \times \epsilon \times SF}{S_c} \right)^{\frac{1}{G_c}} + \left(\frac{R(1-R) \times \epsilon^2 \times SF^2}{S_s \times S_c} \right)^{\frac{1}{G_{sc}}} = 1.00$$

Entering the data in the above, we have

$$\left(\frac{(1-0.68) \times 0.22 \times SF}{0.21} \right)^{\frac{1}{0.048}} + \left(\frac{0.68(1-0.68) \times 0.22^2 \times SF^2}{0.45 \times 0.21} \right)^{\frac{1}{2500}} = 1.00$$

From which we obtain the long-term safety factor $SF = 2.1$. The unified equation predicts a weep long-term safety factor $SF = 2.1$ for polyester pipes after 50 years of continuous service.

The short-term safety factor in this case, as a matter of curiosity, is

$$SF = \frac{0.45\%}{0.22\%} = 2.0$$

This numerical example brings out an interesting point. The computed long-term safety factor (2.1) is higher than the short-term $SF = 2.0$. There is no contradiction in this curious fact, since the two safety factors are not related. The long-term safety factors can be – and often are – larger than the short-term safety factors.

Again, as explained before, the computation of the actual weep time requires use of the pipe weep equation. The ply equation used in this example does not capture the travel time, giving the SF at the onset of weeping

Example 7 – Compute the weep durability of polyester sanitation pipes carrying pressurized water. Assume the following operating conditions.

Static hoop strain component = 0.35%

Cyclic hoop strain component = 0.15%

Desired lifetime $X = 50$ years

Expected number of cycles at 50 years $Y = 10^8$ cycles

The critical ply in this case is the weep barrier of chopped fibers. The pure cyclic weep equation of polyester chopped plies is in table 4 of chapter 13.

$$\log(\epsilon) = \log(0.80) - 0.060 \times \log(N)$$

The pure long-term weep strength is

$$\log S_c = \log(0.80) - 0.060 \times \log 10^8$$

$$S_c = 0.26\%$$

The long-term static strength of polyester chopped ply is

$$S_s = T_w = 0.80\%$$

The operating peak hoop strain is

$$\epsilon = 0.35 + 0.15 = 0.50\%$$

The minimum hoop strain is 0.35%

$$\text{The } R \text{ ratio is } 0.35/0.50 = 0.70$$

The interaction parameter G_{sc} for $R = 0.70$ is not available. We assume $G_{sc} = 10$ for illustration purposes.

The long-term weep safety factor SF comes from the simplified version of the unified equation

$$\left(\frac{(1-R) \times \varepsilon \times SF}{S_c} \right)^{\frac{1}{G_c}} + \left(\frac{R(1-R) \times \varepsilon^2 \times SF^2}{S_s \times S_c} \right)^{\frac{1}{G_{sc}}} = 1.00$$

$$\left(\frac{(1-0.70) \times 0.50 \times SF}{0.26} \right)^{\frac{1}{0.061}} + \left(\frac{0.70(1-0.70) \times 0.50^2 \times SF^2}{0.80 \times 0.26} \right)^{\frac{1}{10}} = 1.00$$

Solving the above, we obtain $SF = 1.4$

The short-term SF is

$$SF = \frac{0.80}{0.50} = 1.6$$

Again, this long-term SF refers to the onset of weeping, not to weeping itself.

Example 8 – Explain why the long-term static strengths S_s of time-independent failures – infiltration, weep and stiffness – are equal to the failure thresholds.

The failure thresholds are short-term strengths. In time-independent failures, the long-term and the short-term strengths have the same value. Therefore, the long-term strengths of time-independent failures are equal to the failure thresholds. See table 6 in the appendix 2 of chapter 13.

6 – Block loading: In deriving the unified equation, we have assumed a constant loading throughout the ply lifetime. We know this never happens in real life. In a lifetime of X years, the laminate may see a variety of loadings and load paths.

To solve this, we group the anticipated complex loading in many blocks of similar R ratios and load paths. We end up with several load blocks of known durations, constant R ratios and constant load paths. The long-term safety factor for the actual loading results from the partial safety factors of each individual load block. The only difficulty in this method is, perhaps, the accurate grouping of the actual loading into representative blocks.

Once the actual loading has been properly broken and grouped into blocks, by whatever means, the solution proceeds based on two assumptions:

- The sequence of the load blocks is not relevant.
- All load blocks operate on pristine, undamaged laminates

These assumptions are not as restrictive as they appear. Recall from the part 2 of chapter 13, that load spikes have little effect on the durability of plies. This implies in the block sequence having little importance.

Example 9 – Consider the design of a pipeline under a variable loading distributed over 45 years of continuous service. A careful analysis of the expected loading identifies the approximate simple load blocks.

1. 10 years peak pressure of 20 bar, minimum pressure of 10 bar at 0.5 Hz.
2. 15 years peak pressure of 20 bar, minimum pressure of 10 bar at 0.1 Hz.
3. 5 years peak pressure of 18 bar, minimum pressure of 12 bar at 1.0 Hz.
4. 10 years peak pressure of 30 bar, minimum pressure 20 bar at 0.01 Hz
5. 5 years at constant pressure of 30 bar. No cyclic loading.

The above load blocks represent a careful estimation of the expected operating conditions. We use the unified equation to compute the long-term safety factors SF of each individual block.

The safety factor SF_1 , corresponding to the load block 1, is computed for a lifetime $X = 10$ years, $R = 0.50$ and a load path of 0.5 Hz. The unified equation readily gives the safety factor SF_1 for this loading.

The safety factor SF_2 for the load block 2 is computed for a lifetime $X = 15$ years, $R = 0.50$ and a load path of 0.1 Hz.

The safety factor SF_3 for the load block 3 is computed for a lifetime $X = 5$ years, $R = 0.67$ and a load path of 1.0 Hz.

The safety factor SF_4 for the load block 4 is computed for a lifetime $X = 10$ years, $R = 0.67$ and a load path of 0.01 Hz.

The safety factor SF_5 for the load block 5 derives from a lifetime of 5 years under a static load of 30 bars.

Each load block produces its own safety factor SF_i .

The overall long-term safety factor SF derives from the partial values SF_i .

The partial life loss of the pipeline in each load block is

$$(\text{partial life loss per block})_i = \frac{\text{total life}}{SF_i}$$

Where SF_i is the partial safety factor of the i_{th} block. The total life loss accumulated over all blocks is

$$\text{life loss} = \sum (\text{partial life loss per block})_i = \sum \frac{\text{total life}}{SF_i} \quad (8)$$

The life loss from the actual loading is

$$\text{life loss} = \frac{\text{total life}}{SF} \quad (9)$$

The overall safety factor SF comes from equating equations (8) and (9)

$$\frac{1}{SF} = \sum \frac{1}{SF_i} \quad (10)$$

In expanded form, the overall SF is

$$\frac{1}{SF} = \frac{1}{SF_1} + \frac{1}{SF_2} + \frac{1}{SF_3} + \frac{1}{SF_4} + \frac{1}{SF_5}$$

The above derivation assumes the individual blocks acting on pristine, non-aged laminates. Equation (10) is valid for all four modes of failure.

Example 10 – The computation of the load-dependent durability of composite equipment assumes pristine plies throughout the ageing process. This assumption does not hold when the critical ply is the last – not the first – to fail. In this situation, the strain in the critical ply will increase along the laminate lifetime, as the weaker plies fail and lose stiffness. This problem is easy to solve with the concept of block loading.

To compute the durability of critical plies that are the last to fail, we discount the failed plies to compute a new strain of a new block. We consider the succession of peak strains on the critical ply as different load blocks. The overall long-term SF of the critical ply comes from the load block concept discussed above.

Example 11 – Explain the difference between the short-term and the long-term safety factors.

The short-term safety factors are multipliers that bring the peak applied strain to the short-term strengths, or failure thresholds.

$$SF = \frac{\text{Failure threshold}}{\text{peak strain}}$$

The short-term safety factors SF do not involve the time variable, or the number of cycles. The classical literature deals exclusively with the short-term safety factor.

The long-term safety factors SF come from the unified equation. They are multipliers that bring the applied peak strain to values that fail the ply at the time X and number of cycles Y. The long-term SF takes into consideration the loading parameters, like R ratio, load path, lifetime X, number of cycles Y, and the interaction between the static and cyclic load components.

$$\left(\frac{R \times \epsilon \times SF}{S_s} \right)^{\frac{1}{G_s}} + \left(\frac{(1-R) \times \epsilon \times SF}{S_c} \right)^{\frac{1}{G_c}} + \left(\frac{R(1-R) \times \epsilon^2 \times SF^2}{S_s \times S_c} \right)^{\frac{1}{G_{sc}}} = 1.00$$

This book deals with the unified equation and the long-term safety factors. The short-term safety factors are not a priority in this book, receiving just a passing mention in a few cases.

The short-term and long-term safety factors are not related. They apply to different situations. It is common to find safety factors larger for the long-term than for the short-term. The prudent engineer computes both values. It is possible to find loading cases with satisfactory long-term safety factors that nevertheless fail the ply in the first cycle.

7 – The unified weep equation for pipes. Failure of the critical ply implies in failure of the entire laminate, regardless of thickness and construction. This is a correction of the well-known concept of first ply failure. In fact, it is not the first ply, but the critical ply, that determines laminate failure. This definition holds for all laminates and all modes of failure, except weeping of pressurized pipes. Ply equations cannot predict weep failures, as they increase with the wall thickness of the pipes.

We should modify the unified equation to predict the weeping of pipes.

To do so, we derive the pipe pure (cyclic and static) weep equations from a simple extension of the concepts discussed in the part 2 of chapter 13.

The pure cyclic weep equation of pipes in the global frame is

$$\log(\epsilon) = \log(A_w) - \frac{\log(A_w/T_0)}{20} \times \log(N)$$

The pure static weep equation of pipes, also in the global frame, is

$$\epsilon = T_w$$

Where

A_w is the weep threshold (or weep strength) of the pipe in the global frame

To is the fatigue threshold of the critical ply in the global frame

Tw is the weep threshold of the critical ply, also in the global frame

We can, if desired, express all thresholds in the local frame. In this case, we have

,
The pure cyclic weep equation of pipes is

$$\log(\epsilon) = \log(A_2) - \frac{\log(A_2/0.05)}{20} \times \log(N)$$

The pure static weep equation of pipes is

$$\epsilon = T_w$$

Where

A_2 is the weep threshold (or weep strength) of the pipe in the local frame
 T_w is the weep threshold of the critical ply, also in the local frame

The above pure equations provide the required inputs to solve for the actual (not the onset) weep durability of pipes using the unified equation. As in the case of plies, the unified equation for pipes is of little practical use in the absence of the interaction parameter G_{sc} . For that reason, we forget the unified equation and move directly to the computation of the intervals for the safety factor SF.

This is clear from the next example.

Example 11 – Let us use the unified equation to compute the actual weep long-term SF for the pipe discussed in the numerical example 7, part 2 of chapter 13. This example holds for a UD70 pipe, with isotropic weep barrier and the same threshold values in the global and local frames.

The following data applies:

$\epsilon = 0.30\%$ is the peak strain in the isotropic ply of chopped fibers
 $R = 0.8$ is the R ratio
 $X = 50$ years is the informed durability
 $N = 10^8$ cycles at 50 years

From the numerical example 7 in the part 2 of chapter 13, the pure cyclic and static equations for the pipe are:

$$\log(\epsilon) = \log(1.60) - 0.075 \times \log(N)$$

$$\epsilon = T_w$$

The long-term cyclic and static strengths of the pipe are

$$\log(S_c) = \log(1.60) - 0.075 \times \log(10^8)$$

$$S_c = 0.40\%$$

$$S_s = T_w = 0.80\%$$

The exponents in the unified equation are

$$G_s = 0.0$$

$$G_c = 0.075$$

$$G_{sc} = \text{unknown}$$

The interaction parameter G_{sc} is not available. For illustration purposes, we assume $G_{sc} = 250$.

Weep failures are time-independent, requiring the simplified form of equation (6)

$$\left(\frac{(1-R) \times \varepsilon \times SF}{S_c} \right)^{\frac{1}{G_c}} + \left(\frac{R(1-R) \times \varepsilon^2 \times SF^2}{S_s \times S_c} \right)^{\frac{1}{G_{sc}}} = 1.00$$

Entering the data in the above, we have

$$\left(\frac{(1-0.80) \times 0.30 \times SF}{0.40} \right)^{1/0.075} + \left(\frac{0.80 \times (1-0.80) \times 0.30^2 \times SF^2}{0.80 \times 0.40} \right)^{1/250} = 1.00$$

From which we obtain the long-term safety factor $SF = 4.0$. The unified equation predicts a weep long-term safety factor $SF = 4.0$ after 50 years of continuous service.

The short-term safety factor in this case is

$$SF = \frac{1.60\%}{0.30\%} = 5.3$$

The reader will note we have used the pure cyclic pipe equation. The computed long-term safety factor SF , therefore, refers to the actual weeping failure.

Appendix 1

The unified equation and the Goodman diagrams

The classical constant life diagrams, or Goodman diagrams, can solve only time-independent durability problems of laminates. These diagrams are essentially graphical representations of the time-independent unified equation.

Figure 1 shows a typical Goodman diagram for plies. The static and cyclic strain components are marked respectively on the horizontal and the vertical axis. The radial lines from the origin contain the experimental failure points corresponding to the load ratios R . The failure points corresponding to constant cycles to failure fall on the marked triangular shaped lines. The Goodman diagrams condense in a single graph the entire failure space of the ply, covering all possible failure points from different R ratios, number of cycles and peak strains. These diagrams are excellent tools to make quick computations of time-independent ply failures, like loss of stiffness, infiltration and weeping.

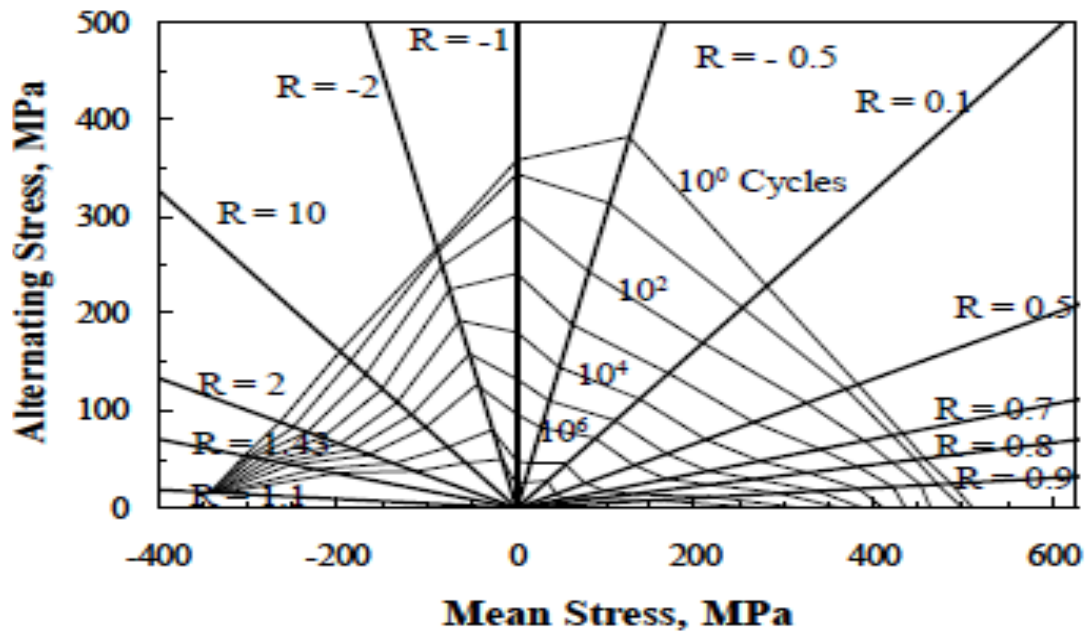


Figure 1

The Goodman diagrams are identical to the cycle-dependent unified equations, giving the correct answers to infiltration, weep and stiffness failures. These diagrams are graphical representation of the time-independent unified equations.

The Goodman diagrams ignore the time variable, a cornerstone of the unified equation. This limitation excludes them from the computation of long-term rupture failures involving the time variable. The long-term rupture durability of composites equipment, so easily captured by the unified equation, is beyond the reach of the Goodman diagrams. The same argument holds for shear strains.

However, for cycle-dependent failures – infiltration, stiffness and weeping – that do not depend on the time variable, the two methods give exactly the same results. This is not surprising, since both derive from the same regression equations.

Chapter 16

Part 2

The interaction parameter Gsc

1 – Introduction: The complete version of the unified equation for tensile strains is

$$\left(\frac{R\varepsilon \times SF}{S_s} \right)^{\frac{1}{G_s}} + \left(\frac{(1-R)\varepsilon \times SF}{S_c} \right)^{\frac{1}{G_c}} + \left(\frac{R(1-R)\varepsilon^2 \times SF^2}{S_s \times S_c} \right)^{\frac{1}{G_{sc}}} = 1.00 \quad (1)$$

The inputs required in this equation are:

- The load ratio R .
- The total peak strain ε on the critical ply.
- The long-term pure static strength of the ply, S_s , for a pre-defined lifetime X
- The long-term pure cyclic strength of the ply, S_c , for a pre-defined Y cycles
- The pure static slope, G_s
- The pure cyclic slope, G_c
- The interaction parameter, G_{sc}

The pre-defined lifetime X , as well as the corresponding number of cycles Y , enter the unified equation indirectly through the ply long-term pure strengths S_s and S_c . The pure static G_s and the pure cyclic G_c exponents are the slopes of the regression lines, listed in chapter 13 for all commercial plies and modes of failure. The only unknowns in equation (1) are the long-term safety factor SF and the interaction parameter G_{sc} .

The focus of this chapter is the numerical computation of the interaction parameter G_{sc} for tensile loads. This is a time-consuming and expensive endeavor, requiring the experimental determination of many regression equations for UD and chopped fiber plies at several R ratios and test frequencies. The effort and cost to produce such equations are too high and will probably never be undertaken. This, of course, prevents the use of the unified equation in the computation of the long-term safety factor SF .

In anticipation of this difficulty, the part 4 of this chapter introduces a method to compute the upper and lower bounds of the long-term safety factor SF , with no need of the interaction parameter G_{sc} . The reader can go directly to the part 4 of this chapter, skipping this part 2 and part 3. The interaction parameter G_{sc} is not absolutely essential to compute the load-dependent durability of composite equipment.

2 – The load paths: The interaction parameters G_{sc} derive directly from the unified equation (1) fed with known experimental failure points. The computation protocol is straightforward.

- *The G_{sc} values come from solving equation (1) fed with experimental failure points.*
- *The experimental failure data come from regression equations developed at known frequencies (load paths) and load ratios R .*
- *Since the failure data points fail the ply, the safety factor is $SF = 1.0$.*
- *That leaves G_{sc} as the only unknown in equation (1).*

There is no difficulty with the computation of the G_{sc} values. The problem is the difficulty to obtain the regression equations to provide the experimental data points. It is unfortunate that experimental regression equations for UD and chopped plies at R ratios other than the pure $R = 1.0$ and $R = 0.0$ values, are simply not available. To my knowledge, the only ply equations available at R ratios other than $R = 0.0$ and $R = 1.0$, are those developed by Guangxu Wei for UD plies at $R = 0.50$.

The number of cycles N relates to the time variable by the expression.

$$N = (\text{test frequency}) \times (\text{time}) \quad (2)$$

Guangxu Wei did his cyclic tests at the test frequency of 20 Hertz. Such a high frequency is good to shorten the test duration, but does not capture the time effect very well. Taking as an example the 20 Hz test frequency used by Guangxu Wei, we have

$$N = (20) \times (\text{seconds}) \quad (3)$$

From equation (3), Guangxu Wei completed his tests to $N = 10^8$ cycles in just 1400 hours. Such a short test duration, less than 60 days, is hardly long enough to capture the time-dependent fiber strain-corrosion.

Taking logarithms of both members in equation (3), we have

$$\log N = 1.30 + \log (\text{time}) \quad (4)$$

In equation (4), the time variable enters in seconds. Figure 1 shows a plot of this equation, illustrating the linear load path for 20 Hz test frequency. This is the only load path tested at this time for UD plies. In fact, the only load path for any ply. Obviously, we need more tests, conducted at different frequencies. We will have more to say about this as we move on.

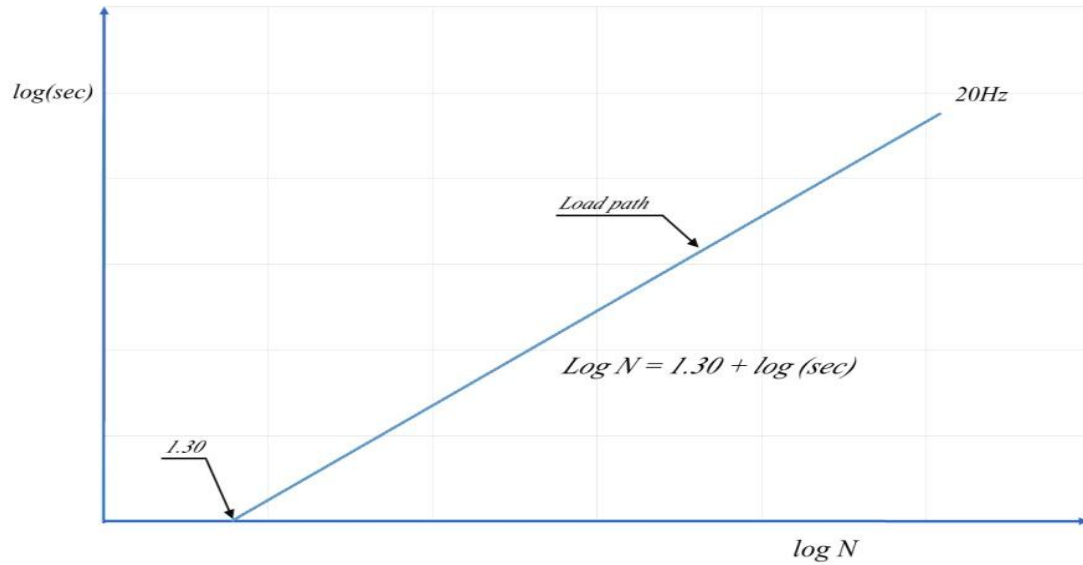


Figure 1

Load path for the test frequency 20 Hz. This is the only load path reported in the literature. The pure static load path is not cycle-dependent, plotting along the vertical axis. Likewise, the pure cyclic load path is not time-dependent, plotting along the horizontal axis.

3 – Computing the Gsc values: This section describes the protocol to compute the interaction parameter Gsc from experimental data. The experimental failure data come from regression equations developed at constant test frequencies and R ratios. The test frequency defines the load path, as shown in figure 1.

In using the unified equation, the known pure regression lines give the exponents Gs and Gc, as well as the long-term strengths Ss and Sc. The experimental failure points come from regression equations developed at constant R ratios and loading frequencies. These data enter the unified equation with the safety factor SF = 1.0, allowing the computation of the interaction parameter Gsc.

As we see, the protocol to compute Gsc uses experimental failure data from regression lines. To cover a wide range of situations, the computation requires many regression lines, at several R ratios and load paths. As an example, the experimental ply regression lines could be developed for R = 0.10, R = 0.30, R = 0.50, R = 0.70 and R = 0.9. For each of these R ratios, the test frequencies could be 10 Hertz, 5 Hertz, 1 Hertz and 0.1 Hertz.

The different test frequencies produce load paths similar to the one described in equation (4) and figure 1. A test frequency of 0.1 Hertz would produce the load path

$$N = 0.1 \times \text{seconds}$$

$$\log N = -1 + \log (\text{seconds})$$

The load path for a test frequency of 10 Hz is

$$N = 10 \times \text{seconds}$$

$$\log N = 1 + \log (\text{seconds})$$

And so on.

In figure 2, we plot several load paths for a fixed load ratio R . There is a G_{sc} value for every point on the load path. The value G_{sc}^1 corresponds to the failure point t_1 and N_1 on the load path 1. The value G_{sc}^2 corresponds to the failure point t_2 and N_2 on the load path 2.

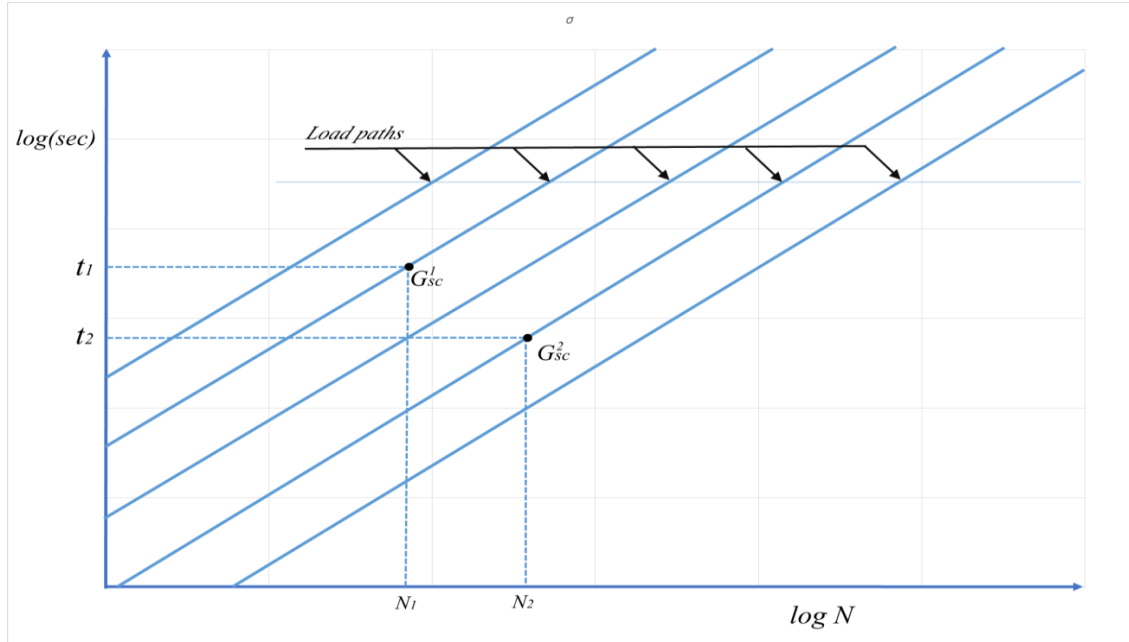


Figure 2

Shows two G_{sc} values of two failure points.

The protocol to compute the G_{sc} values falling on experimental load paths is as follows:

- Define the failure points by arbitrarily choosing the time “ t ” and the number of cycles “ N ”.
- Compute the peak failure strains for each pair of “ t ” and “ N ” using the ply regression equation.
- The peak strains and the corresponding N are known experimental failure points.
- The static and cyclic pure slopes G_s and G_c come from the pure regression equations.
- The long-term ply strengths S_s and S_c come from the pure regression equations and each failure point “ t ” and “ N ”.
- Enter the above data in the unified equation.
- The G_{sc} value is computed by setting the safety factor $SF = 1.0$.
- The G_{sc} values are tabulated for every load path as a function of N and t .

The following chapter gives some numerical examples to clarify the computation protocol just outlined. The important point to have in mind at this time is the recognition of different G_{sc} values for every point on the load paths. The G_{sc} value to use in the unified equation (1) falls on the load path corresponding to the user-informed durability X and Y . As a rule, the informed durability, represented by the time $t = X$ and the number of cycles $N = Y$ never fall on known load paths. Rather, they fall between known load paths, as shown in figure 2. The required value of the interaction parameter G_{sc} is obtained by interpolation.

4 – Interpolating the Gsc values: The R ratios and load paths in actual service are never the same as those that have been tested and tabulated. The Gsc value to use in equation (1) is therefore not readily available, requiring interpolation between tabulated values. The interpolation follows two steps. The first step interpolates is between load paths. The second interpolates is between R ratios. It works like this.

The Gsc value we are seeking interpolates between the four values determined where the user-defined X and Y axes meet the neighboring load paths. See figure 3. The interpolated Gsc value is

$$G_{sc} = \frac{G_1 + G_2 + G_3 + G_4}{4}$$

The same applies to interpolation between R ratios.

As we see, the computation of the Gsc value requires two interpolations, one between load paths and another between R ratios.

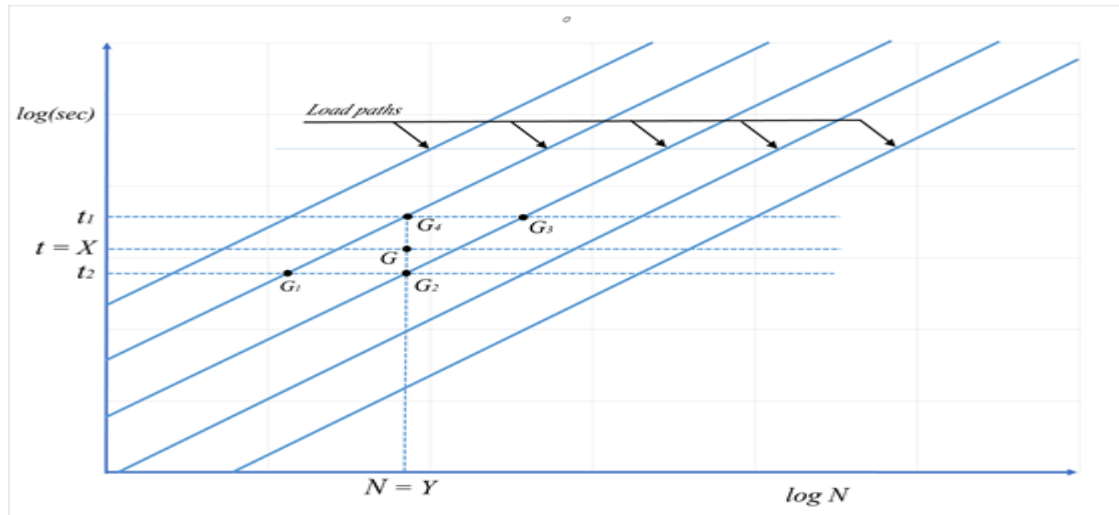


Figure 3

The figure shows load paths for a given R ratio. The parameter Gsc is known along the load paths. Outside the load paths, the Gsc values are obtained by interpolation. Good interpolations require many load paths.

5 – Computing the fiber-dominated durability. The time-dependent fiber failures require the complete version of the unified equation. The computation is like follows.

- Compute the total peak and minimum strains in the critical ply.
- Compute the R ratio.
- Compute the long-term pure static Ss and pure cyclic Sc strengths of the critical ply.
- The static and cyclic pure slopes Gs and Gc come from the pure rupture equations of the ply.
- Mark the target durability X and Y in figure 17.3.
- Interpolate between load paths as in figure 17.3.
- Derive the actual Gsc by interpolating between R ratios.
- Enter the actual Gsc in the unified equation
- Compute the long-term SF

The generation of good G_{sc} values requires many experimental regression equations developed for several R ratios and load frequencies. The equations to do this are not available at this time. The current lack of data limits the application of the unified equation in many practical situations involving fiber-dominated rupture. The computation protocol to compute time-dependent rupture durability seems very complicated. Indeed, it is.

6 – Computing resin-dominated durability: The computation of time-independent, resin-dominated ply failures uses a simplified version of the unified equation. The computation is like follows.

- Compute the total peak and minimum strains on the critical ply
- Compute the actual R ratio.
- Choose the applicable regression line (infiltration, weep or stiffness). See chapter 9.
- The long-term static strength S_s is equal to the infiltration, stiffness or weep threshold, as the case may be.
- Compute the long-term pure cyclic ply strength S_c using the pure regression line.
- The pure slope G_c comes from the pure regression equation.
- Derive the actual G_{sc} by interpolating between the tabulated values.
- Enter the interpolated G_{sc} in the simplified unified equation
- Compute the long-term SF

The time-independence and the absence of load paths facilitate the interpolation in the time-independent durability cases.

7 – A few G_{sc} values: The numerical computation of the G_{sc} values from experimental regression equations is discussed in the next chapter. The sample values in tables 1 and 2 are from the next chapter.

<i>Rupture G_{sc} in the fiber direction</i>						
<i>Load path 20 Hertz</i>						
<i>Lifetime (seconds)</i>	<i>N</i>	<i>R</i>				
		<i>0.0</i>	<i>0.1</i>	<i>0.5</i>	<i>0.9</i>	<i>1.0</i>
<i>5x10</i>	<i>10³</i>	<i>0.0</i>	<i>8.70</i>	<i>683.23</i>	<i>NA</i>	<i>0.0</i>
<i>5x10²</i>	<i>10⁴</i>	<i>0.0</i>	<i>8.76</i>	<i>348.48</i>	<i>NA</i>	<i>0.0</i>
<i>5x10³</i>	<i>10⁵</i>	<i>0.0</i>	<i>8.84</i>	<i>177.41</i>	<i>NA</i>	<i>0.0</i>
<i>5x10⁴</i>	<i>10⁶</i>	<i>0.0</i>	<i>8.91</i>	<i>90.44</i>	<i>NA</i>	<i>0.0</i>
<i>5x10⁵</i>	<i>10⁷</i>	<i>0.0</i>	<i>9.00</i>	<i>45.40</i>	<i>NA</i>	<i>0.0</i>
<i>5x10⁶</i>	<i>10⁸</i>	<i>0.0</i>	<i>9.07</i>	<i>22.78</i>	<i>NA</i>	<i>0.0</i>
<i>5x10⁷</i>	<i>10⁹</i>	<i>0.0</i>	<i>9.15</i>	<i>11.16</i>	<i>NA</i>	<i>0.0</i>
<i>5x10⁸</i>	<i>10¹⁰</i>	<i>0.0</i>	<i>9.22</i>	<i>6.08</i>	<i>NA</i>	<i>0.0</i>

50 years	3.17×10^{10}	0.0	9.26	3.70	NA	0.0

Table 1

Rupture G_{sc} for tensile strains in the fiber direction and a load path of 20 Hz.

<i>G_{sc} of transverse loaded vinyl ester UD plies</i> <i>The time variable and the loading frequency are not relevant</i>						
<i>Lifetime (Seconds)</i>	<i>N</i>	<i>R</i>				
		<i>0.0</i>	<i>0.1</i>	<i>0.5</i>	<i>0.9</i>	<i>1.0</i>
----	10^3	0.0	16.89	53 100	NA	0.0
----	10^4	0.0	17.67	28 160	NA	0.0
----	10^5	0.0	18.47	13 800	NA	0.0
----	10^6	0.0	19.23	7 950	NA	0.0
----	10^7	0.0	20.02	4 450	NA	0.0
----	10^8	0.0	20.81	2 250	NA	0.0
----	10^9	0.0	21.57	1 275	NA	0.0
----	10^{10}	0.0	22.35	670	NA	0.0
----	3.17×10^{10}	0.0	22.76	465	NA	0.0

Table 2

G_{sc} for transverse loaded vinyl ester UD plies.

Chapter 16

Part 3

Numerical computations of G_{sc}

1 – Introduction: This part 3 of chapter 16 deals with the numerical computation of the interaction parameter G_{sc} . The computation protocol consists in feeding the unified equation with known long-term failure points, setting the $SF = 1$, and then solving for G_{sc} . The failure points come from experimental regression equations measured at several R ratios and loading frequencies. There are seven experimental ply equations available at this time.

- *One pure static ($R = 1.0$) rupture line for UD plies loaded in the fiber direction.*
- *Two cyclic rupture equations ($R = 0.1$ and $R = 0.5$) for UD plies loaded in the fiber direction at 20 Hz.*
- *Two cyclic infiltration equations ($R = 0.1$ and $R = 0.5$) for transverse loaded UD plies at 20 Hz.*
- *One pure static ($R = 1.0$) rupture equation for chopped fibers.*
- *One cyclic ($R = 0.1$) rupture equation for chopped fibers.*

The pure cyclic equations ($R = 0.0$) are not available due to technical difficulties. In their absence, we consider as pure the equations derived at $R = 0.1$.

There is a specific regression equation for every ply, every R ratio and every loading frequency. The cost and effort to measure these equations are considerable. However, as explained in chapter 13, the cyclic regression lines converge on the fatigue point defined by $T_o = 0.05\%$ and the number of cycles N_o . Knowledge of the ply short-term failure thresholds (infiltration, weep, stiffness and rupture) and the long-term fatigue point (T_o and N_o) allows the direct derivation of all regression equations.

For every R ratio, we need to measure just one regression line, usually the rupture line, from which we determine the number of cycles N_o . Knowledge of N_o allows the computation of all regression lines. The regression equations capture all variables related to load-dependent long-term ply failures. They include the peak strain, the time variable, the load path, the load ratio R , and number of cycles N . The G_{sc} values computed from these regression equations are exact.

In chapter 13 we list the pure tensile equations of UD and chopped plies for polyester and vinyl ester resins. The regression equations for R ratios other than R = 0.0 and R = 1.0 are very scarce. The only non-pure regression equations available at this time are those developed by Guangxu Wei for polyester UD plies at 20 Hz, R = 0.5 and R = 0.1. See reference 25.

$$\log \epsilon = \log (3.0) - 0.065 \times \log N \quad (\text{UD ply, fiber direction, rupture, } R = 0.5, 20 \text{ Hz})$$

$$\log \epsilon = \log (3.0) - 0.089 \times \log N \quad (\text{UD ply, fiber direction, rupture, } R = 0.1, 20 \text{ Hz})$$

$$\log \epsilon = \log (0.40) - 0.041 \times \log N \quad (\text{UD ply, transverse loaded, } R = 0.5, 20 \text{ Hz})$$

$$\log \epsilon = \log (0.40) - 0.045 \times \log N \quad (\text{UD ply, transverse loaded, } R = 0.1, 20 \text{ Hz})$$

These are the only non-pure ply equations available. The scarcity of such equations is a serious impediment to do the required interpolations and use the unified equation. A special effort to develop a comprehensive set of such equations, covering several load ratios R and load-paths, is essential. The above four equations, developed for R = 0.10, R = 0.50 and 20 Hz, are clearly not enough to compute a comprehensive set of Gsc values.

2 – Computing the time-dependent Gsc values: The computation of the Gsc values is very simple. The time-dependent protocol uses the full version of the unified equation, setting SF = 1.0.

$$\left(\frac{R\epsilon}{S_s} \right)^{\frac{1}{G_s}} + \left(\frac{(1-R)\epsilon}{S_c} \right)^{\frac{1}{G_c}} + \left(\frac{R(1-R)\epsilon^2}{S_s \times S_c} \right)^{\frac{1}{G_{sc}}} = 1.00$$

Solving for Gsc, we have

$$G_{sc} = \frac{\log \left[\frac{R(1-R) \times \epsilon^2}{S_s \times S_c} \right]}{\log \left[1 - \left(\frac{R \times \epsilon}{S_s} \right)^{1/G_s} - \left(\frac{(1-R) \times \epsilon}{S_c} \right)^{1/G_c} \right]}$$

The peak strain ϵ comes from the non-pure rupture equation. The cyclic Sc and static Ss strengths come from the pure rupture equations listed in chapter 13. The slopes Gs and Gc of the pure static and pure cyclic rupture equations are also available in chapter 13. The pure regression equations are available. The problem is the scarcity of non-pure equations.

Let us derive a few Gsc values for UD plies loaded in the fiber direction. The available regression lines are

$$\log \epsilon = 0.477 - 0.065 \times \log N \quad (\text{UD ply, fiber direction, rupture, } R = 0.5, 20 \text{ Hz})$$

$$\log \epsilon = 0.477 - 0.089 \times \log N \quad (\text{UD ply, fiber direction, rupture, } R = 0.1, 20 \text{ Hz})$$

We illustrate the computation of Gsc for N = 10⁹ cycles, load path 20Hz and load ratio R = 0.5. The test duration in hours, corresponding to N = 10⁹ cycles and a load path of 20 Hertz is

$$\text{hours} = \frac{10^9}{20 \times 60 \times 60} = 14000 \text{ hours}$$

The peak strain comes from the rupture regression equation for R = 0.50.

$$\log \epsilon = 0.477 - 0.065 \times \log N$$

In the above, ϵ is the peak strain in the fiber direction and N is the number of cycles to failure. The equation is valid for $R = 0.5$ and the load path of 20 Hz. The peak strain that fails the ply in $N = 10^9$ cycles is

$$\log \epsilon = 0.477 - 0.065 \times \log 10^9$$

$$\epsilon = 0.780\%$$

This is the peak strain that fails the ply in the fiber direction at $R = 0.5$, $N = 10^9$ cycles and $t = 14\,000$ hours. This peak strain comes from an experimental rupture equation and is accurate. Next, we compute the ply long-term pure static and pure cyclic strengths. The long-term pure static strength comes from the pure static equation listed in chapter 13 for UD plies loaded in the fiber direction.

$$\log \epsilon = 0.400 - 0.077 \log(\text{hours}) \quad (\text{Pure static})$$

The time variable corresponding to $N = 10^9$ cycles is 14 000 hours. The long-term static strength is

$$\log S_s = 0.400 - 0.077 \log(14000)$$

$$S_s = 1.204\%$$

The long-term pure cyclic strength comes from the pure cyclic equation of UD plies loaded in the fiber direction. This equation is in chapter 13.

$$\log \epsilon = 0.477 - 0.089 \times \log N \quad (\text{Pure cyclic})$$

$$\log S_c = 0.477 - 0.089 \times \log(10^9)$$

$$S_c = 0.474\%$$

Entering the above in the equation to compute G_{sc} , we have

$$G_{sc} = \frac{\log \left[\frac{R(1-R) \times \epsilon^2}{S_s \times S_c} \right]}{\log \left[1 - \left(\frac{R \times \epsilon}{S_s} \right)^{1/G_s} - \left(\frac{(1-R) \times \epsilon}{S_c} \right)^{1/G_c} \right]}$$

$$G_{sc} = \frac{\log \left[\frac{0.5(1-0.5) \times 0.780^2}{1.204 \times 0.474} \right]}{\log \left[1 - \left(\frac{0.5 \times 0.780}{1.204} \right)^{1/0.077} - \left(\frac{(1-0.5) \times 0.780}{0.474} \right)^{1/0.089} \right]}$$

From which we obtain $G_{sc} = 11.16$ for $N = 10^9$ cycles and a loading frequency of 20Hz.

The same procedure leads to other values of N and t along the load path of 20 Hz and $R = 0.5$. The results are in table 1. The G_{sc} values computed in this way are exact and produce the same results as the regression equations that originated them.

N	Seconds	$\epsilon(\%)$	R	$S_s(\%)$	$S_c(\%)$	G_{sc}

10^3	5×10	1.914	0.5	3.492	1.622	683.23
10^4	5×10^2	1.648	0.5	2.924	1.321	348.48
10^5	5×10^3	1.419	0.5	2.449	1.076	177.41
10^6	5×10^4	1.222	0.5	2.051	0.877	90.44
10^7	5×10^5	1.052	0.5	1.718	0.714	45.40
10^8	5×10^6	0.906	0.5	1.439	0.582	22.78
10^9	5×10^7	0.780	0.5	1.205	0.474	11.16
10^{10}	5×10^8	0.671	0.5	1.009	0.386	6.08
3.17×10^{10}	50 years	0.623	0.5	0.924	0.349	3.70

Table 1 – Rupture Gsc values for UD plies loaded in the fiber direction at 20 Hertz and $R = 0.5$. The rupture Gsc values in the fiber direction are resin-independent.

The Gsc values in the fiber direction for $R = 0.1$ and 20 Hz are computed in same way. The regression equation to use in this case is

$$\log \epsilon = 0.477 - 0.089 \times \log N \quad (\text{UD ply, fiber direction, rupture, } R = 0.1, 20 \text{ Hz})$$

The load ratio is $R = 0.1$ and the load path is 20 Hz. The results are in table 2.

N	Seconds	$C(\%)$	R	$Ss(\%)$	$Sc(\%)$	Gsc
10^3	5×10	1.622	0.1	3.492	1.622	8.70
10^4	5×10^2	1.321	0.1	2.924	1.321	8.76
10^5	5×10^3	1.076	0.1	2.449	1.076	8.84
10^6	5×10^4	0.877	0.1	2.051	0.877	8.91
10^7	5×10^5	0.714	0.1	1.718	0.714	9.00
10^8	5×10^6	0.582	0.1	1.439	0.582	9.07
10^9	5×10^7	0.474	0.1	1.205	0.474	9.15
10^{10}	5×10^8	0.386	0.1	1.009	0.386	9.22
3.17×10^{10}	50 years	0.349	0.1	0.924	0.349	9.26

Table 2

Rupture Gsc values for UD plies loaded in the fiber direction at 20 Hertz and $R = 0.1$. The peak strain ϵ and the long-term cyclic strength S_C come from the same regression equation and have equal values. This

is because we have assumed the pure equation ($R = 0.0$) equal to the equation for $R = 0.1$. Such an approximation is required in the absence of the true regression equation for $R = 0.0$. The above G_{sc} values are fiber-dominated and resin-independent.

3 – Computing the time-independent G_{sc} : The weep, infiltration and stiffness failures of chopped and transverse loaded UD plies are resin-dominated and time-independent. The G_{sc} values in these cases derive from the simplified version of the unified equation.

$$\left(\frac{(1-R) \times \epsilon \times SF}{S_c} \right)^{\frac{1}{G_c}} + \left(\frac{R(1-R) \times \epsilon^2 \times SF^2}{S_s \times S_c} \right)^{\frac{1}{G_{sc}}} = 1.00$$

Setting $SF = 1.0$ and solving for G_{sc} , we have

$$G_{sc} = \frac{\log \left[\frac{R(1-R) \times \epsilon^2}{S_s \times S_c} \right]}{\log \left[1 - \left(\frac{(1-R) \times \epsilon}{S_c} \right)^{1/G_c} \right]}$$

The long-term static strengths, S_s , in time-independent failures is set equal to the corresponding short-term failure threshold. For example, the long-term static strengths for weep and infiltration failures are

$S_s = T_i = 0.40\%$ (Infiltration and weep thresholds, tough polyester, UD plies)

$S_s = T_i = 0.50\%$ (Infiltration and weep thresholds, vinyl ester, UD plies)

$S_s = T_i = 0.40\%$ (Infiltration threshold, tough polyester, chopped fibers)

$S_s = T_i = 0.50\%$ (Infiltration threshold, vinyl ester, chopped plies)

$S_s = T_w = 0.80\%$ (Weep threshold, tough polyester, chopped plies)

$S_s = T_w = 1.10\%$ (Weep threshold, vinyl ester, chopped plies)

The resin-dominance removes the time variable from the simplified unified equation, making the stiffness, weep and infiltration failures fully cycle-dependent. The cycle-dependent interaction parameters G_{sc} are a function of N alone. As explained elsewhere, this simplification allows the use of the Goodman diagrams to estimate the ply stiffness, weep and infiltration durability on an equal standing with the unified equation. The reader will understand that the Goodman diagrams coincide with the unified equation only in the solution of time-independent cases involving infiltration, weep and stiffness failures. They differ from the unified equation in the solution of time-dependent long-term ruptures associated with fiber strain-corrosion.

Example 1 – Compute the time-independent weep G_{sc} value for transverse loaded tough polyester UD plies at $N = 10^9$ cycles and $R = 0.5$. This is a time-independent situation in which the load path (loading frequency) is not relevant.

The experimental weep line of transverse loaded tough polyester UD plies at $R = 0.5$ is

$$\log \epsilon = -0.399 - 0.041 \times \log N$$

The peak strain that weeps the ply at $R = 0.5$ and $N = 10^9$ is

$$\log \epsilon = -0.399 - 0.041 \times \log 10^9$$

Table 3

Weep G_{sc} values for transverse loaded tough polyester UD plies at $R = 0.5$. The time variable and the loading frequency are not relevant. The computed G_{sc} values are surprisingly high, indicating a large interaction at the load ratio $R = 0.5$.

Table 4 lists the weep G_{sc} values for transverse loaded polyester UD plies at $R = 0.1$. In deriving these values, we have used the weep regression equation for $R = 0.1$ as a substitute for the pure equation. This is so, because the “true” pure line at $R = 0.0$ is not available.

The pure cyclic equation for transverse loaded tough polyester UD plies is in chapter 9.

$$\log \epsilon = \log(0.40) - 0.051 \times \log N$$

The long-term pure cyclic weep strengths S_c and the peak strains ϵ come from this equation. The pure static transverse strength is $S_s = T_w = 0.40\%$. The load ratio is $R = 0.1$. The pure slope for the weep line is $G_c = 0.051$. To compute the G_{sc} values at $R = 0.1$ we simply enter these inputs into the simplified version of the unified equation.

$$G_{sc} = \frac{\log \left[\frac{R(1-R) \times \epsilon^2}{S_s \times S_c} \right]}{\log \left[1 - \left(\frac{(1-R) \times \epsilon}{S_c} \right)^{1/G_c} \right]}$$

N	Seconds	ϵ	R	S_s	S_c	G_{sc}
10^3	----	0.281	0.1	0.40	0.281	20.38
10^4	----	0.249	0.1	0.40	0.249	21.27
10^5		0.222	0.1	0.40	0.222	22.12
10^6	----	0.197	0.1	0.40	0.197	23.00
10^7	----	0.175	0.1	0.40	0.175	23.87
10^8	----	0.156	0.1	0.40	0.156	24.72
10^9	----	0.139	0.1	0.40	0.139	25.57
10^{10}	----	0.123	0.1	0.40	0.123	26.47
3.17×10^{10}	----	0.116	0.1	0.40	0.116	26.91
The experimental equation is: $\log(\epsilon) = -0.399 - 0.051 \times \log(N)$ The pure cyclic equation is: $\log(\epsilon) = -0.399 - 0.051 \times \log(N)$						

Table 4

Weep Gsc values for transverse loaded tough polyester UD plies at $R = 0.1$. The time variable and the loading frequency are not relevant. The peak strain ϵ and the long-term cyclic strength S_c come from the same regression equation.

Tables 5 and 6 list the weep Gsc values for transverse loaded vinyl ester UD plies at $R = 0.1$ and $R = 0.5$. The weep regression equation for transverse loaded vinyl ester UD plies at $R = 0.5$ is, as explained in chapter 13.

$$\log(\epsilon) = \log(T_w) - \frac{\log(T_w/T_0)}{\log N_0} \times \log(N)$$

Where $T_w = 0.50\%$ and $T_0 = 0.05\%$. The value of N_0 depends on the R ratio. From table 6, in chapter 13, for $R = 0.5$ we have $N_0 = 10^{24}$.

$$\log(\epsilon) = \log(0.50) - \frac{\log(0.50/0.05)}{\log 10^{24}} \times \log(N)$$

$$\log(\epsilon) = -0.301 - 0.040 \times \log(N) \quad (\text{Vinyl ester, transverse loaded UD plies, } R = 0.5)$$

The reader should pause for a moment and ponder what we have just done. We have derived the cyclic regression equation for a vinyl ester resin from experimental measurements on polyester. This reflects the power of the fatigue threshold concept.

For technical reasons, we use the equation for $R = 0.1$ as a replacement to the pure regression equation.

$$\log \epsilon = \log(0.50) - 0.056 \log N \quad (\text{Vinyl ester, transverse loaded UD plies, } R = 0.1)$$

The regression equations compute the peak strains ϵ that weep the plies as a function of N , at $R = 0.5$ and $R = 0.1$. The long-term cyclic strength S_c come from the known pure equation. The long-term static strength for vinyl ester resins is $S_s = T_w = 0.50\%$. Furthermore, from the pure cyclic equation for vinyl esters, $G_c = 0.056$.

Example 2 – Compute the time-independent weep Gsc value for vinyl ester transverse loaded UD plies at $N = 10^9$ cycles and $R = 0.5$. This is a time-independent situation in which the load path (loading frequency) is not relevant.

The experimental weep line of transverse loaded vinyl ester UD plies at $R = 0.5$ is

$$\log(\epsilon) = -0.301 - 0.040 \times \log(N) \quad (\text{Vinyl ester, transverse loaded, } R = 0.5)$$

The peak strain that weeps the ply at $R = 0.5$ and $N = 10^9$ is

$$\log \epsilon = -0.301 - 0.040 \times \log 10^9$$

$$\epsilon = 0.218\%$$

The long-term pure static strength of transverse loaded polyester UD plies is equal to the weep threshold

$$S_s = T_w = 0.50\%$$

The long-term pure cyclic strength of transverse loaded tough polyester UD plies comes from taking $N = 10^9$ in the pure regression equation listed in chapter 13.

$$\log \epsilon = \log(0.50) - 0.056 \times \log N$$

$$\log S_c = -0.301 - 0.056 \times \log 10^9$$

$$S_c = 0.157\%$$

Feeding the above in the simplified equation, we have

$$G_{sc} = \frac{\log \left[\frac{R(1-R) \times \epsilon^2}{S_s \times S_c} \right]}{\log \left[1 - \left(\frac{(1-R) \times \epsilon}{S_c} \right)^{1/G_c} \right]}$$

$$G_{sc} = \frac{\log \left[\frac{0.5(1-0.5) \times 0.218^2}{0.50 \times 0.157} \right]}{\log \left[1 - \left(\frac{(1-0.5) \times 0.218}{0.157} \right)^{1/0.056} \right]} = 1\,275$$

Where $G_c = 0.056$ is the slope of the pure cyclic weep line for transverse loaded vinyl ester UD plies.

N	Seconds	ϵ	R	S_s	S_c	Gsc
10^3	----	0.379	0.5	0.50	0.340	53 100
10^4	----	0.346	0.5	0.50	0.299	28 160
10^5		0.316	0.5	0.50	0.262	13 800
10^6	----	0.288	0.5	0.50	0.231	7 950
10^7	----	0.262	0.5	0.50	0.203	4 450
10^8	----	0.239	0.5	0.50	0.178	2 250
10^9	----	0.218	0.5	0.50	0.157	1 275
10^{10}	----	0.199	0.5	0.50	0.138	670
3.17×10^{10}	----	0.190	0.5	0.50	0.129	465
The experimental equation is: $\log(\epsilon) = -0.301 - 0.040 \times \log(N)$ The pure cyclic equation is: $\log(\epsilon) = -0.301 - 0.056 \times \log(N)$						

Table 5

Weep Gsc values for transverse loaded vinyl ester UD plies at $R = 0.5$. The time variable and the loading frequency are not relevant.

N	Seconds	ϵ	R	S_s	S_c	Gsc
10^3	----	0.340	0.1	0.50	0.340	16.89
10^4	----	0.299	0.1	0.50	0.299	17.67

10^5		0.262	0.1	0.50	0.262	18.47
10^6	----	0.231	0.1	0.50	0.231	19.23
10^7	----	0.203	0.1	0.50	0.203	20.02
10^8	----	0.178	0.1	0.50	0.178	20.81
10^9	----	0.157	0.1	0.50	0.157	21.57
10^{10}	----	0.138	0.1	0.50	0.138	22.35
3.17×10^{10}	----	0.129	0.1	0.50	0.129	22.76
The experimental equation is: $\log(\epsilon) = -0.301 - 0.056 \times \log(N)$ The pure cyclic equation is: $\log(\epsilon) = -0.301 - 0.056 \times \log(N)$						

Table 6

Weep Gsc values for transverse loaded vinyl ester UD plies at $R = 0.1$. The time variable and the loading frequency are not relevant. The peak strain ϵ and the long-term cyclic strength Sc come from the same regression equation.

Tables 7, 8 and 9 show the previously computed Gsc values for vinyl ester and polyester UD plies at several load ratios. The values at $R = 1.0$ and $R = 0.0$ are both equal to zero, as there is no interaction in pure loadings. In the neighborhood of $R = 0.5$ the parameter Gsc increases substantially for low N and short time durations, reflecting a very strong interaction. Such an increased interaction suggests the existence of a singularity. To avoid this singularity the unified equation is not applicable to low values of N and short life-times. This limitation holds for the classical regression equations as well, as explained in the appendix 2 of chapter 13.

Rupture Gsc in the fiber direction						
Load-path 20 Hertz						
Lifetime (seconds)	N	R				
		0.0	0.1	0.5	0.9	1.0
5×10	10^3	0.0	8.70	683.23	NA	0.0
5×10^2	10^4	0.0	8.76	348.48	NA	0.0
5×10^3	10^5	0.0	8.84	177.41	NA	0.0
5×10^4	10^6	0.0	8.91	90.44	NA	0.0
5×10^5	10^7	0.0	9.00	45.40	NA	0.0
5×10^6	10^8	0.0	9.07	22.78	NA	0.0

5×10^7	10^9	0.0	9.15	11.16	NA	0.0
5×10^8	10^{10}	0.0	9.22	6.08	NA	0.0
50 years	3.17×10^{10}	0.0	9.26	3.70	NA	0.0

Table 7

Rupture G_{sc} for tensile strains in the fiber direction and a load path of 20Hz. These values are resin-independent.

Gsc of transverse loaded tough polyester UD plies at several R ratios						
The time variable and the loading frequency are not relevant						
Lifetime (Seconds)	N	R				
		0.0	0.1	0.5	0.9	1.0
----	10^3	0.0	20.38	332 400	NA	0.0
----	10^4	0.0	21.27	204 300	NA	0.0
----	10^5	0.0	22.12	147 000	NA	0.0
----	10^6	0.0	23.00	98 400	NA	0.0
----	10^7	0.0	23.87	61 600	NA	0.0
----	10^8	0.0	24.72	44 900	NA	0.0
----	10^9	0.0	25.57	27 900	NA	0.0
----	10^{10}	0.0	26.47	18 000	NA	0.0
----	3.17×10^{10}	0.0	26.91	14 400	NA	0.0

Table 8

G_{sc} for transverse loaded tough polyester UD plies at several R ratios.

Gsc of transverse loaded vinyl ester UD plies at several R ratios						
The time variable and the loading frequency are not relevant						
Lifetime (Seconds)	N	R				
		0.0	0.1	0.5	0.9	1.0
----	10^3	0.0	16.89	53 100	NA	0.0
----	10^4	0.0	17.67	28 160	NA	0.0

----	10^5	0.0	18.47	13 800	NA	0.0
----	10^6	0.0	19.23	7 950	NA	0.0
----	10^7	0.0	20.02	4 450	NA	0.0
----	10^8	0.0	20.81	2 250	NA	0.0
----	10^9	0.0	21.57	1 275	NA	0.0
----	10^{10}	0.0	22.35	670	NA	0.0
----	3.17×10^{10}	0.0	22.76	465	NA	0.0

Table 9

G_{sc} values of transverse loaded vinyl ester UD plies at several R ratios. These G_{sc} values enter the unified equation to compute the weep durability of oil pipes.

Winding angle	$\alpha = \pm 55$ degrees
Static hoop strain component	$\epsilon_y = 0.20\%$
Static axial strain component	$\epsilon_x = 0.10\%$
Cyclic hoop strain component	$\Delta\epsilon_y = 0.10\%$
Cyclic axial strain component	$\Delta\epsilon_x = 0.05\%$
Expected number of cycles in 20 years	$Y = 10^8$
Target lifetime	$X = 20$ years

Table 10

Operating conditions for the oil pipes discussed in the numerical example 4.

Example 4 – Suppose an oil pipeline operating as indicated in table 10. It is required to determine the safety factor SF against rupture failure for the lifetime $X = 20$ years.

First, we note that all pipe strains are in the global frame.

Second, we add the static and cyclic strain components to obtain the peak global hoop and axial strains.

$$\epsilon_y = 0.20 + 0.10 = 0.30\% \quad (\text{Peak hoop})$$

$$\epsilon_x = 0.10 + 0.05 = 0.15\% \quad (\text{Peak axial})$$

The minimum hoop and axial strains are equal to the static components

$$\epsilon_y = 0.20\% \quad (\text{Minimum hoop})$$

$$\epsilon_x = 0.10\% \quad (\text{Minimum axial})$$

Next, we use equation $\epsilon_1 = \epsilon_x \cos^2 \alpha + \epsilon_y \sin^2 \alpha$ to rotate the global strains to the fiber direction.

The maximum and the minimum strains in the fiber direction are

$$\epsilon_1^{\max} = 0.15 \times \cos^2 55 + 0.30 \times \sin^2 55 = 0.25\%$$

$$\epsilon_1^{\min} = 0.10 \times \cos^2 55 + 0.20 \times \sin^2 55 = 0.17\%$$

The R ratio in the fiber direction is

$$R = \frac{\epsilon_{\min}}{\epsilon_{\max}} = \frac{0.17}{0.25} = 0.68$$

The long-term pure static strength of boron-free glass in 20 years is

$$\log(S_s) = 0.400 - 0.077 \log(20 \times 365 \times 24)$$

$$S_s = 1.20\%$$

The long-term pure cyclic strength of boron-free glass in $Y = 10^8$ cycles is

$$\log(S_c) = 0.519 - 0.089 \log 10^8$$

$$S_c = 0.64\%$$

Next, we compute the loading frequency

$$\text{frequency} = \frac{10^8}{20 \times 365 \times 24 \times 60 \times 60} = 0.15 \text{ cycles/sec}$$

The interaction parameter G_{sc} for the load ratio $R = 0.68$ and the load path of 0.15 Hz in the fiber direction is not known. For illustration purposes we assume $G_{sc} = 10$. The parameters G_s and G_c come from the pure static and the pure cyclic equations. For a pipeline of boron-free glass the safety factor SF after 20 years is

$$\left(\frac{0.68 \times 0.25 \times SF}{1.20} \right)^{\frac{1}{0.077}} + \left(\frac{(1 - 0.68) \times 0.25 \times SF}{0.64} \right)^{\frac{1}{0.089}} + \left(\frac{0.68 \times (1 - 0.68) \times 0.25^2 \times SF^2}{1.20 \times 0.64} \right)^{\frac{1}{10}} = 1.00$$

$$SF = 5.0$$

The unified equation predicts excellent performance against long-term rupture.

The short-term safety factor against rupture is

$$SF = \frac{3.0\%}{0.25\%} = 12$$

Example 5 – Compute the 20 years weep safety factor for the pipeline of example 4. The computation protocol is similar to the one in example 4, except that now we deal with transverse strains. The maximum global hoop and axial strains are the same as in example 4.

$$\epsilon_y = 0.20 + 0.10 = 0.30\% \quad (\text{Peak hoop})$$

$$\epsilon_x = 0.10 + 0.05 = 0.15\% \quad (\text{Peak axial})$$

The minimum global strains are, as before

$$\epsilon_y = 0.20\% \quad (\text{Minimum hoop})$$

$$\epsilon_x = 0.10\% \quad (\text{Minimum axial})$$

Next, we use the equation $\epsilon_2 = \epsilon_x \sin^2 \alpha + \epsilon_y \cos^2 \alpha$ to compute the transverse strain in the local frame. The maximum and the minimum transverse strains are

$$\epsilon_2^{\max} = 0.15 \times \sin^2 55 + 0.30 \times \cos^2 55 = 0.20\%$$

$$\epsilon_2^{\min} = 0.10 \times \sin^2 55 + 0.20 \times \cos^2 55 = 0.13\%$$

The R ratio for the transverse loading is

$$R = \frac{\epsilon_{\min}}{\epsilon_{\max}} = \frac{0.13}{0.20} = 0.65$$

The long-term pure static strength in the transverse direction is set equal to the weep threshold. Assuming a tough polyester resin, we have

$$T_w = 0.40\%$$

The long-term pure cyclic weep strength of transverse loaded polyester UD plies at $Y = 10^8$ cycles is

$$\log S_C = -0.398 - 0.051 \log 10^8$$

$$S_C = 0.16\%$$

The loading frequency is not relevant in time-independent weep failures. The transverse interaction parameters G_{sc} for $R = 0.65$ and $Y = 10^8$ cycles is not known. For illustration purposes, we assume $G_{sc} = 1.000$.

Entering the above in the simplified form of the unified equation, we have

$$\left(\frac{(1 - 0.65) \times 0.20 \times SF}{0.16} \right)^{\frac{1}{0.051}} + \left(\frac{0.65 \times (1 - 0.65) \times 0.20^2 \times SF^2}{0.40 \times 0.16} \right)^{\frac{1}{1000}} = 1.00$$

$$SF = 1.8$$

The unified equation predicts a reasonable weep performance in 20 years of continuous service.

The weep short-term safety factor is

$$SF = \frac{0.40\%}{0.20\%} = 2.0$$

Example 6 – Continuing with the analysis of the previous pipe, let us assume it has a weep barrier of vinyl ester chopped ply. The mode of failure is weep. The informed lifetime parameters are $X = 20$ years and $Y = 10^8$ cycles. The computation protocol is the same as in examples 4 and 5, except the critical ply is the weep barrier of chopped fibers.

The maximum and minimum hoop strains are

$$\epsilon_y = 0.20 + 0.10 = 0.30\% \quad (\text{Peak hoop})$$

$$\epsilon_y = 0.20\% \quad (\text{Minimum hoop})$$

The corrosion barrier is isotropic and needs no strain rotation. The R ratio for the weep barrier is

$$R = \frac{\epsilon_{min}}{\epsilon_{max}} = \frac{0.20}{0.30} = 0.66$$

The long-term weep pure static strength is set equal to the weep threshold. Assuming vinyl ester resin we have

$$S_s = T_w = 1.10\%$$

The weep pure regression equation of vinyl ester chopped plies is in chapter 9

$$\log \epsilon = \log(1.10) - 0.068 \times \log N$$

The long-term weep pure cyclic strength of the chopped ply is

$$\log S_c = \log(1.10) - 0.068 \times \log 10^8$$

$$S_c = 0.31\%$$

The loading frequency is not relevant in weep failures. The interaction parameters G_{sc} for vinyl ester chopped plies at the loading ratio $R = 0.66$ and $Y = 10^8$ is not known. For illustration purposes we assume $G_{sc} = 5\,000$.

Entering the above into the simplified form of the unified equation, we have

$$\left(\frac{(1 - 0.66) \times 0.30 \times SF}{0.31} \right)^{1/0.068} + \left(\frac{0.66 \times (1 - 0.66) \times 0.30^2 \times SF^2}{0.31 \times 1.10} \right)^{1/5000} = 1.0$$

$$SF = 2.2$$

The unified equation predicts good weep performance for the pipeline with a weep barrier of vinyl ester chopped ply after 20 years of continuous service. The long-term safety factor for the pipes without a weep barrier, from the previous example, is $SF = 1.8$.

The weep short-term safety factor for the vinyl ester chopped ply is

$$SF = \frac{1.10\%}{0.30\%} = 3.7$$

The above results do not reflect the true behavior of the pipe. This is not because the unified equation is wrong, but because the assumed interaction parameters G_{sc} are certainly very inaccurate. Not until the interaction parameters G_{sc} are accurately measured can the unified equation give reliable results. As explained in chapters 13 and in the part 2 of this chapter, the accurate determination of G_{sc} requires the development of many regression equations at several frequencies and load ratios R .

Furthermore, the above analysis based on plies are good to compute the onset of weeping, not the actual pipe weeping. For a computation of the actual pipe weeping, we need the pipe weep equations.

Example 7 – Estimate the minimum number of regression equations needed to develop reliable values for the G_{sc} parameter.

Just how many regression equations are required will depend on the desired accuracy. Perhaps, as a minimum, the following suffices:

- The fiber loaded UD plies need a minimum of 5 load paths (loading frequency) for each load ratio R . Taking 5 load ratios, we need $5 \times 5 = 25$ equations.
- The transverse loaded UD plies are time-independent. Taking 5 load ratios, gives $1 \times 5 = 5$ equations.
- The chopped fiber plies fail in weep or infiltration, which are time-independent. Taking 5 load ratios, gives $1 \times 5 = 5$ equations.

The required number of experimental regression equations is not very large. The measurement of just 35 equations would allow the accurate solution of any load-dependent durability problem for any laminate and loading.

Example 8 – Explain the conditions of applicability for the unified equation and the experimental regression equations.

The experimental regression equations contain all information needed to compute the ply long-term load-dependent failure. When available, the regression equations are the way to go. However, they are never available. The unified equation has general applicability and finds use in all situations. The unified equation is a major step forward in the analysis of load-dependent durability of composites.

Example 9 – Explain the condition of applicability for the Goodman diagrams.

The Goodman diagrams are as good as the unified equation to compute the easy cases of time-independent durability. Both methods give excellent results in such cases. However, unlike the unified equation, the Goodman diagrams do not capture the time variable and cannot solve cases involving fiber-dominated failures. The computation of load-dominated time-dependent durability requires the unified equation.

Chapter 16

Part 4

Long-term SF

Upper and lower bounds

1 – Introduction – The enormous effort to determine the experimental equations required to accurately interpolate for the values of G_{sc} will probably never be undertaken. As a result, the unavailability of the interaction parameter G_{sc} prevents the use of the unified equation. We need an approximate solution that eliminate the need for G_{sc} .

This solution is the topic of this chapter. It consists in establishing the upper and lower bounds for the long-term safety factor SF, regardless of the interaction parameter G_{sc} . The equipment is safe to operate if its long-term safety factor is less than the computed lower bound. The upper bound, although sometimes computed, is not of great interest in the study of the load-dependent durability of composites equipment.

To simplify the presentation, the solution developed in this chapter will focus on tensile strains. The next chapter will deal with shear strain failures.

The development of the upper and lower bounds for SF start from the full version of the time-dependent unified equation.

$$\left(\frac{R \times \epsilon \times SF}{S_s}\right)^{\frac{1}{G_s}} + \left(\frac{(1-R) \times \epsilon \times SF}{S_c}\right)^{\frac{1}{G_c}} + \left(\frac{R \times (1-R) \times \epsilon^2 \times SF^2}{S_s \times S_c}\right)^{\frac{1}{G_{sc}}} = 1.00$$

Later we will be addressing the particular features of the upper and lower bounds of the time-independent version of the unified equation

$$\left(\frac{(1-R) \times \epsilon \times SF}{S_c}\right)^{\frac{1}{G_c}} + \left(\frac{R \times (1-R) \times \epsilon^2 \times SF^2}{S_s \times S_c}\right)^{\frac{1}{G_{sc}}} = 1.00$$

2 – The SF lower bound – The two possible lowest values for the long-term safety factor SF are those measured in pure cyclic and pure static loadings. A close examination of figure 1 will clarify this statement for the pure cyclic case.

Figure 1 shows that pure cyclic loadings ($R = 0.0$) are more damaging than any other R value. Therefore, the cyclic lower bound of the safety factor SF is obtained by setting $R = 0.0$ in the unified equation (1).

Entering $R = 0.0$ in the complete unified equation, we have

$$\left[\frac{\epsilon \times SF}{S_c} \right]^{\frac{1}{G_c}} = 1.00$$

Which gives

$$SF = \frac{S_c}{\epsilon}$$

Where

SF is the safety factor at $R = 0.0$.

S_c is the long-term cyclic strength of the critical ply

ϵ is the peak strain.

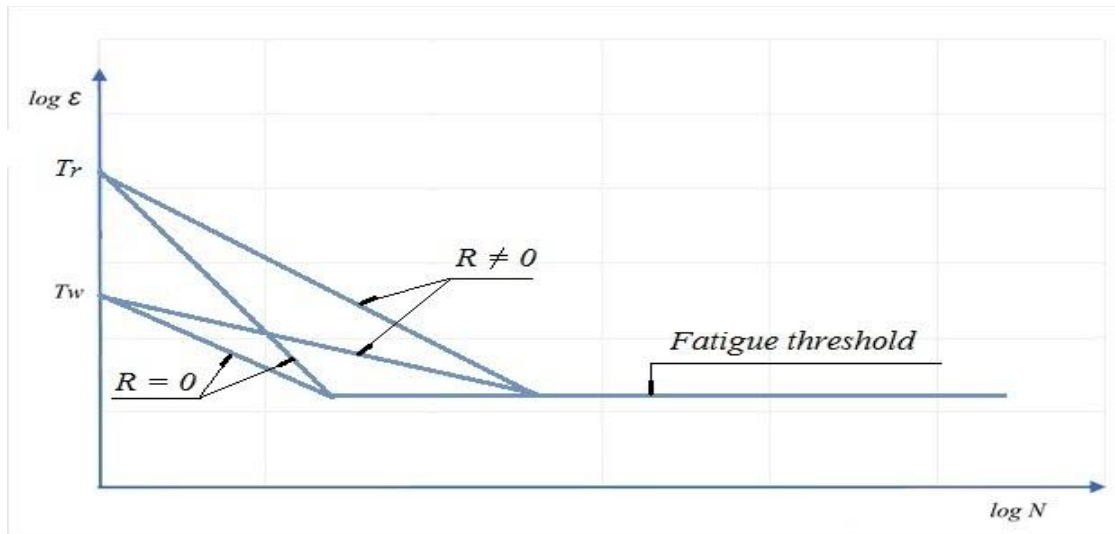


Figure 1

This is the same figure 5 of chapter 13. The slopes of the cyclic regression lines decrease with increasing R ratios. The pure cyclic loadings ($R = 0.0$) are the most damaging, producing the least values for the long-term safety factor SF.

A similar reasoning for pure static loads leads to the static-dominated lower limit for SF. Setting $R = 1.0$ in the unified equation, we have

$$\left[\frac{\epsilon \times SF}{S_s} \right]^{\frac{1}{G_s}} = 1.00$$

Which gives

$$SF = \frac{S_s}{\epsilon}$$

Where

SF is the safety factor at $R = 1.0$.

S_s is the long-term static strength of the critical ply

ϵ is the peak strain.

We have just computed two values for the lower limits of the long-term safety factor SF , one for fully cyclic ($R = 0.0$) another for fully static ($R = 1.0$) loadings. The combined lower bound for the safety factor SF varies with the load ratio R along a straight line connecting these two limits. See figure 2.

$$SF = \frac{R \times S_s}{\epsilon} + \frac{(1 - R) \times S_c}{\epsilon}$$

This equation gives the lower bound of the long-term safety factor SF for any load ratio R . No long-term safety factor SF can be less than this lower bound, regardless of the interaction parameter G_{sc} .

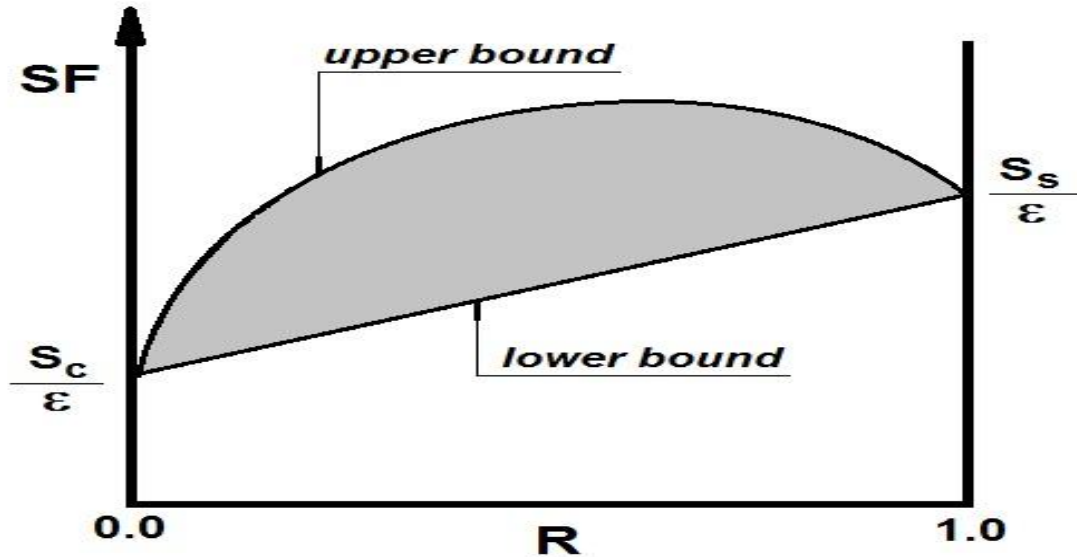


Figure 2

The upper and lower bounds for the long-term safety factor SF vary with the load ratio R . The reader will note the upper and lower bounds coinciding at $R = 0.0$ and $R = 1.0$.

3 – The SF upper bound – To derive the highest possible value for the long-term safety factor SF we simply drop the interaction term in the unified equation. The elimination of the interaction damages certainly increase the long-term SF.

$$\left[\frac{R \times \epsilon \times SF}{S_s} \right]^{\frac{1}{G_s}} + \left[\frac{(1 - R) \times \epsilon \times SF}{S_c} \right]^{\frac{1}{G_c}} = 1.00$$

This equation gives the highest possible value for the long-term safety factor SF as a function of the load ratio R. The derivation assumed absence of all interaction between the static and cyclic load components. The highest possible value of SF will occur somewhere near R = 0.5. As shown in figure 2, the upper bounds have the same values as the lower bounds at R = 0.0 and R = 1.0.

The last statement can be easily demonstrated computing the upper bounds at R = 1.0 and R = 0.0. The computed values are the same as those for the lower bounds.

These considerations are correct and capable to compute the lower and upper bounds of SF for the time-dependent unified equation. However, as shown later, they do not work for the upper bound of the time-independent unified equation.

We cannot compute the upper bound SF for time-independent failures, such as infiltration and weeping.

4. The upper and lower bounds for SF – The foregoing discussion produced an upper and a lower bound for the long-term SF of time-dependent failures. Unfortunately, for time-independent failures, we have only the lower bound. This is true regardless of the interaction parameter Gsc.

To compute the upper and lower bounds for the long-term SF, we use the same inputs required in the unified equation. These inputs, as explained in the part 1 of chapter 16, derive from two known pure regression equations valid for the critical ply.

The pure cyclic equation gives the long-term cyclic strength (Sc) and the slope Gc

The pure static equation gives the long-term static strength (Ss) and the slope Gs

These are the four inputs required to compute the upper and lower bounds according to the theory supported by the unified equation.

The time-independent modes of failure do not have static regression lines to compute the long-term strength Ss and the slope Gs. The absence of these parameters preclude the computation of the upper bound. That is the reason why time-independent failures, such as infiltration and weep, do not have upper bounds.

5. Time-independent modes of failure. To compute the upper and lower bounds for cycle-dependent failures, we substitute the short-term *failure threshold* for the long-term static strength Ss. That is the meaning of Ss, anyway. This substitution implies in constant

values for the long-term static strength (S_s). As we know, the failure thresholds are ply properties with constant values independent of the time variable.

Therefore, for time-independent failures

$$S_s = \text{failure threshold.}$$

Likewise, in the absence of time deterioration, we have

$$G_s = 0.0$$

Incidentally, these are the same assumptions introduced in the part 1 of chapter 16 to derive the time-independent version of the unified equation. These assumptions are correct, working well to compute not only the actual SF, but also its lower bound. What they cannot do is compute the upper bound. The impossibility to compute the upper bound derives from the assumptions of constant $S_s = \text{failure threshold}$ and $D_s = 0.0$.

These assumptions invalidate the computation of the upper SF, as shown by entering them in the equations to compute the lower and upper bounds.

5.1. For the lower bound

$$SF = \frac{R \times S_s}{\epsilon} + \frac{(1 - R) \times S_c}{\epsilon}$$

$$SF = \frac{R \times (\text{failure threshold})}{\epsilon} + \frac{(1 - R) \times S_c}{\epsilon}$$

The short-term failure thresholds for the two time-independent modes of failure are

$$\text{Failure threshold} = T_i \text{ (infiltration failure)}$$

$$\text{Failure threshold} = T_w \text{ (weep failure)}$$

$$\text{Failure threshold} = T_w \text{ (weep failure of pipes. We decided to include the study of pipes)}$$

The equation to compute the time-independent lower bound SF is the same used in time-dependent cases, with $S_s = \text{failure threshold}$.

5.2. For the upper bound

Taking $G_s = 0.0$ in the equation for the upper bound, we have

$$\left[\frac{R \times \epsilon \times SF}{S_s} \right]^{\frac{1}{G_s}} + \left[\frac{(1 - R) \times \epsilon \times SF}{S_c} \right]^{\frac{1}{G_c}} = 1.00$$

$$\left[\frac{(1 - R) \times \epsilon \times SF}{S_c} \right]^{\frac{1}{G_c}} = 1.00$$

$$SF = \frac{S_c}{(1 - R) \times \epsilon}$$

This equation has two problems. First, it gives infinite SF for $R = 1.0$. Besides, it does not capture the long-term static strength. This is not correct. A simple inspection of figure 2 shows the upper bound taking values varying between the long-term static (S_s) and the long-term cyclic (S_c) strengths. The fact that $G_s = 0.0$ and $S_s = \text{constant}$ destroys the capability of the unified equation to compute the upper bound for time-independent modes of failure.

This failure, however, is of no practical consequence, given the little importance of the upper bound. The practicing engineers make their decisions based on the lower bound, not the upper bound.

With these provisos in mind, we conclude for the impossibility to compute the upper bounds for time-independent failures

Table 1 lists the following equations to compute the lower and upper bounds for the long-term safety factor SF.

	<i>Time-dependent failures</i>	<i>Time-independent failures</i>
<i>Upper bound</i>	$\left[\frac{R \times \epsilon \times SF}{S_s} \right]^{\frac{1}{G_s}} + \left[\frac{(1-R) \times \epsilon \times SF}{S_c} \right]^{\frac{1}{G_c}} = 1.00$	<i>There is no equation to compute the upper bound</i>
<i>Lower bound</i>	$SF = \frac{R \times S_s}{\epsilon} + \frac{(1-R) \times S_c}{\epsilon}$	$SF = \frac{R \times (\text{failure threshold})}{\epsilon} + \frac{(1-R) \times S_c}{\epsilon}$

Table 1

The upper and lower bounds of the long-term safety factor SF

Example 1 – Compute the upper and lower bounds for the long-term safety factors SF for the pipes in the numerical examples 4 and 5, discussed in the previous chapter. As no mention is made of shear strains, we assume tensile-dominated failures.

In the numerical example 4 of the previous chapter, we discussed a time-dependent problem with the following data.

$S_s = 1.20\%$, $S_c = 0.64\%$, $R = 0.68$ and $\epsilon = 0.25\%$.

The lower bound for the long-term SF is

$$SF = \frac{R \times S_s}{\epsilon} + \frac{(1-R) \times S_c}{\epsilon}$$

$$SF = \frac{0.68 \times 1.20}{0.25} + \frac{(1-0.68) \times 0.64}{0.25} = 4.08$$

The upper bound for SF is

$$\left[\frac{R \times \epsilon \times SF}{S_s} \right]^{\frac{1}{G_s}} + \left[\frac{(1 - R) \times \epsilon \times SF}{S_c} \right]^{\frac{1}{G_c}} = 1.00$$

$$\left[\frac{0.68 \times 0.25 \times SF}{1.20} \right]^{\frac{1}{0.077}} + \left[\frac{(1 - 0.68) \times 0.25 \times SF}{0.64} \right]^{\frac{1}{0.089}} = 1.00$$

Which gives

$$SF = 6.50$$

The interval for the long-term safety factor SF is

$$[4.08 < SF < 6.50]$$

The long-term SF falls in the range $[4.08 - 6.50]$. This is the best we can do, given the current state of our knowledge. The determination of the true SF requires knowledge of the G_{sc} values, which are not available at this time. The value $SF = 5.0$, computed in example 4 of the previous chapter, assumed $G_{sc} = 10$.

In the numerical **example 5 of the previous chapter**, we have a cycle-dependent weep problem with the following data

$$S_s = 0.40\%, S_c = 0.16\%, R = 0.65 \text{ and } \epsilon = 0.20\%.$$

The lower bound for the long-term SF is

$$SF = \frac{R \times S_s}{\epsilon} + \frac{(1 - R) \times S_c}{\epsilon}$$

$$SF = \frac{0.65 \times 0.40}{0.20} + \frac{(1 - 0.65) \times 0.16}{0.20} = 1.58$$

This is a time-independent problem for which we cannot compute the upper bound.

The lower bound $SF > 1.58$ is the best we can do at this time. The value $SF = 1.8$ computed in example 5 of the previous chapter assumed $G_{sc} = 1\ 000$.

Example 2. This numerical example completes the solution of the pipeline durability problem discussed in the example 4 of chapter 7. As discussed there, the consulting firm computed the pipe rated pressure as $P = 4.3$ bar, slightly less than the peak pressure $P = 5$ bar. Based on this, and the desired durability of 50 years, the consulting firm ruled out the pipes out as unfit for service. We proceed now to present our solution to this problem.

The following data is available:

- The critical ply is the sand-filled core
- The applied pressure oscillates in the range 3 – 5 bar
- The pipe diameter is $D = 1000$ mm
- The durability established by the owner is $X = 50$ years
- The number of cycles in 50 years is $Y = 1\ 000\ 000$ cycles
- The threshold failure of the critical sand-filled core is $Tw = 0.30\%$

From chapter 7, the equation to compute the total hoop strain in the sand core is

$$\epsilon_y = 3.77 \times 10^{-6} \times P \times D - 0.0002 + 0.0001$$

The peak hoop strain in the sand core is

$$\epsilon_{\max} = \epsilon = 3.77 \times 10^{-6} \times 5 \times 100 - 0.0002 + 0.0001 = 0.0020 \text{ (0.20\%)}$$

The minimum hoop strain in the sand core is

$$\epsilon_{\min} = 3.77 \times 10^{-6} \times 3 \times 100 - 0.0002 + 0.0001 = 0.0010 \text{ (0.10\%)}$$

The R ratio is

$$R = \frac{0.10}{0.20} = 0.50$$

To compute the load-dependent durability of the pipeline, we need the pure static and the pure cyclic regression equations for the sand-filled core, the critical ply in this case.

We assume the grains of sand immune to strain-corrosion in the presence of water. This very reasonable assumption results in a flat static equation in which $G_s = 0.0$. The time-independent long-term static resistance of the sand core ply is

$$S_s = T_w = 0.30\%.$$

The pure cyclic regression equation for the sand core is, as discussed in chapter 9

$$\log \epsilon = \log(T_w) - \frac{\log(T_w/T_0)}{\log(N_0)} \times \log N$$

$$\log \epsilon = \log(0.30) - \frac{\log(0.30/0.05)}{\log(10^{20})} \times \log N$$

$$\log \epsilon = \log(0.30) - 0.039 \times \log N$$

From this equation, the long-term cyclic strength of the core ply is

$$\log S_c = \log(0.30) - 0.039 \times \log(1\,000\,000)$$

$$S_c = 0.17\%$$

We now have all necessary information to solve this time-independent durability problem using the unified equation.

The lower bound for the long-term safety factor SF as a function of the load ratio R is

$$SF = \frac{R \times S_s}{\epsilon} + \frac{(1 - R) \times S_c}{\epsilon}$$

$$SF = \frac{0.50 \times 0.30}{0.20} + \frac{(1 - 0.50) \times 0.17}{0.20} = 1.2$$

The upper bound is not available.

Therefore, we have for the lower bound

$$SF > 1.2$$

The short-term safety factor is $SF = 0.30/0.20 = 1.5$.

A simple analysis using the sand core as the critical ply indicates the pipeline adequate for the intended application. The pipe weep equations would give still better results.

Appendix 1

The brittle pipe paradox

Let us assume two identical UD55 pipes, made of different resins, one a tough vinyl ester and the other a brittle polyester. This appendix will evaluate the durability of both under varying cycles to failure and R ratios. Table 2 summarizes the relevant data for the two pipes.

<i>Pipe parameters to compute the weep durability. All properties reported in the global frame, assuming 2:1 loading.</i>		
<i>Pipe property</i>	<i>Tough Derakane 411</i>	<i>Brittle polyester</i>
<i>Construction</i>	<i>UD55 oil pipe</i>	<i>UD55 oil pipe</i>
<i>Mechanical properties</i>	<i>Same</i>	<i>Same</i>
<i>Diameter and thickness</i>	<i>Same</i>	<i>Same</i>
<i>Weep barrier</i>	<i>No</i>	<i>No</i>
<i>Weep strength of pipe in the global frame</i>	$A_w = 1.8\%$	$A_w = 2.5\%$
<i>Weep threshold of the critical ply in the global frame</i>	$T_w = 1.10\%$	$T_w = 0.40\%$
<i>Fatigue threshold in the global frame</i>	$T_o = 0.10\%$	$T_o = 0.10\%$

Table 2

The short-term weep strength of pipes, A_w , is the lowest hoop strain that causes detectable leakages in short-term pressure tests. By contrast, the weep threshold, T_w , is the lowest hoop strain that starts the cracks coalescence that initiates the weeping process. T_w measures the onset of crack coalescence, not the

actual weeping. All hoop strains in the global frame of UD55 pipes under 2:1 loadings are twice as large as the local transverse strains.

Table 2 shows the brittle pipe weeping at a short-term hoop strain $A_w = 2.5\%$, a value 40% higher than the $A_w = 1.8\%$ for the tough pipe. This paradoxical result indicates the brittle pipe sustaining a 40% higher short-term weep pressure than the tough pipe. This is not expected. All expectations would point to the opposite result. The explanation for this paradox lies in the way that crack densities grow and coalesce into pathways for the passage of water.

The tougher resins certainly fail later, forming fewer long cracks and smaller crack densities than brittle ones. The fewer long cracks can easily coalesce into pathways for the passage of water. By contrast, the higher densities of smaller cracks in brittle pipes develop many short pathways terminating mostly in dead ends that retard the passage of water. The extremely high crack densities present in brittle pipes require higher pressures to weep. That is a short version of the explanation offered in chapter 9 to justify this paradox. For details, see chapter 9.

However, despite this elegant explanation, short-term weep tests indicate tougher pipes failing sooner than brittle ones. The question we ask is how would the two pipes behave in the long-term under a variety of loadings? To answer this question, we develop the upper and lower values for the long-term SF of pipes in weep.

As discussed in the part 2 of chapter 13, the long-term cyclic weep strength of pipes (S_c) is

$$\log(S_c) = \log(A_w) - \frac{\log(A_w/T_0)}{20} \times \log(Y)$$

Where Y is the user-informed number of cycles to failure. As usual in the study of pipes, all above strains are in the global frame.

From table 1, the pipe weep equations to compute the lower bound for the long-term safety factor SF is

$$SF = \frac{R \times T_w}{\epsilon} + \frac{(1 - R) \times S_c}{\epsilon}$$

Example 3. *Compute the durability of the brittle and tough UD55 pipes described in table 2 assuming the following operating conditions*

$Y = 1\,000\,000$ is the expected number of cycles

$\epsilon = 0.50\%$ is the peak hoop strain

$R = 0.0$ (pure cyclic loading)

First, we compute the long-term cyclic strength of the pipes.

$$\log(S_c) = \log(A_w) - \frac{\log(A_w/T_0)}{20} \times \log(Y)$$

For the tough pipe, we have

$$\log(S_c) = \log(1.8) - \frac{\log(1.8/0.1)}{20} \times \log(1\,000\,000)$$

$$S_c = 0.76\%$$

For the brittle pipe, we have

$$\log(S_c) = \log(2.5) - \frac{\log(2.5/0.1)}{20} \times \log(1\,000\,000)$$

$$S_c = 0.95\%$$

Next, we compute the lower bounds for $R = 0.0$.

$$SF = \frac{R \times T_w}{\epsilon} + \frac{(1 - R) \times S_c}{\epsilon}$$

For the tough pipe

$$SF = \frac{S_c}{\epsilon} = \frac{0.76}{0.50} = 1.52$$

For the brittle pipe

$$SF = \frac{0.95}{0.50} = 1.90$$

Example 4. Compute the durability of brittle and tough UD55 pipes described in table 2 assuming the following operating conditions

$Y = 10\,000\,000$ is the expected number of cycles

$\epsilon = 0.50\%$ is the peak hoop strain

$R = 0.0$ (pure cyclic loading)

This situation is the same as in example 3, except the number of cycles grows from 1 million to 10 million.

First, we compute the long-term cyclic strength of the pipes.

$$\log(S_c) = \log(A_w) - \frac{\log(A_w/T_0)}{20} \times \log(Y)$$

For the tough pipe, we have

$$\log(S_c) = \log(1.8) - \frac{\log(1.8/0.1)}{20} \times \log(10\,000\,000)$$

$$S_c = 0.66\%$$

For the brittle pipe, we have

$$\log(S_c) = \log(2.5) - \frac{\log(2.5/0.1)}{20} \times \log(10\,000\,000)$$

$$S_c = 0.81\%$$

Next, we compute the lower bounds for $R = 0.0$

$$SF = \frac{R \times T_w}{\epsilon} + \frac{(1 - R) \times S_c}{\epsilon}$$

For the tough pipe

$$SF = \frac{S_c}{\epsilon} = \frac{0.66}{0.50} = 1.32$$

For the brittle pipe

$$SF = \frac{0.81}{0.50} = 1.62$$

Example 5. Compute the durability of brittle and tough UD55 pipes described in table under 2:1 pure cyclic loadings assuming the following conditions

$$R = 0.5$$

$$Y = 10\,000\,000 \text{ cycles}$$

$$\epsilon = 0.50\% \text{ is the peak hoop strain}$$

The long-term cyclic weep strengths computed at $Y = 10\,000\,000$ cycles are

$$S_c = 0.66\% \text{ for the tough pipe}$$

$$S_c = 0.81\% \text{ for brittle pipe}$$

The long-term static weep strengths are in table 2

$$T_w = 1.10\% \text{ for the tough pipe}$$

$$T_w = 0.40\% \text{ for brittle pipe.}$$

The lower bounds are

$$SF = \frac{R \times T_w}{\epsilon} + \frac{(1 - R) \times S_c}{\epsilon}$$

For the tough pipe

$$SF = \frac{0.5 \times 1.1}{0.50} + \frac{(1 - 0.50) \times 0.66}{0.50} = 1.76$$

For the brittle pipe

$$SF = \frac{0.5 \times 0.40}{0.50} + \frac{(1 - 0.50) \times 0.81}{0.50} = 1.21$$

Example 6. Compute the durability of brittle and tough UD55 pipes described in table 2 assuming pure static loadings. The following conditions apply

$$R = 1.0$$

$$Y = 0$$

$\epsilon = 0.50\%$ is the peak hoop strain

The lower bounds are

$$SF = \frac{R \times T_w}{\epsilon} + \frac{(1 - R) \times S_c}{\epsilon} = \frac{T_w}{\epsilon}$$

$$SF = \frac{T_w}{\epsilon}$$

For the tough pipe, we have

$$SF = \frac{1.10}{0.50} = 2.20$$

For the brittle pipe, we have

$$SF = \frac{0.40}{0.50} = 0.80$$

The weep threshold T_w controls pipe failures under static loads. The high value $T_w = 1.10\%$ for the tough pipe explains its healthy safety factor $SF = 2.20$. Likewise, the low $T_w = 0.40\%$ indicates failure of the brittle pipe at $SF = 0.80$. The static failure of the brittle pipe is not immediate, since the weeping water may take a long time to travel the pipe thickness.

Table 4 summarizes the above results

Lower bounds for the long-term SF of pipes under 2:1 loadings in weep failure			
Operating conditions	Brittle pipe	Tough pipe	Comments
$Y = 10^9$ $\epsilon = 0.50\%$ $R = 0.0$	$SF > 1.90$	$SF > 1.52$	The brittle pipe is better at 1 000 000 cycles in pure cyclic loads
$Y = 10^{10}$ $\epsilon = 0.50\%$ $R = 0.0$	$SF > 1.62$	$SF > 1.32$	The brittle pipe is better at 10 000 000 cycles in pure cyclic load
$Y = 10^{10}$ $\epsilon = 0.50\%$ $R = 0.5$	$SF > 1.21$	$SF > 1.76$	The tough pipe is better at $R = 0.5$ and 10 000 000 cycles
$Y = 10^{10}$ $\epsilon = 0.50\%$ $R = 1.0$	$SF > 0.80$	$SF > 2.20$	The tough pipe is significantly better in pure static loads.

The overall conclusion is....

The brittle pipe is better at low R ratios, especially pure cyclic loadings. This superior performance decreases at low Y values.

The tough pipe is better at high R ratios, especially pure static loadings. This superior performance increases at low Y values.

Table 4.

Comparison of the weep failure of brittle and tough pipes

Chapter 16

Part 5

The complete unified equation

Introduction. Under cyclic loadings, UD55 oil pipes may exhibit weep failures from shear strains. Such unusual failures require special loadings, mostly those involving high hoop or high axial forces. The usual 2:1 loadings are of no concern. The unified equation discussed earlier will need modification to include damages from simultaneous shear and transverse strains.

Since these are time-independent weep failures, the damage from transverse tensile strains is

$$\left[\text{Damage from transverse strain} \right] = \left[\frac{(1-R) \times \epsilon}{S_c} \right]^{\frac{1}{G_c}} + \left[\frac{R \times (1-R) \times \epsilon^2}{S_s \times S_c} \right]^{\frac{1}{G_{sc}}}$$

Likewise, the damage from the shear strain component is

$$\left[\text{Damage from shear strain} \right] = \left[\frac{(1-R) \times \gamma}{\gamma_c} \right]^{\frac{1}{G_c}} + \left[\frac{R \times (1-R) \times \gamma^2}{\gamma_s \times \gamma_c} \right]^{\frac{1}{G_{sc}}}$$

The accumulated pipe damage is the sum of the above

$$\left[\text{Total accumulated damage} \right] = \left[\text{transverse tensile damage} \right] + \left[\text{shear damage} \right]$$

In this discussion, we assume the shear and transverse strains acting independently, without interaction. This is so as long as the crack densities are low enough to assure no loss of stiffness. Given our interest in failures prior to loss of stiffness, we may safely assume absence of interaction.

The complete unified equation for weep failure of UD55 oil pipes is

$$\left[\frac{(1-R) \times \epsilon \times SF}{S_c} \right]^{\frac{1}{G_c}} + \left[\frac{R \times (1-R) \times \epsilon^2 \times SF^2}{S_s \times S_c} \right]^{\frac{1}{G_{sc}}} + \left[\frac{(1-R) \times \gamma \times SF}{\gamma_c} \right]^{\frac{1}{G_c}} + \left[\frac{R \times (1-R) \times \gamma^2 \times SF^2}{\gamma_s \times \gamma_c} \right]^{\frac{1}{G_{sc}}} = 1.0$$

At failure, the unified equation should include a long-term safety factor SF

2 – The lower bound SF. Before deriving the cycle-dependent shear equation, we emphasize four points.

- *Shear cracks develop along the UD fibers, just like those from transverse tensile loads.*
- *Shear cracks do not open by themselves. To weep, they require the presence of some transverse strains.*
- *The shear failure threshold of UD plies is assumed at $\tau = 2.0\%$ for all resins and modes of failure*
- *The fatigue threshold point for pure cyclic shear is $T_0 = 0.05\%$ and $N_0 = 10^{20}$.*

From the above, and taking into consideration the concepts put forth in the part 1 of chapter 13, the pure cyclic shear equation for UD55 plies is

$$\log(\gamma) = \log(\tau) - \frac{\log(\tau/T_0)}{\log(N_0)} \times \log N$$

$$\log(\gamma) = \log(2.0) - \frac{\log(2.0/0.05)}{\log(10^{20})} \times \log N$$

$$\log(\gamma) = 0.301 - 0.080 \times \log N$$

This cyclic equation applies to all four modes of failure, infiltration, weep, stiffness and weep. Our interest in this chapter is restricted to weep failure.

Table 1 lists the shear equation alongside the transverse weep equations for vinyl ester UD plies and pipes.

Type loading	Pure cyclic equations for vinyl ester UD plies	Static strengths	Cyclic strength at $Y = 10^8$ cycles
Shear failure (ply weep)	$\log(\gamma_c) = 0.301 - 0.080 \times \log N$	$\gamma_s = 2.00\%$	$\gamma_c = 0.46\%$
Transverse failure (ply weep)	$\log(S_c) = -0.260 - 0.052 \times \log N$	$S_s = 0.55\%$	$S_c = 0.21\%$
Transverse failure (pipe weep)	$\log(A_c) = \log(A_2) - \frac{\log(A_2/0.05)}{20} \times \log N$	A_c Depends on wall thickness	Depends on A_2

Table 1

Shows the pure cyclic regression equations and the long-term strengths of UD plies at $N = 10^8$ cycles. Also shown is the weep cyclic weep equation of pipes.

The combined lower bound comes from the following relation

$$\frac{1}{SF} = \left(\frac{1}{SF}\right)_{shear} + \left(\frac{1}{SF}\right)_{tensile}$$

$$\frac{1}{SF} = \frac{\epsilon}{R \times S_s + (1 - R) \times S_c} + \frac{\gamma}{R \times \gamma_s + (1 - R) \times \gamma_c}$$

Example 1. Derive the pure cyclic equations and the long-term strengths of vinyl ester UD pipes and plies.

The weep equation for transverse strain is

$$\log(\epsilon) = \log(T_w) - \frac{\log(T_w/T_0)}{\log(N_0)} \times \log N$$

$$\log(\epsilon) = \log(0.55) - \frac{\log(0.55/0.05)}{\log(10^{20})} \times \log N$$

$$\log(\epsilon) = -0.260 - 0.052 \times \log N$$

For a lifetime of $N = Y = 10^8$ cycles, the long-term cyclic strength is

$$S_C = -0.260 - 0.052 \times \log 10^8$$

$$S_C = 0.21\%$$

The long-term static strength for cycle-dependent weep failures is

$$S_S = T_w = 0.55\%$$

The weep equation for shear strain is

$$\log(\gamma) = \log(\tau) - \frac{\log(\tau/T_0)}{\log(N_0)} \times \log N$$

$$\log(\gamma) = \log(2.0) - \frac{\log(2.0/0.05)}{\log(10^{20})} \times \log N$$

$$\log(\gamma) = 0.301 - 0.080 \times \log N$$

This shear equation dominates the weep behavior of pipes loaded with high ratios of hoop to axial stresses.

For a lifetime of $N = Y = 10^8$ cycles, the long-term ply strengths are

$$\gamma_C = 0.301 - 0.080 \times \log 10^8$$

$$\gamma_C = 0.46\%$$

The long-term static strength for cycle-dependent weep failures is

$$S_S = \tau = 2.00\%$$

The pipe weep equation is

$$\log(\epsilon) = \log(A_2) - \frac{\log(A_2/T_0)}{\log(N_0)} \times \log N$$

The pipe weep threshold A_2 depends on the wall thickness

Figure 1 shows the pure cyclic lines for UD pipes and plies.

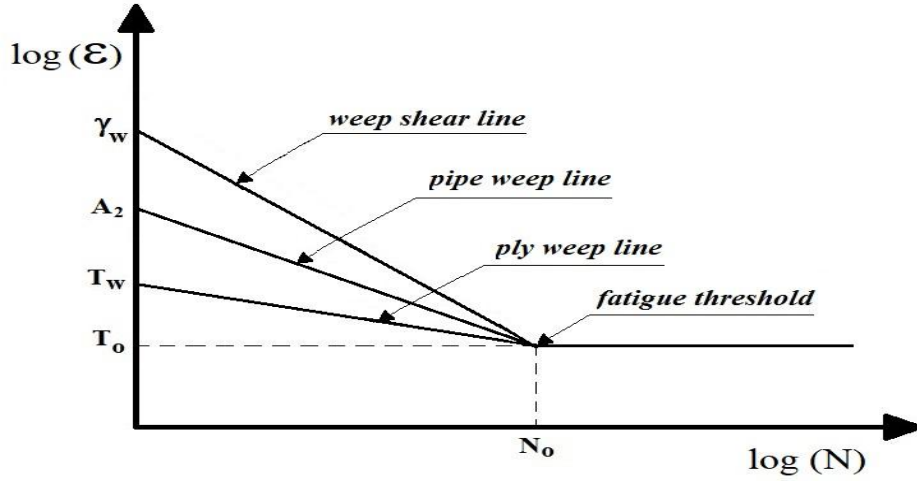


Figure 1

The weep shear lines usually plot above the pipe weep lines. All strains in the local system

Example 2. Let us use the complete unified equation to compute the wall thickness and the long-term SF for weep failure of UD55 oil pipes.

The following data is applicable:

- Resin matrix is vinyl ester Derakane 411
- Diameter $D = 200$ mm
- 2:1 loading
- Internal pressure $P = [30 \text{ to } 50]$ bar
- The owner-informed lifespan is $N = 10^8$ cycles in 50 years
- The operating temperature is $OT = RT = 30C$.
- The stress-free peak temperature is $PT = 100C$
- The water absorption by the resin matrix is $\Delta m = 0.01$
- The transverse weep threshold for the UD plies is $T_w = 0.55\%$
- The shear weep threshold for UD plies is $\tau = 2.0\%$

From chapter 5, the total strains on UD55 pipes under 2:1 pressure loading are

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 0.015 \\ 0.010 \\ \pm 0.007 \end{bmatrix} \times 10^{-4} \times \frac{PD}{t} + \begin{bmatrix} 0.262 \\ -0.104 \\ \pm 0.024 \end{bmatrix} \times 10^{-4} \times (OT - PT) + \begin{bmatrix} 800 \\ -700 \\ \mp 300 \end{bmatrix} \times 10^{-4} \times \Delta m$$

Entering the operating conditions and $P = 50$ bar, we have

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 0.015 \\ 0.010 \\ \pm 0.007 \end{bmatrix} \times 10^{-4} \times \frac{50 \times 200}{t} + \begin{bmatrix} 0.262 \\ -0.104 \\ \pm 0.024 \end{bmatrix} \times 10^{-4} \times (30 - 100) + \begin{bmatrix} 800 \\ -700 \\ \mp 300 \end{bmatrix} \times 10^{-4} \times 0.01$$

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} \frac{0.015}{t} \\ \frac{0.010}{t} \\ \pm \frac{0.007}{t} \end{bmatrix} + \begin{bmatrix} -0.002 \\ +0.001 \\ \pm 0.0002 \end{bmatrix} + \begin{bmatrix} +0.0008 \\ -0.0007 \\ \mp 0.0003 \end{bmatrix}$$

The minimum wall thickness for transverse failure is determined to keep ϵ_2 below half the weep threshold.

$$\epsilon_2 = \frac{0.010}{t} + 0.001 - 0.0007 = \frac{0.0055}{2}$$

From which we have

$$t = 4.1 \text{ mm}$$

The minimum wall thickness for shear failure is determined to keep the shear strain γ_{12} less than half the shear failure threshold $\tau = 2.0\%$.

$$\gamma_{12} = \frac{0.007}{t} + 0.0002 - 0.0003 = \frac{0.02}{2}$$

From which we have

$$t = 0.7 \text{ mm}$$

The minimum wall thickness is $t = 4.1 \text{ mm}$.

The combined time-independent unified equation is

$$\left[\frac{(1-R) \times \epsilon \times SF}{S_c} \right]^{\frac{1}{G_c}} + \left[\frac{R \times (1-R) \times \epsilon^2 \times SF^2}{S_s \times S_c} \right]^{\frac{1}{G_{sc}}} + \left[\frac{(1-R) \times \gamma \times SF}{\gamma_c} \right]^{\frac{1}{G_c}} + \left[\frac{R \times (1-R) \times \gamma^2 \times SF^2}{\gamma_s \times \gamma_c} \right]^{\frac{1}{G_{sc}}} = 1.0$$

The R ratios have slightly different values for transverse and shear strains. To simplify the computation, we assume the R ratio for both strains as

$$R = \frac{(\text{Minimum pressure})}{(\text{Maximum pressure})} = \frac{30}{50} = 0.60$$

Entering the data in the unified equation, we have

$$\left[\frac{(1-0.60) \times 0.28 \times SF}{0.21} \right]^{\frac{1}{0.052}} + \left[\frac{0.60 \times (1-0.60) \times 0.28^2 \times SF^2}{0.55 \times 0.21} \right]^{\frac{1}{G_{sc}}} + \left[\frac{(1-0.60) \times 0.16 \times SF}{0.46} \right]^{\frac{1}{0.080}} + \left[\frac{0.60 \times (1-0.60) \times 0.16^2 \times SF^2}{2.0 \times 0.46} \right]^{\frac{1}{G_{sc}}} = 1.0$$

The combined unified equation takes into account the damages from the shear and the transverse strains. We cannot use it to compute SF , since the interaction parameters G_{sc} are unknown.

The lower bound for the long-term safety factor is.

$$\frac{1}{SF} = \frac{\epsilon}{R \times S_s + (1-R) \times S_c} + \frac{\gamma}{R \times \gamma_s + (1-R) \times \gamma_c}$$

$$\frac{1}{SF} = \frac{0.28}{0.6 \times 0.55 + (1-0.6) \times 0.21} + \frac{0.16}{0.6 \times 2.0 + (1-0.6) \times 0.46}$$

From which we have $SF > 1.23$

Example 3. Compare the combined lower bound with the lower bound from transverse strains.

Taking $\gamma = 0$, we have

$$SF = \frac{R \times (\text{failure threshold})}{\epsilon} + \frac{(1 - R) \times S_c}{\epsilon}$$

$$SF = \frac{0.60 \times 0.55}{0.28} + \frac{(1 - 0.60) \times 0.21}{0.28} = 1.5$$

Ignoring the shear strains, we have $SF > 1.5$. This is close to the combined lower bound. The weep failure of UD55 pipes under 2:1 loading is dominated by the transverse strains.

Example 4. Compute the required wall thickness of the pipe in example 2, allowing for interaction between the transverse and shear strains

The required equations are, from chapter 14

$$\epsilon_2 = \frac{0.010}{t} + 0.001 - 0.0007 = \frac{T_w}{2} \times \sqrt{1 - \left(\frac{\gamma_{12}}{\gamma_w}\right)^2}$$

$$\gamma_{12} = \frac{0.007}{t} + 0.0002 - 0.0003 = \frac{\gamma_w}{2} \times \sqrt{1 - \left(\frac{\epsilon_2}{T_w}\right)^2}$$

Entering the known threshold values, we have

$$\frac{0.010}{t} + 0.001 - 0.0007 = \frac{0.0055}{2} \times \sqrt{1 - \left(\frac{\gamma_{12}}{0.02}\right)^2}$$

$$\frac{0.007}{t} + 0.0002 - 0.0003 = \frac{0.02}{2} \times \sqrt{1 - \left(\frac{\epsilon_2}{0.0055}\right)^2}$$

From which we take the larger of the two values.

Example 5. Compute the wall thickness and the lower bound SF for the UD55 pipes discussed in example 2, assuming 1:0 loading.

The operating conditions are

- Resin matrix is vinyl ester Derakane 411
- Diameter $D = 200$ mm
- 1:0 loading
- Internal pressure $P = 40 \pm 10$ bar
- The owner-informed lifespan is $N = 10^8$ cycles in 50 years
- The operating temperature is $OT = RT = 30C$.
- The stress-free peak temperature is $PT = 100C$
- The water pick-up of the resin matrix is $\Delta m = 0.01$
- The weep threshold of vinyl ester UD plies is $T_w = 0.55\%$
- The failure shear threshold of UD plies is $\tau = 2.0\%$

From chapter 5, example 7, the total strains on plies of UD55 pipes under 1:0 loadings are

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 0.0130 \\ -0.0047 \\ \pm 0.0492 \end{bmatrix} \times 10^{-4} \times \left(\frac{PD}{t} \right) + \begin{bmatrix} 0.262 \\ -0.104 \\ \pm 0.024 \end{bmatrix} \times 10^{-4} \times (OT - PT) + \begin{bmatrix} 800 \\ -700 \\ \mp 300 \end{bmatrix} \times 10^{-4} \times \Delta m$$

Entering the data with $P = 50$ bar, we have

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 0.0130 \\ -0.0047 \\ \pm 0.0492 \end{bmatrix} \times 10^{-4} \times \frac{50 \times 200}{t} + \begin{bmatrix} 0.262 \\ -0.104 \\ \pm 0.024 \end{bmatrix} \times 10^{-4} \times (30 - 100) + \begin{bmatrix} 800 \\ -700 \\ \mp 300 \end{bmatrix} \times 10^{-4} \times 0.01$$

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} \frac{0.0130}{t} \\ -\frac{0.0047}{t} \\ \pm \frac{0.0492}{t} \end{bmatrix} + \begin{bmatrix} -0.002 \\ +0.001 \\ \pm 0.0002 \end{bmatrix} + \begin{bmatrix} +0.0008 \\ -0.0007 \\ \mp 0.0003 \end{bmatrix}$$

This equation shows a negative (compressive) transverse strain that obviously cannot cause weep failures. UD55 pipes under 1:0 loadings never weep from transverse tensile strains.

The next possible cause of weep failure is shear strain. As a design criterion, we establish that the shear strain do not exceed half the shear threshold $\gamma_S = 2.0\%$.

$$\frac{0.0492}{t} + 0.0002 - 0.0003 = \frac{0.020}{2}$$

$$t = 4.9 \text{ mm}$$

The required pipe thickness under 1:0 loadings is determined by shear strains. Shear dominated weep failures may ignore the transverse strains. Therefore, we have

For the load ratio

$$R = \frac{\text{min. pressure}}{\text{max. pressure}} = \frac{30}{50} = 0.60$$

The lower bound SF is computed for shear failure, entering the peak strain $\gamma = 0.01$ and the load ratio $R = 0.60$ in the appropriate equation.

$$SF = \frac{R \times \gamma_S}{\gamma} + \frac{(1 - R) \times \gamma_C}{\gamma}$$

$$SF = \frac{0.6 \times 2.0}{1.0} + \frac{(1 - 0.6) \times 0.46}{1.0} = 1.38$$

Example 6. Compute the wall thickness and the lower bound SF for the UD55 pipes discussed in example 2, assuming 0:1 loading.

The following conditions apply.

- Resin matrix is vinyl ester Derakane 411
- Diameter $D = 200$ mm
- 0:1 loading
- The internal pressure $P = 40 \pm 10$ bar
- The owner-informed lifespan is $N = Y = 10^8$ cycles in 50 years
- The operating temperature is $OT = RT = 30^\circ\text{C}$.

- The stress-free peak temperature is $PT = 100C$
- The water pick-up of the resin matrix is $\Delta m = 0.01$
- The weep threshold of vinyl ester UD plies is $T_w = 0.55\%$
- The failure shear threshold of vinyl ester UD plies is $\tau = 2.0\%$

From chapter 5, example 8, the total strains on UD55 plies under 0:1 loadings are

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 0.0015 \\ 0.0144 \\ \mp 0.0356 \end{bmatrix} \times 10^{-4} \times \left(\frac{PD}{t} \right) + \begin{bmatrix} 0.262 \\ -0.104 \\ \pm 0.024 \end{bmatrix} \times 10^{-4} \times (OT - PT) + \begin{bmatrix} 800 \\ -700 \\ \mp 300 \end{bmatrix} \times 10^{-4} \times \Delta m$$

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 0.0015 \\ 0.0144 \\ \mp 0.0356 \end{bmatrix} \times 10^{-4} \times \frac{50 \times 200}{t} + \begin{bmatrix} 0.262 \\ -0.104 \\ \pm 0.024 \end{bmatrix} \times 10^{-4} \times (30 - 100) + \begin{bmatrix} 800 \\ -700 \\ \mp 300 \end{bmatrix} \times 10^{-4} \times 0.01$$

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} \frac{0.0015}{t} \\ \frac{0.0144}{t} \\ \mp \frac{0.0356}{t} \end{bmatrix} + \begin{bmatrix} -0.002 \\ +0.001 \\ \pm 0.0002 \end{bmatrix} + \begin{bmatrix} +0.0008 \\ -0.0007 \\ \mp 0.0003 \end{bmatrix}$$

The minimum thickness to accommodate the transverse tensile strain is

$$\frac{0.0144}{t} + 0.001 - 0.0007 = \frac{0.0055}{2}$$

From which we have

$$t = 5.8 \text{ mm}$$

We next compute the minimum thickness for shear strain. As a design criterion, we establish that the shear strain do not exceed half the shear failure threshold $\gamma_s = 2.0\%$.

$$\frac{0.0356}{t} - 0.0002 + 0.0003 = \frac{0.020}{2}$$

$$t = 3.6 \text{ mm}$$

The minimum pipe thickness for shear failure is $t = 3.6 \text{ mm}$.

The required pipe thickness should be $t = 5.8 \text{ mm}$. This thickness gives the following strains

$$\epsilon = \frac{0.0055}{2} = 0.003$$

$$\frac{0.0356}{5.8} - 0.0002 + 0.0003 = \gamma$$

$$\gamma = 0.0062 \text{ (0.62\%)}$$

The durability analysis for this loading requires the combined equation. The lower bound is

$$\frac{1}{SF} = \frac{\epsilon}{R \times S_s + (1 - R) \times S_c} + \frac{\gamma}{R \times \gamma_s + (1 - R) \times \gamma_c}$$

$$\frac{1}{SF} = \frac{0.28}{0.6 \times 0.55 + (1 - 0.6) \times 0.21} + \frac{0.62}{0.6 \times 2.0 + (1 - 0.6) \times 0.46}$$

From which we have $SF > 0.89$, indicating possible failure

The above results are in table 2

Weep failure of UD55 oil pipes under several loadings $P = 40 \pm 10$ bar $D = 200$ mm $N = Y = 100\ 000\ 000$ cycles				
Loading	Mode of failure	Dominating strain	Wall thickness	Lower bound SF
2:1	Weep	Transverse tensile	4.1 mm	$SF > 1.23$
0:1	Weep	Shear and transverse	5.8 mm	$SF > 0.89$
1:0	Weep	Shear	4.9 mm	$SF > 1.38$

Table 2

Lower bound SF for several loadings. The wall thickness for the 0:1 loading should be higher than 5.8 mm.

Example 7. This numerical example will illustrate the application of the complete unified equation to UD55 pipes under 2:1 loadings.

The following conditions apply

- Resin matrix is vinyl ester Derakane 411
- 2:1 loading
- The owner-informed lifespan is $N = 10^8$ cycles in 50 years
- The weep threshold of vinyl ester UD plies is $T_w = 0.55\%$
- The failure shear threshold of vinyl ester UD plies is $\tau = 2.0\%$
- The pipe weep threshold is $A_w = 2.5\%$
- The operating peak cyclic hoop strain in the pipe is 0.40%
- The R ratio is $R = 0.7$

We will do the analysis in the ply frame. The peak operating strain and the pipe weep threshold in the ply frame are

$$\epsilon = \frac{0.40}{2} = 0.20\%$$

$$A_2 = \frac{A_w}{2} = \frac{2.5}{2} = 1.25\%$$

The pipe weep equation in the local frame is

$$\log(\epsilon) = \log(1.25) - \frac{\log\left(\frac{1.25}{0.05}\right)}{20} \times \log(N)$$

$$\log(\epsilon) = \log(1.25) - 0.070 \times \log(N)$$

The shear weep equation in the local frame is

$$\log(\gamma) = \log(2.0) - \frac{\log\left(\frac{2.0}{0.05}\right)}{20} \times \log(N)$$

$$\log(\gamma) = \log(2.0) - 0.080 \times \log(N)$$

The pipe long-term transverse strength is

$$\log(S_c) = \log(1.25) - 0.070 \times \log(10^8)$$

$$S_c = 0.34\%$$

The pipe long-term shear strength is

$$\log(\gamma_c) = \log(2.0) - 0.080 \times \log(10^8)$$

$$\gamma_c = 0.45\%$$

From the equations in chapter 5, the peak shear strain at the operating pressure is

$$\gamma = 0.7 \times \epsilon = 0.7 \times 0.20 = 0.14\%$$

The lower bound for the combined long-term safety factor is.

$$\frac{1}{SF} = \frac{\epsilon}{R \times S_s + (1 - R) \times S_c} + \frac{\gamma}{R \times \gamma_s + (1 - R) \times \gamma_c}$$

$$\frac{1}{SF} = \frac{0.20}{0.7 \times 0.55 + (1 - 0.7) \times 0.34} + \frac{0.14}{0.7 \times 2.0 + (1 - 0.7) \times 0.45}$$

From which we have $SF > 2.00$

Example 7. *This numerical example will solve the above example 6 ignoring the shear strains. The lower bound SF is*

$$SF = R \times \frac{S_s}{\epsilon} + (1 - R) \times \frac{S_c}{\epsilon}$$

$$SF = 0.7 \times \frac{0.55}{0.20} + (1 - 0.7) \times \frac{0.34}{0.20} = 2.4$$

The lower bound SF, ignoring the shear strains, is 2.4. This is practically the same value 2.00 from the complete unified equation. The shear strains have a negligible effect on the durability of UD55 pipes at 2:1 loadings

4 – Conclusion. Depending on the loading, either the shear strains or the transverse strains governs the weep failure of UD55 pipes. More often than not, we will need only one of the two terms in the complete unified equation.

We have the following rules for the weep failure of UD55 pipes.

- *For stresses near 1:0 loadings, the shear strains dominate*

- *For stresses near 2:1 loadings, the transverse strains dominate*
- *For stresses near 0:1 loadings, the complete unified equation is required*

Chapter 17

Durability of API 15HR pipes

1 – Introduction: This chapter illustrates the application of the unified equation to justify the API 15HR long-term cyclic requirement for composite pipes. The requirement is....

“...no pipe failure in 20 years of continuous operation under nominal pressure and $N = 10^9$ cycles at a strain ratio $R = 0.9$ ”.

We address the problem from the perspectives of weep and rupture failures, the two meaningful modes of long-term failure in this case.

2 – Operating strains: First, we compute the strains at “nominal pressure”, assuming UD55 pipes of vinyl ester Derakane 411 under 2:1 loading. From the numerical example 1 in the part 2 of chapter 13, we have

$$\epsilon_y = 2.0 \times \epsilon_2$$

$$\epsilon_1 = 1.5 \times \epsilon_2$$

We define as “nominal” the pressure at the weep threshold divided by the short-term safety factor $SF = 2$. Therefore

$$[\text{nominal transverse strain}] = \epsilon_2 = \frac{T_w}{2} = \frac{0.55\%}{2} = 0.275\%$$

$$[\text{nominal hoop strain}] = \epsilon_y = 2.0 \times 0.275\% = 0.55\%$$

$$[\text{nominal fiber strain}] = \epsilon_1 = 1.5 \times 0.275\% = 0.41\%$$

These are the “nominal” mechanical strains on UD55 oil pipes under the requirements of API 15HR. To simplify the discussion, the residual thermal and hydric strains were not included in the analysis. Furthermore, we have ignored the effect of shear strains. A more accurate analysis should include these effects.

3 – Long-term rupture from fiber failure: We proceed to check the long-term burst failure of the pipe.

The fiber rupture durability from the unified equation is

$$\left(\frac{R \times \varepsilon \times SF}{S_s}\right)^{\frac{1}{G_s}} + \left(\frac{(1-R) \times \varepsilon \times SF}{S_c}\right)^{\frac{1}{G_c}} + \left(\frac{R \times (1-R) \times \varepsilon^2 \times SF^2}{S_s \times S_c}\right)^{\frac{1}{G_{sc}}} = 1.00$$

Where

$\varepsilon = 0.41\%$ is the peak strain in the fiber direction

$R = 0.9$ is the strain ratio

S_s = is the pure long-term (20 years) static strength in the fiber direction

S_c = is the pure long-term (10^9 cycles) cyclic strength in the fiber direction

$G_s = 0.077$ is the pure static slope for boron-free glass

$G_s = 0.130$ is the pure static slope for E glass

$G_c = 0.089$ is the pure cyclic slope, any glass

G_{sc} is the interaction parameter for the target lifetime $X = 20$ years and $Y = N = 10^9$ cycles.

The long-term pure static and pure cyclic strengths of UD plies loaded in the fiber direction come from the appropriate pure regression lines.

The 20 years pure static strength for boron-free glass is

$$\log(S_s) = 0.400 - 0.077 \log(20 \times 365 \times 24)$$

$$S_s = 0.99\% \quad (\text{Boron-free glass})$$

The 20 years pure static strength for E glass is

$$\log(S_s) = 0.347 - 0.130 \log(20 \times 365 \times 24)$$

$$S_s = 0.46\% \quad (\text{E glass})$$

The long-term pure cyclic strength of UD plies in the fiber direction for $N = 10^9$ cycles is

$$\log(S_c) = 0.477 - 0.089 \log(N)$$

$$\log(S_c) = 0.477 - 0.089 \log(10^9)$$

$$S_c = 0.474\% \quad (\text{Boron-free and E glass})$$

Now we come to the interaction parameter G_{sc} . The loading frequency prescribed by API 15HR is

$$\text{frequency} = \frac{10^9}{20 \times 365 \times 24 \times 60 \times 60} = 1.6 \text{ Hz}$$

The G_{sc} values for this load-path are not known. The only G_{sc} values available at this time for fiber-loaded UD plies come from the Guangxu Wei cyclic tests conducted at 20 Hz. Unfortunately, the G_{sc} value for the load path 1.6 Hz and $R = 0.9$ is not available.

However, for illustration purposes, we arbitrarily choose $G_{sc} = 9.0$. This is an arbitrary choice, for illustration purposes only.

Example 1 – Compute the long-term rupture durability of boron-free oil pipes under the stated API 15HR conditions. Time-dependent rupture failures require the full version of the unified equation.

The mode of failure is rupture, and the focus is on the fiber direction. The peak strain in the fiber direction is 0.41% under the rated operating conditions. We enter the data in the unified equation

$$\left(\frac{0.9 \times 0.41 \times SF}{0.99}\right)^{1/0.077} + \left(\frac{0.1 \times 0.41 \times SF}{0.474}\right)^{1/0.089} + \left(\frac{0.1 \times 0.9 \times 0.41^2 \times SF^2}{0.474 \times 0.99}\right)^{1/9.0} = 1.0$$

Solving for SF , we have

$$SF = 2.5$$

The unified equation predicts that oil pipes of boron-free glass have a long-term (20 years) safety factor $SF = 2.5$ against rupture. This SF assumed $G_{sc} = 9.0$.

Next, we compute the upper and lower bounds of the long-term safety factor, independent of the interaction parameter G_{sc} . The lower bound of SF is

$$SF = \frac{R \times S_s}{\epsilon} + \frac{(1 - R) \times S_c}{\epsilon}$$

$$SF = \frac{0.90 \times 0.99}{0.41} + \frac{(1 - 0.90) \times 0.474}{0.41} = 2.3$$

The upper bound is

$$\left[\frac{R \times \epsilon \times SF}{S_s}\right]^{\frac{1}{G_s}} + \left[\frac{(1 - R) \times \epsilon \times SF}{S_c}\right]^{\frac{1}{G_c}} = 1.00$$

$$\left[\frac{0.90 \times 0.41 \times SF}{0.99}\right]^{\frac{1}{0.077}} + \left[\frac{(1 - 0.90) \times 0.41 \times SF}{0.474}\right]^{\frac{1}{0.089}} = 1.00$$

Which gives

$$SF = 2.8$$

The interval for the long-term safety factor SF in rupture is

$$[2.3 < SF < 2.8]$$

The actual long-term SF falls in the range $[2.3 - 2.8]$. This is the best we can do at this time, given the current state of our knowledge. The computation of the true SF requires knowledge of G_{sc} values not available at this time. The $SF = 2.5$ computed previously arbitrarily assumed $G_{sc} = 9.0$.

Example 2 – We next solve the same long-term rupture problem for pipes of standard E glass.

$$\left(\frac{0.9 \times 0.41 \times SF}{0.46}\right)^{1/0.130} + \left(\frac{0.1 \times 0.41 \times SF}{0.474}\right)^{1/0.089} + \left(\frac{0.1 \times 0.9 \times 0.41^2 \times SF^2}{0.474 \times 0.46}\right)^{1/9.0} = 1.0$$

$$SF = 1.2$$

The pipes of regular E glass barely meet the API 15HR requirements for long-term rupture. The superior performance of boron-free glass ($SF = 2.5$) versus regular E glass ($SF = 1.2$) is clear. The long-term safety factors computed in this numerical example are illustrative, since we have assumed a fictitious value $G_{sc} = 9.0$ for the interaction parameter.

For E glass, the upper and lower bounds of SF are

The lower bound is

$$SF = \frac{R \times S_s}{\epsilon} + \frac{(1 - R) \times S_c}{\epsilon}$$

$$SF = \frac{0.90 \times 0.46}{0.41} + \frac{(1 - 0.90) \times 0.474}{0.41} = 1.2$$

The upper bound is

$$\left[\frac{R \times \epsilon \times SF}{S_s}\right]^{\frac{1}{G_s}} + \left[\frac{(1 - R) \times \epsilon \times SF}{S_c}\right]^{\frac{1}{G_c}} = 1.00$$

$$\left[\frac{0.90 \times 0.41 \times SF}{0.46}\right]^{\frac{1}{0.120}} + \left[\frac{(1 - 0.90) \times 0.41 \times SF}{0.474}\right]^{\frac{1}{0.089}} = 1.00$$

Which gives

$$SF = 1.3$$

The interval for the long-term safety factor in rupture is

$$[1.2 < SF < 1.3]$$

We next address the weep durability.

4 – Weep failure: Weep failures result from water leakage through cracked walls. The following facts are undisputed:

- Under static pressure, the pipe develops many small stationary cracks parallel to the UD fibers.
- Under cyclic pressure, the otherwise stationary cracks grow in number and in length along the UD fibers.
- Cracks grow only under cyclic loads. Static loads do not grow cracks.
- Under cyclic pressure, the initially small cracks increase in length and in density but not in opening. The constant pipe stiffness controls the crack openings, static or cyclic.

In the weeping process, the water moves along many narrow and long cracks running parallel to the fibers. The cracks in the innermost + UD55 ply cross with similar cracks in the – UD55 ply immediately above. The water migrates from the inner to the outer plies at these crossing points, eventually weeping out. The cracks are narrow, the path-

ways are tortuous, and the travel distances are long. The travel time from the innermost to the outermost ply may be very long. It is obvious that the weep time depends on the pipe's wall thickness.

For best accuracy, we use the pipe pure cyclic weep equation, as explained in the part 2 of chapter 13

$$\log(\epsilon) = \log(A_2) - \frac{\log(A_2/0.05)}{20} \times \log(N)$$

The above equation requires knowledge of the pipe weep threshold A_w , measured in the hoop direction. For consistency, we rotate A_w from the global to the ply frame, to obtain the value A_2 . Another approach, preferred by most engineers, express all strains in the global frame. This is a matter of preference.

The short-term weep strength A_w takes different values, depending on the pipe thickness. In the absence of the pipe parameter A_w , we will use the overly conservative ply pure cyclic weep equation

$$\log(\epsilon) = \log(T_w) - \frac{\log(T_w/0.05)}{20} \times \log(N)$$

The above ply equation predicts the onset of the weeping process, i.e., the onset of crack coalescence. To obtain the actual weep time, we should use the pipe equation.

Example 3 – Compute the short-term weep safety factor of vinyl ester and polyester oil pipes operating under the specified API 15HR conditions.

The mode of failure is weep, and the focus is on the transverse strains of the UD plies. The peak strain in the transverse direction, under the rated operating conditions, is 0.275%. The short-term safety factors are

For polyester pipes

$$\text{short-term safety factor} = \frac{T_w}{\epsilon_2} = \frac{0.45}{0.27} = 1.6$$

For vinyl ester pipes

$$\text{short-term safety factor} = \frac{T_w}{\epsilon_2} = \frac{0.55}{0.27} = 2.0$$

The peak strain does not exceed the weep thresholds.

Example 4 – Compute the long-term SF for the weep failure of vinyl ester oil pipes under the specified API 15HR conditions.

Since weep is a cycle-dependent failure, we use the simplified version of the unified equation.

$$\left(\frac{(1-R) \times \epsilon \times SF}{S_c} \right)^{\frac{1}{G_c}} + \left(\frac{R \times (1-R) \times \epsilon^2 \times SF^2}{S_c \times S_s} \right)^{\frac{1}{G_{sc}}}$$

Where

$\epsilon = 0.27\%$ is the peak transverse strain

S_C = Long-term cyclic transverse strength ($N = 10^9$ cycles).

S_S = Long-term static transverse strength, equal to the weep threshold

$R = 0.9$

G_{SC} = Interaction parameter for $R = 0.9$ and $N = 10^9$ cycles

$G_C = 0.052$ is the slope of transverse pure cyclic weep lines of vinyl ester UD plies.

The G_{SC} values in the unified equation are not available for $R = 0.9$. We will illustrate the case assuming $G_{SC} = 21$.

The long-term cyclic strength S_C comes from the weep pure cyclic regression line of transverse loaded vinyl ester UD plies.

$$\log \epsilon = \log(T_w) - \frac{\log(T_w/0.05)}{20} \times \log N$$

Entering $T_w = 0.55\%$, we have

$$\log(\epsilon) = \log(0.55) - 0.052 \times \log(N)$$

The API requirement calls for $N = 10^9$ cycles. The long-term weep cyclic strength S_C in this case is

$$\log(S_C) = \log(0.55) - 0.052 \times \log(10^9)$$

$$S_C = 0.18\%$$

The long-term static strength S_S in the transverse direction is the same as the weep threshold T_w of the ply. See figure 1 in the part 2 of chapter 13. Therefore, for vinyl ester UD plies

$$S_S = T_w = 0.55\%$$

Entering the above in the unified equation, we have

$$\left(\frac{(1-R) \times \epsilon \times SF}{S_C} \right)^{\frac{1}{G_C}} + \left(\frac{R \times (1-R) \times \epsilon^2 \times SF^2}{S_C \times S_S} \right)^{\frac{1}{G_{SC}}} = 1.00$$

$$\left(\frac{(1-0.9) \times 0.27 \times SF}{0.18} \right)^{\frac{1}{0.052}} + \left(\frac{0.9 \times (1-0.9) \times 0.27^2 \times SF^2}{0.18 \times 0.55} \right)^{\frac{1}{21}} = 1.00$$

Solving for SF , we obtain

$$SF = 3.9$$

This is the long-term weep SF assuming $G_{SC} = 21$ for the interaction parameter. Since this parameter is not known, we proceed to compute the lower bound of the safety factor SF .

The lower bound SF is

$$SF = \frac{R \times S_S}{\epsilon} + \frac{(1-R) \times S_C}{\epsilon}$$

$$SF = \frac{0.9 \times 0.55}{0.27} + \frac{(1-0.9) \times 0.18}{0.27}$$

$$SF > 1.9$$

The unified equation predicts that, for a target lifetime $Y = 10^9$ cycles, vinyl ester UD55 pipes under the loading prescribed by API 15HR do not weep.

Example 5 – Repeat the above computations for UD55 polyester pipes.

Again, this is a cycle-dependent problem, requiring the simplified version of the unified equation.

$$\left(\frac{(1-R) \times \epsilon \times SF}{S_C} \right)^{\frac{1}{G_C}} + \left(\frac{R \times (1-R) \times \epsilon^2 \times SF^2}{S_C \times S_S} \right)^{\frac{1}{G_{SC}}} = 1.00$$

Where

$\epsilon = 0.27\%$ is the peak transverse strain

S_C = Long-term cyclic transverse strength ($N = 10^9$ cycles).

S_S = Long-term static transverse strength, taken equal to the weep threshold

$R = 0.9$

G_{SC} = Interaction parameter for $R = 0.9$ and $N = 10^9$ cycles

$G_C = 0.048$ is the slope of the transverse pure cyclic weep line

The G_{SC} values entering the simplified version of the unified equation are not available for $R = 0.9$. To illustrate the case, we arbitrarily assume $G_{SC} = 21$.

The long-term cyclic strength S_C comes from the weep pure cyclic regression line for transverse loaded UD plies.

$$\log \epsilon = \log(T_w) - \frac{\log(T_w/0.05)}{20} \log N$$

Entering $T_w = 0.45\%$ for transverse loaded polyester UD plies, we have

$$\log(\epsilon) = \log(0.45) - 0.048 \times \log(N)$$

The API requirement calls for $N = 10^9$ cycles. The long-term weep cyclic strengths S_C for UD plies are

$$\log(S_C) = \log(0.45) - 0.048 \times \log(10^9)$$

$$S_C = 0.16\%$$

The long-term static strength S_S in the transverse direction is the same as the weep threshold. Therefore, for polyester UD plies

$$S_S = 0.45\%$$

Entering the above in the simplified unified equation

$$\left(\frac{(1-R) \times \epsilon \times SF}{S_C} \right)^{\frac{1}{G_C}} + \left(\frac{R \times (1-R) \times \epsilon^2 \times SF^2}{S_C \times S_S} \right)^{\frac{1}{G_{SC}}} = 1.00$$

$$\left(\frac{(1-0.9) \times 0.27 \times SF}{0.16} \right)^{\frac{1}{0.048}} + \left(\frac{0.9 \times (1-0.9) \times 0.27^2 \times SF^2}{0.16 \times 0.45} \right)^{\frac{1}{21}} = 1.00$$

Solving for SF , we obtain

$$SF = 3.3$$

Next, we compute the lower bound of SF. The computation in this case involves only the cyclic load component, since the weeping failure is cycle-dominated and time-independent.

The SF lower bound for cycle-dependent weep failure is

$$SF = \frac{R \times S_s}{\epsilon} + \frac{(1 - R) \times S_c}{\epsilon}$$

$$SF = \frac{0.9 \times 0.45}{0.27} + \frac{(1 - 0.9) \times 0.16}{0.27}$$

$$SF > 1.6$$

The unified equation predicts that UD55 polyester pipes under the specified API loading do not weep in 20 years. We can be sure that, for a target lifetime of $X = 20$ years and $Y = 10^9$ cycles, the polyester pipes under the loading prescribed by API 15HR will not weep.

Example 6 – The preceding weep analyses used the pure cyclic ply equations. From the part 2 of chapter 13, we know that the weep equation of pipes gives far better results.

In this example, we assume a short-term weep hoop strength $A_w = 2.50\%$ for a “representative” pipe. The hoop strain A_w rotated to the ply frame is

$$A_2 = \frac{A_w}{2} = \frac{2.50}{2} = 1.25\%$$

The pure cyclic weep regression equation for this UD55 vinyl ester pipe is

$$\log(\epsilon) = \log(A_2) - \frac{\log(A_2/T_0)}{\log(N_0)}$$

Where $A_2 = 1.25\%$, $T_0 = 0.05\%$ and $N_0 = 10^{20}$

$$\log(\epsilon) = \log(1.25) - \frac{\log(1.25/0.05)}{20} \times \log(N)$$

$$\log(\epsilon) = \log(1.25) - 0.070 \times \log(N)$$

The long-term hoop cyclic weep strength of the pipe comes by entering $N = 10^9$ in this equation

$$\log(S_c) = \log(1.25) - 0.070 \times \log(10^9)$$

$$S_c = 0.29\%$$

The long-term static weep strength of the pipe is set equal to the weep threshold T_w of the UD ply

$$S_s = T_w = 0.55\%$$

The lower bound of SF for the pipe is

$$SF = \frac{R \times S_s}{\epsilon} + \frac{(1 - R) \times S_c}{\epsilon}$$

$$SF = \frac{0.9 \times 0.55}{0.275} + \frac{(1 - 0.9) \times 0.29}{0.275} = 1.9$$

$SF > 1.9$

The lower bound SF has the same value for pipes and plies. This is so because the API requirement prescribes a nearly static load ratio $R = 0.9$.

This completes our analysis of the API 15HR cyclic requirement.

Chapter 18

Impermeable pipes

1 – Introduction: The composite pipes currently used in oil and sanitation service have three minor issues in need of attention. The first is their high permeability to solvents and gases. The second is their low tolerance to impact-cracking of liners. The third is the need to pressure test every item from fear of manufacturing defects. It would be desirable to find ways to eliminate these issues.

The insertion of a thin aluminum foil behind the resin-rich liner of the pipes immediately eliminates the permeability issue and the need of pressure test. However, the effect of such thin foils on the impact resistance is still debatable.

The aluminum foil chemically bonds to the resin matrix, becoming an integral part of the pipe wall. The thin metal film is barely perceptible, maintaining all original pipe specification, while eliminating the need of test pressure and the issue of high permeability. The improved impermeable pipes open new opportunities for composites in applications involving the transmission of gases and solvents.

2 – Permeability, diffusivity and solubility: The transmission of fluids in underground plastic pipelines is a cause of concern for two reasons.

- *Fear of soil contamination from exfiltration of carried products.*
- *Fear of product contamination from infiltration of soil pollutants.*

The risk of soil or product contamination has always been an environmental concern in applications involving underground plastic pipelines. Furthermore, the high permeability of composite pipes is the cause of a new and unsuspected type of the long-term anomalous failure, discussed later in this chapter. Before delving into these issues, let us say a few words about permeability, diffusivity and solubility.

The permeability of a system consisting of composites pipes and diffusing unreactive chemicals is

$$P = DS$$

Where P is the permeability, D is the diffusivity and S is the solubility of the diffusing chemical in the resin matrix. Each combination of diffusing material and laminate construction has its own P, D and S, which are highly dependent on the specific resin matrix. It is quite possible that the multiplicity of values taken by P, D and S for systems involving composite materials has discouraged the efforts to measure them. Be it as it may, the numerical values of P, D and S are scarce in the published literature. The rest of this section is dedicated to a brief discussion of P, D and S.

The diffusivity D measures the ease of unreactive molecules to slide through the intermolecular spaces in resin matrices. It depends on the matrix-fluid interaction, the dimensions of the permeating molecule and the size of the intermolecular matrix spaces.

Note: As discussed in chapter 10, the slow rate of penetration of reactive chemicals derive from their strong interaction with the resin matrix. The diffusivity concept is not strictly applicable to reactive chemicals. For details, see chapter 10.

Table 1 shows values of the diffusivity D of high polarity water molecules in laminates at 37 C.

Laminate	$D \left(\times 10^{-12} \frac{\text{m}^2}{\text{s}} \right)$	
	Polyester	Novolac vinyl ester
Resin casting	4.0 – 6.0	1.0
Chopped fiber plies	2.0 – 3.0	1.0
UD plies	2.0 – 3.0	1.0

Table 1

The diffusivity D of water is higher in polyester resins than in highly cross-linked vinyl ester novolac.

3 – Solvents: The solubility S designates the quantity of a substance that saturates a given mass of resin. The solubility is, therefore, the amount of fluid absorbed by the matrix at saturation. The solubility controls the maximum working temperature and the load bearing capability of laminates in contact with solvents. High values of S implies in high solvent intake and high resin swelling, which reduces the laminate structural and temperature capabilities. The decreased performance of laminates in the presence of solvents is well known and documented.

There are three categories of solvents.

- **Low solubility** – *The small swelling from solvents in this category has little effect on the structural capability of composite laminates. The most notorious representative in this category is water, with a solubility $S = 1.0\%$. The low swelling caused by such small water absorption does little harm to laminates, allowing their use at high temperatures and extremely demanding load-bearing applications.*
- **Medium solubility** - *The swelling from solvents in this category may restrict the use of composites to non-load bearing and low temperature services. There is a variety of or-*

ganic solvents falling in the category. The most notorious are, maybe, ethanol and methanol, with solubility values ranging from $S = 10.0\%$ to $S = 14.0\%$.

- **High solubility** - The high swelling from these solvents may cause resin cracking and laminate failure, even in non-structural service. Acetone is an example.

Table 2 illustrates the solubility S of water, ethanol and toluene in castings of two premium resin matrices. The reported solubility values are valid for resin castings at 37 C.

<i>Solubility – S</i>	<i>Water</i>	<i>Methanol</i>	<i>Toluene</i>
<i>High reactivity polyester</i>	<i>1.0%</i>	<i>10.0%</i>	<i>18.0%</i>
<i>Novolac vinyl ester</i>	<i>1.2%</i>	<i>13.0%</i>	<i>10.0%</i>

Table 2

The small water pickup (1.0%) has no effect on the structural capability and a modest (10C) decrease in the maximum operating temperature of composites. The higher solubility of methanol and toluene limit the use of composite laminates to non-structural low temperature services.

In conclusion:

1. The low solubility $S = 1\%$ of water in resin matrices has no appreciable effect on composites pipes, other than a minor, say 10C, decrease in the maximum operating temperature.
2. The medium solubility of some organic solvents may reduce the working temperature/pressure rating of composite pipes to very low values.
3. Composite pipes are not good for direct contact with highly soluble solvents.

The traditional composite pipelines are not adequate in services involving highly soluble solvents, like acetone and many others. Besides, they find limited use in medium solubility solvents, such as ethanol. The recently developed impermeable composite pipes do not have such restrictions. The impermeable aluminum liner of such pipes bars the ingress of solvents, causing the pipe to perform as if in water service. As an example, in the transmission of methanol or ethanol, the traditional permeable pipes are limited to room temperature and gravity service. The impermeable pipes, however, are good in methanol or ethanol at any temperature and pressure. The impermeable pipes carry solvents as they carry water.

The permeability P is a property of the system pipe-solvent, obtained by multiplying the diffusivity D and the solubility S .

$$P = D.S$$

The permeability P is a good metric of the flow of solvents through laminates. The following sections highlight some benefits of the aluminum foil in liners of impermeable pipes.

4 – Anomalous failure: The anomalous failure is a long-term deterioration of composite pipes and tanks, identified and explained in 2012. This section describes the causes and prevention of the anomalous failure.

The amount of unreactive solvents diffusing through the wall of composite laminates derives from Fick's first equation

$$[flow] = \frac{D \times S \times C}{t}$$

Where

[flow] is the amount of solvent passing a unit area of the wall per unit time.

D is the coefficient of diffusion

S is the solubility,

C is the solvent concentration in the outside solution

t is the wall thickness.

Most anomalous failures occur in the presence of water. The solubility *S* and the diffusivity *D* of water in a few laminates are in tables 1 and 2. The water permeability of the system is the product $P = D.S$.

Example 1 – Compute the flow of water through the wall of a polyester pipe at 37°C. Assume the following conditions

- Wall thickness $t = 5.0 \text{ mm}$.
- $D = 3.0 \times 10^{-12} \frac{\text{m}^2}{\text{s}}$
- $S = 0.01$ (Solubility of water in polyester).
- $C = 1 \text{ g/cm}^3$ (the pipe carries pure water).

Entering the above in Fick's equation, we obtain

$$[flow] = \frac{3.0 \times 10^{-12} \frac{\text{m}^2}{\text{s}} \times 0.01 \times 1 \text{ g/cm}^3}{5.0 \text{ mm}}$$

$$[flow] = 190 \text{ g/m}^2.\text{year}$$

The water lost by permeation amounts to just about 3 liters per square meter in 15 years. As a rule, the pipe owner is not aware of such a loss, since the water that permeates the pipe dissipates harmlessly into the atmosphere. However, before evaporating into the atmosphere, the permeating water first fills all voids and delamination in its way. The anomalous failure results from water accumulation in delamination.

Let us discuss the mechanism leading to the anomalous failure. The water pressure inside the voids drop off linearly from a maximum at the inner surface, to zero at the outer surface of the equipment. Under steady operating conditions, there is nothing unusual or damaging with the presence of pressurized water in voids. The problem arises if the pressure or temperature oscillates in the fluid carried inside the pipe. Let us explain this.

Figure 1 illustrates the discussion that follows. Suppose a void or delamination in the wall or core of a pipe carrying pressurized water. This void or delamination, initially dry and empty, eventually fills up with pressurized water. The water pressure is just enough to equilibrate the inner pressure in the pipe. Nothing happens at this equilibrium condition. Suppose now a sudden pressure drop inside the pipe. This drop upsets the equilibrium, causing the pressurized delamination to expand by a small amount, just enough to balance the lower pressure inside the pipe. As the pressure in the pipe rises to its previous value, the delamination refills to reach a new equilibrium. Again, nothing happens at equilibrium. The process repeats itself, with a series of pressure drops and expansions in the void or delamination. After many cycles of pressure oscillation, the water-filled void or delamination may accumulate considerable enlargements. This process continues unchecked, growing the initially small delamination into enormous blisters filled with pressurized water.

The anomalous blister continues its unchecked growth until rupture of the pipe. The recognition and explanation of this unsuspected mode of failure occurred recently, in 2012, when we had a chance to observe the enormous blisters that formed on a large diameter pressurized water line. Our surprise and amazement on that occasion was so great that we named this phenomenon as “anomalous failure”, to emphasize its abstruse origin and fundamental difference from the well-known osmotic blisters.

Note: The anomalous failure result from oscillations in the pressure or temperature of the transmitted or stored fluid. Pressure oscillations is prevalent in pipes, while temperature oscillations occur mostly in process equipment.



Figure 1a

Note the delaminated sand core that sites the water accumulation.

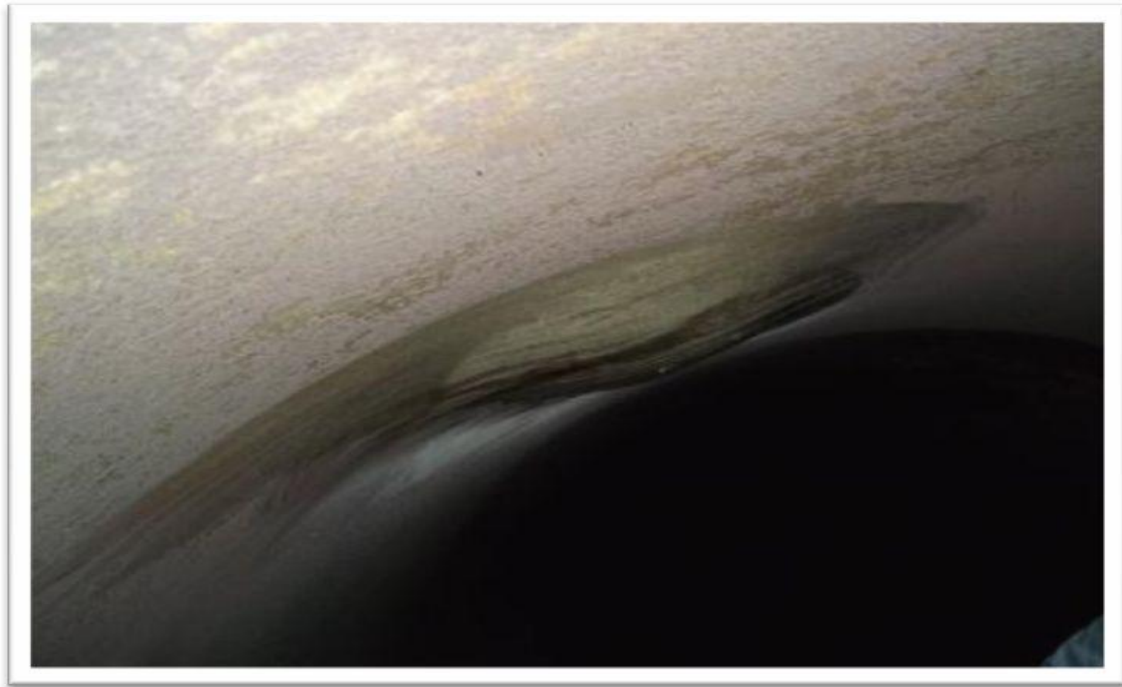


Figure 1b

The water-filled blister may grow to large sizes, like 500 mm in length and 100 mm in height, before bursting.



Figure 1c

There is no mechanism to check the growth of the anomalous blisters. Eventually they burst the pipe.

The sand cores in underground pipes are particularly susceptible to large delamination from mishandling or impact. Such delamination provide the sites for water accumulation and the development of the anomalous failure. The use of an impermeable alumi-

num foil under the liner bars the ingress of water and prevents the formation of the anomalous blisters that would otherwise occur.

The following concerns are always present on the minds of contractors involved with the installation of sanitation pipes.

- *Have the unavoidable impacts from shipping and receiving damaged the pipe?*
- *Has that hard fall of the pipe on that rock caused any damage?*
- *Have we at some point inadvertently mishandled the pipe?*
- *What about the sand core? Has it delaminated? Will anomalous blisters develop at some point?*

There are no simple answers to these questions. A visual inspection will indicate no apparent damage to the pipe. Nevertheless, the worry remains. The presence of the aluminum foil would eliminate all these worries. The insertion of thin aluminum foils under the pipe liners eliminates the worries from damages and prevent the formation of anomalous blisters.

5 – Sanitation and oil pipes: The aluminum foils, although impermeable, cannot control weep failures. The foils are too thin and not strong enough to arrest the large crack densities that develop in pipes pressurized above the weep threshold. The large coalesced cracks in this situation, mostly those in UD55 pipes, will rupture the thin foil, causing the pipes to weep. That aluminum foils may increase the weep threshold, but not arrest it.

The assured benefits from the impermeable foil in sanitation and oil pipes are fourfold:

- *Improved tolerance to minor manufacturing and mishandling damages.*
- *Slight increase in the weep threshold*
- *Guaranteed absence of weeping from manufacturing defects.*
- *Prevention of anomalous failure.*

While the aluminum foils may not protect composite pipes from weeping or major impact damage, they certainly eliminate the possibility of premature weep failures from manufacturing defects. The premature weeping from manufacturing defects is a major concern in sanitation and oil pipes. The detection of such defects involves expensive short-term (30 seconds) pressure testing of every pipe. Let us expand on this.

The current pipe standards for sanitation and oil pipes require two quality control pressure tests.

- *A high-pressure test to ascertain pressure rating. This is a destructive test conducted on representative samples at 4 times the rated pressure for 10 minutes with no weep. The aluminum foil do not affect this test and will not interfere in the assessment of the rated pressure.*
- *A 30 seconds test at 1.5 times the rated pressure, to ascertain the absence of manufacturing defects. The rationale here is that all defective pipes weep when pressurized at 1.5 times the rated pressure for 30 seconds. This non-destructive test is mandatory on all pipes less than 1400 mm for sanitation or oil service. The impermeable pipes never weep at pressures less than the weep threshold, even in the presence of manufacturing defects. Therefore, the impermeable pipes do not require any quality control hydrostatic tests.*

The insertion of an aluminum foil under the liner eliminates the expensive quality control hydrostatic pressure testing currently required on sanitation and oil pipes. This benefit sometimes raise the objection that the impermeable foil would mask major manufacturing defects, forcing the acceptance of faulty pipes. This argument is not valid, since the gross defects hidden by the foil are not crack-like and do not grow under cyclic loads. Such manufacturing defects cause no harm other than weeping the pipe at low pressures. The defective pipes live normal lives, as if free from any defects.

6 – Underground fuel tanks: The impermeable aluminum foil allows the use low cost polyester resins in underground storage tanks for solvents and ethanol.

7 – Solvent storage and transmission: One of the evident shortcomings of composite pipes and tanks is their inability to work in solvent service at high pressure/temperature. The aluminum foil allows the use of low cost pipes and tanks in solvent service at high pressures and temperatures.

8 – Industrial effluent: The aluminum foil allows the use of regular sanitation pipelines (designed to carry water) in service involving solvent-laden industrial effluents at high pressures and temperatures.

9 – Gas transmission: The impermeable aluminum foil eliminates the risk of anomalous failure and permeation in services involving the transmission of gases. The fully gas-tight impermeable pipeline would be immune to corrosion and anomalous failure.

10 – Chemical service: The impermeable pipes, originally developed for the transmission of ethanol, evolved to include underground storage tanks, oil pipes and sanitation pipes. The impermeable pipes may also find application in chemical service.

The insertion of aluminum foils between the liners and the corrosion barriers of chemical equipment may have benefits. The impermeable foil would bring the presence of aggressive chemicals to zero in the equipment wall. This desirable situation would fully protect the equipment for as long as the foil maintains its integrity. The corrosion resistance of the thin aluminum foil in such cases will range from very good in sanitation, oil and solvent service, to perhaps not so good in aggressive chemicals. However, whatever its durability, the foil would still do some good, while having no bad effects. At worse, assuming instant chemical destruction of the foil, the original corrosion barrier would still be there.

The best applications of aluminum foils would be in extremely aggressive chemicals, those not contemplated by composites. In these cases, the foil would certainly give full protection to the equipment wall for some time, delaying its destruction. This would certainly extend the service life of the equipment. The inclusion of two foils would give even better protection. The use impermeable aluminum foils in chemical equipment may be a wise decision to improve their chemical lives, especially in nasty environments.

11 – Market acceptance of aluminum foils: The proposed impermeable pipes consist essentially of a low cost thin aluminum foil inserted behind the liner of equipment. This is a simple solution, straightforward and market-ready. No extensive and expensive qualification testing is required. In fact, these are not “new equipment” but the same “old ones”, already qualified and augmented by the presence of the impermeable foil.

The chemically bonded foils are an integral part of the structure. The impermeable liner opens new opportunities for composites in applications like solvents, industrial effluents, gas transmission and perhaps chemical service.

Figures 2, 3 and 4 illustrates the manufacture of impermeable pipes. The helically wound foil goes directly on top of the resin-rich liner. The regular structural laminate goes on top of the foil. To improve the pipe's impact resistance and eliminate the risk of damage from careless handling and transportation, we suggest the insertion of a ply of chopped fibers on top of the foil, before the filament wound structure. The ply of chopped fibers protects the pipe from cracks originating in the UD plies. The foil will have perfect adhesion to the resin and become an integral part of the pipe.

The “new impermeable pipes” are in essence the “old pipes”, with the added feature of impermeability. Market acceptance is not expected to be a major problem.

The impermeable pipes bring the following confirmed benefits to the composites industry:

- *Zero contamination in underground service*
- *Good for solvent service at high pressure/temperature*
- *Raise the weep threshold*
- *Good for industrial effluent service at high pressure/temperature*
- *Good for gas pipeline service*
- *Elimination of hydrostatic pressure tests*
- *Elimination of anomalous failure*
- *Elimination of worries*

In addition to the above, the following are likely benefits:

- *Improved durability in chemical service*
- *Protection against high impact damages*



Figure 2

Overwrapping aluminum foil on wet resin

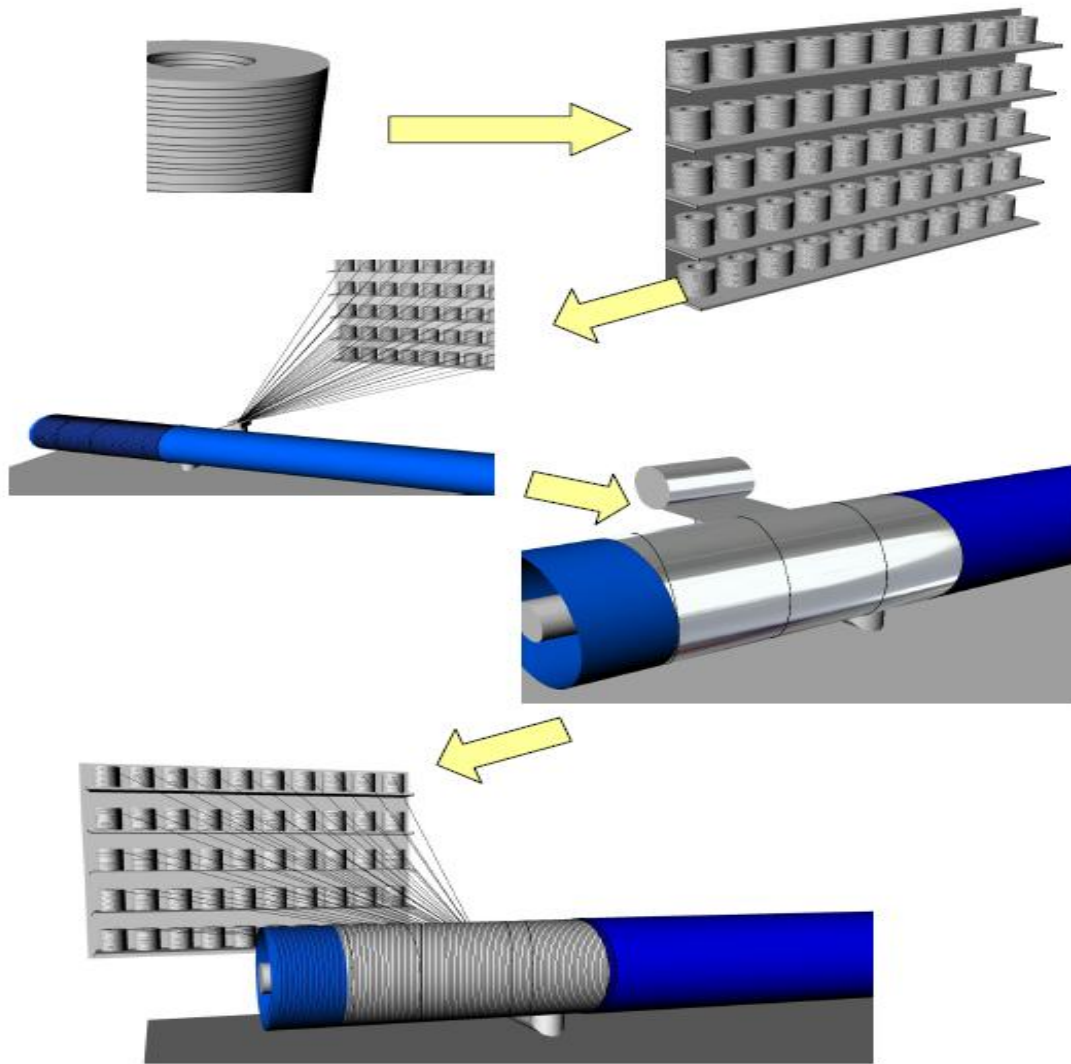


Fig. 3

The manufacturing process is the same for regular and impermeable pipes, with the aluminum foil helically wound on the rotating mandrel.



Fig 4.

The aluminum foil on the mandrel, prior to lamination of the structural plies.

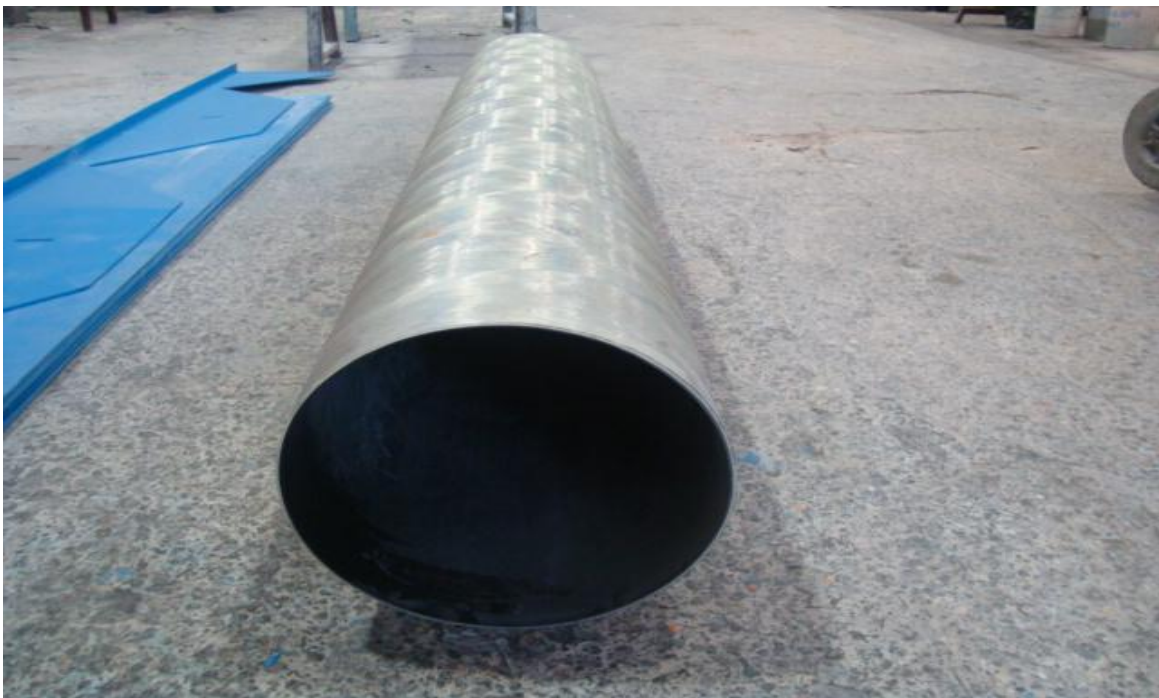


Fig. 5

The pipe in the picture has the following layup:

- *Conductive inner layer. The dark color is imparted by graphite. Liner conductivity is a requirement for hydrocarbon solvents.*

- *Tape wound impermeable aluminum foil.*
- *Filament wound UD55 angle-ply.*

Chapter 19

Sudden death and the regression equations

1 – Introduction. The foregoing chapters presented the necessary information to compute the chemical and load-dependent durability of composite equipment. In retrospect, however, I see a few points in need of further clarification. These points are:

- *A detailed description of the differences between the mechanical and the chemical regression equations*
- *The meaning of such equations*

This chapter is an attempt to explain and clarify these points.

We start with three statements:

- *The chemical and load-dependent durability of composite equipment are uncoupled and independent. Mechanical loads do not affect the chemical durability in the same way that aggressive chemicals have no effect on the load-dependent lifespans. This simplification is a direct consequence of the concept of infiltration threshold, introduced in chapter 8.*
- *Knowledge of the aged infiltration threshold and the aged tensile modulus suffice to compute the chemical durability of any equipment.*
- *Knowledge of the pure static and the pure cyclic regression equations suffice to solve any load-controlled durability problem. The parameters required in the unified equation derive directly from these two regression lines.*

The regression equations are available in either exponential or power formats. The preference for the power representation is ease of mathematical treatment, since their data plot as straight lines in log-log space. Therefore, for ease of mathematical treatment, we usually write all regression equations in the power law format.

$$\epsilon = A \times t^{-B}$$

$$\epsilon = A \times N^{-B}$$

Or

$$\log(\epsilon) = \log(A) - B \times \log(t)$$

$$\log(\epsilon) = \log(A) - B \times \log(N)$$

2 – Chemical and load-dependent durability. As explained in chapter 8, the load-induced failures result from increasing crack densities reaching critical values. The crack coalescence at the critical values explains the failures. Below the stiffness threshold, the small relative volumes of the increased crack densities are not enough to affect the laminate moduli. That is the explanation for the sudden death principle.

The sudden death principle establishes that all load-dependent failures (except rupture) take place under constant modulus. The laminates finish their load-carrying lives suddenly, without warning, in full retention of their modulus. This is the essence of the sudden-death principle.

By contrast, the equipment dying chemical deaths give ample warning of their impending failure in terms of reduced modulus. The decreasing aged modulus implies in increasing strain that eventually fails the equipment on reaching the aged infiltration threshold.

The following table summarizes the differences between chemical and load-dependent failures.

<i>Mechanical failures</i>	<i>Chemical failures</i>
<i>Constant strains and constant modulus</i>	<i>Increasing strains from decreasing modulus</i>
<i>Constant failure thresholds</i>	<i>Decreasing infiltration threshold</i>
<i>Time or cycle-dependent</i>	<i>Time-dependent</i>
<i>Regression equations expressed in strain</i>	<i>Regression equations expressed in modulus</i>
<i>Controlled by crack densities at the failure thresholds</i>	<i>Controlled by crack densities at the aged infiltration threshold</i>
<i>Presence of all four modes of failure, infiltration, weeping, stiffness and rupture</i>	<i>Infiltration is the only mode of failure</i>
<i>Durability predicted by the unified equation</i>	<i>Durability and residual lives predicted by the residual modulus and the aged infiltration threshold</i>

The estimation of both chemical and mechanical lifetimes require knowledge of regression equations.

- *The mechanical durability requires knowledge of the regression equations linking the failure strains with the time or number of cycles.*
- *The chemical durability requires knowledge of the regression equations linking the retained modulus with the exposure time. Knowledge of the aged infiltration threshold is also required.*

3 – Load-dependent regression equations. The load-dependent regression equations relate the strain to the time or the number of cycles at failure. Therefore, from the power law representation, we have

$$\log(\epsilon) = A_c - G_c \times \log(N) \quad (\text{Pure cyclic equation})$$

$$\log(\epsilon) = A_s - G_s \times \log(t) \quad (\text{Pure static equation})$$

In the above equations the strain, the time and number of cycles are those at failure. That is the meaning of the load-dependent regression equations. They describe the loci of all failure combinations of strain, time and number of cycles. The regression equations needed are the pure static and the pure cyclic. As discussed in chapter 16, the inputs to the unified equation derive directly from these two equations.

There are five pure load-dependent regression lines, one for each mode of failure.

- *1 pure static line, describing rupture failures from strain-corrosion of the glass fibers.*
- *4 pure cyclic lines, describing failures by infiltration, weep, stiffness and rupture.*

Figure 1 shows the four pure cyclic equations connecting the short-term failure thresholds to the fatigue threshold $T_o = 0.05\%$ and $N_o = 10^{20}$ cycles

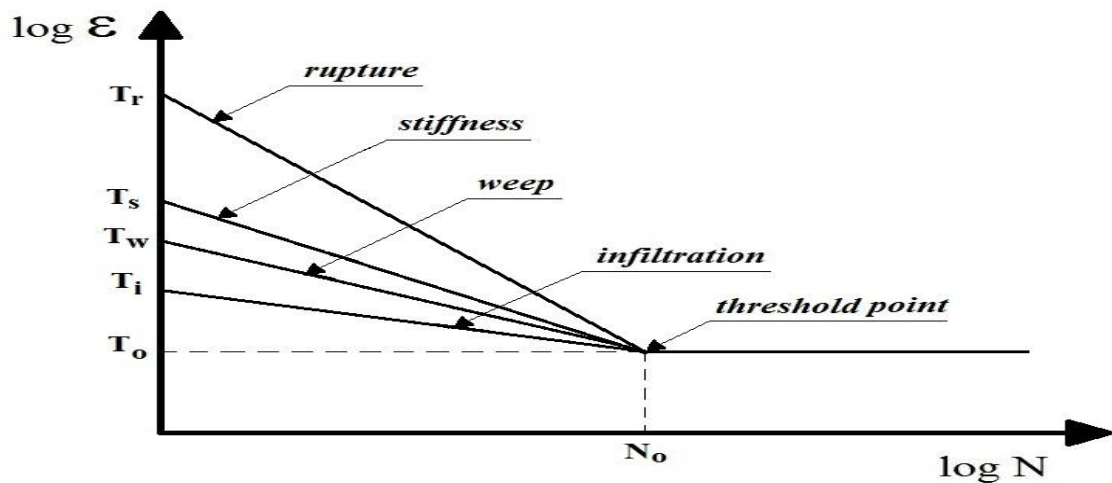


Figure 1

The pure cyclic equations relate the constant applied peak strains to the number of cycles to failure. Since cyclic loads grow both fiber and resin cracks, they include the four modes of failure. The modulus remains constant throughout the lifespan of the ply or laminate, giving no indication of ageing. This “sudden death” is typical of all load-controlled failures.

Figure 2 shows the pure static rupture equation.

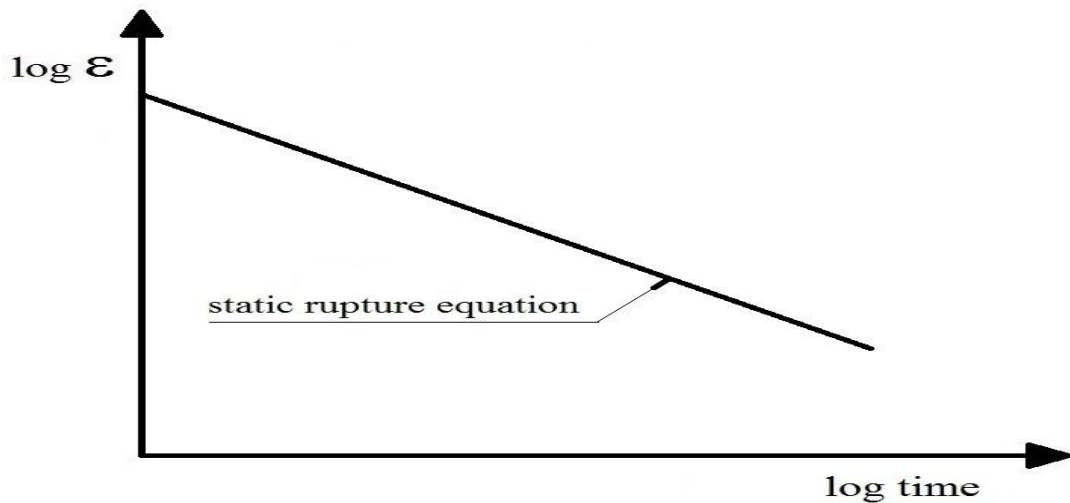


Figure 2

The pure static rupture equation relates the constant applied strain to the time to rupture. As in figure 1, the laminate modulus remains constant throughout its lifespan, as typical of load-controlled failures.

The reader may wonder why load-induced damages have no appreciable effect on the laminate modulus. This is easy to explain, considering the extremely small volume fraction occupied by the damaged sites (crack densities) in any given laminate. The smallness of the damaged volume fraction explains the constant modulus throughout the equipment structural life. The loss of modulus from load-induced damages goes unnoticed until the stiffness threshold, when it is barely perceptible.

Figure 3 illustrates this, for both cyclic and static loads.

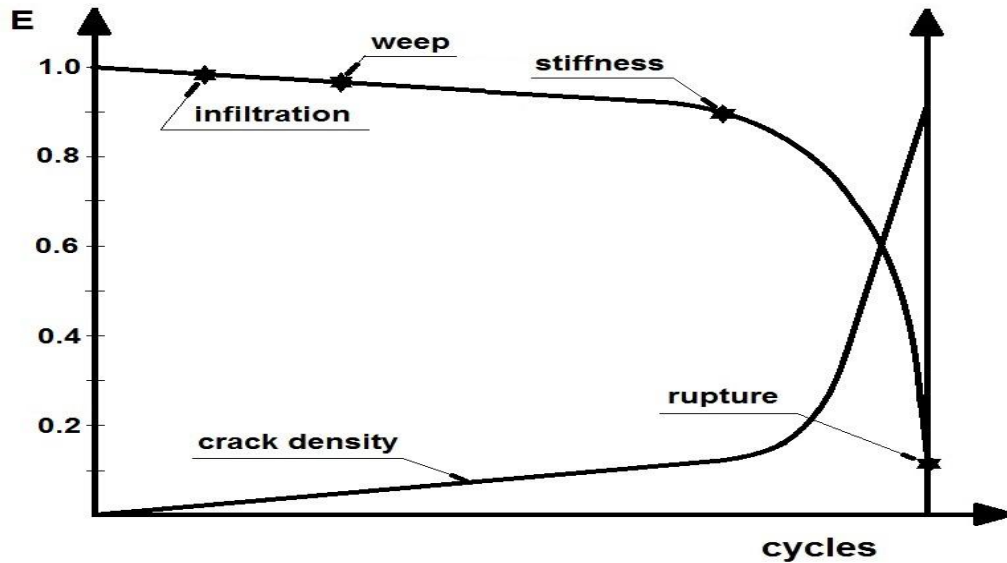


Figure 3

The small volume fraction occupied by the ruptured fiber points and resin cracks causes little loss of modulus in laminates. The laminate retains most of its original stiffness throughout the ageing process, even at large crack densities. As indicated in the figure, loss of modulus is noticeable near rupture, at extremely high crack densities. The laminates under load die a sudden death, showing no significant sign of deterioration throughout most of their lives. This is typical of load-induced laminate failures.

Figure 4 shows the modulus retention and rupture durability for several constant applied strains. We see that higher strains cause earlier rupture. The load-induced regression equations relate the constant strains with the time/cycles to failure.

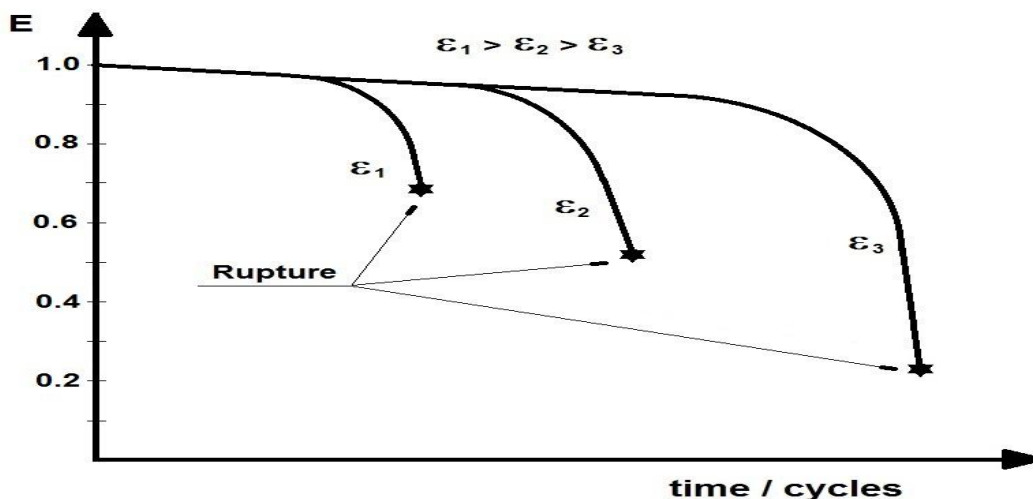


Figure 4

Modulus loss becomes apparent near rupture, at extremely high crack densities. Higher applied strains produce shorter lifetimes. The load-induced regression equations plot the constant strains as a function of time/cycles to failure.

4 – Chemical regression equations. The slowly diffusing aggressive chemicals gradually attack the resin matrix and fibers, thereby reducing the laminate modulus. This is in direct contrast with the load-induced ageing process, which grows crack densities with no attack on either the resin or the fibers. The small volume fraction occupied by the cracks are in deep contrast to the large volume of laminate penetrated and deteriorated by the aggressive chemical. We have our first conclusion:

There is a considerable reduction in the laminate modulus throughout the chemical ageing process

Furthermore, the chemical processes do not depend on the nature of the applied load (cyclic or static). We have our second conclusion:

The chemical regression equations plot the residual modulus as a function of time only, independent of the applied loads.

These conclusions are in figure 5.

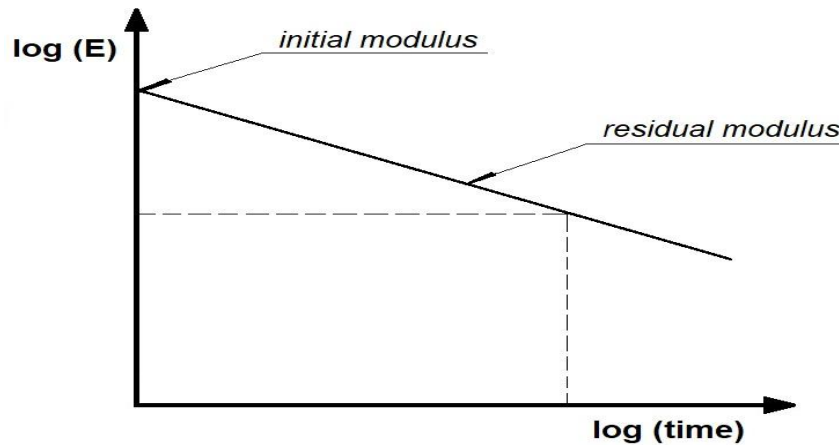


Figure 5

The regression equations for chemical durability link the residual tensile modulus E_r to the exposure time. The expected chemical durability and the residual lives are strictly time-dependent.

As explained in chapter 10, the expected chemical durability and the residual lives of equipment derive from equations expressed in terms of the residual tensile modulus. This is in striking contrast to the load-controlled durability equations, in which the modulus remains essentially constant throughout the ageing process.

In addition to the decreasing modulus, the chemical life depends also on the aged infiltration threshold. Given the tremendous difficulty to measure the residual infiltration threshold, we assume it to decrease as the modulus.

Example 3 – Explain the difference between chemical-controlled and load-controlled durability of composite equipment.

The chemical life is a direct consequence of the drop in tensile modulus and infiltration threshold observed in laminates under chemical attack. The decaying modulus grows the laminate strain that eventually reaches the aged infiltration threshold. The chemical regression equation, linking the residual tensile modulus with the exposure time, directly estimates the chemical durability and the residual life.

By contrast, the load-dependent failures come from growth in crack densities. The laminate modulus and failure thresholds remain essentially constant in the entire ageing process. The regression equations linking the constant strains to the time/cycles to failure supply the necessary inputs to the unified equation.

The above statements indicate that measurement of the residual tensile modulus is valid to compute the chemical, not the load-dependent, durability. They also indicate that apparently healthy laminates are not immune against load-induced sudden death. The same situation applies in living organisms, where excellent check-up test results are no guarantee against sudden death.

The unified equation is the only protective tool available to warn us against sudden mechanical failure.

Chapter 20

Short-term strengths of UD55 pipes

1 – Introduction: The failure envelope proposed by ISO 14692 for the long-term analysis of composite oil pipes is cumbersome, artificial and inaccurate. In this chapter, we discuss an easier and far more accurate analytical tool than those failure envelopes.

2 – The general equation: The general equation (1) computes the global strains on composite circular cylinders under the simultaneous action of internal pressure, torque, hoop and axial loads

$$\begin{bmatrix} \frac{P \times \Phi}{4} + N_x \\ \frac{P \times \Phi}{2} + N_y \\ \frac{2 \times T}{\pi \times \Phi^2} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \times \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \quad (1)$$

Where

x is the axial direction

y is the hoop direction

P is the internal fluid pressure

Φ is the pipe diameter

N_x is the axial force per unit length

N_y is the hoop force per unit length

T is the torque

[A] is the laminate tensile stiffness matrix.

[C] is the matrix of global strains.

Equation (1) applies to any circular cylinder under owner-informed mechanical forces. The protocol to compute the stiffness matrix [A] of the pipe is in the part 1 of this book. The only unknowns in equation (1) are the global strains.

Equation (1) gives the global strains for the most general membrane loading possible on composite circular pipes. To facilitate the presentation, we ignore the non-membrane bending loads.

3 – Angle-ply laminates: For angle-ply and balanced circular laminates, equation (1) reduces to

$$\begin{bmatrix} \frac{P \times \Phi}{4} + N_x \\ \frac{P \times \Phi}{2} + N_y \\ \frac{2 \times T}{\pi \times \Phi^2} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ 0 & 0 & A_{33} \end{bmatrix}_{55} \times \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \quad (2)$$

The composite pipes used in oil and gas service use a combination of balanced UD55 plies. From chapter 5, the stiffness matrix [A] of standard UD55 laminates is

$$[A] = \begin{bmatrix} 132470 & 96220 & 0 \\ 96220 & 235070 & 0 \\ 0 & 0 & 101220 \end{bmatrix} \times [t] \quad kg/cm$$

In the above, “t” is the laminate thickness. Entering this matrix [A] in equation (2), we obtain

$$\begin{bmatrix} \frac{P \times \Phi}{4} + N_x \\ \frac{P \times \Phi}{2} + N_y \\ \frac{2 \times T}{\pi \times \Phi^2} \end{bmatrix} = \begin{bmatrix} 132470 & 96220 & 0 \\ 96220 & 235070 & 0 \\ 0 & 0 & 101220 \end{bmatrix} \times [t] \times \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \quad (3)$$

Dividing through by the laminate thickness “t” we obtain equation (4), expressed in terms of average stresses

$$\begin{bmatrix} \frac{P \times \Phi}{4 \times t} + \sigma_x \\ \frac{P \times \Phi}{2 \times t} + \sigma_y \\ \frac{2 \times T}{\pi \times \Phi^2 \times t} \end{bmatrix} = \begin{bmatrix} 132470 & 96220 & 0 \\ 96220 & 235070 & 0 \\ 0 & 0 & 101220 \end{bmatrix} \times \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \quad (4)$$

Equation (4) links the applied internal pressure, external torque and average stresses to the global strains of standard UD55 pipes.

4 – Strain analysis: The analysis of composite pipes in terms of strains is much simpler than in terms of average stresses. The reasons for the use of strains in place of stresses are:

- 1 – The failure strains are independent of Poisson effects. The failure envelope of plies in the strain space is rectangular, not elliptical as in the stress space.
- 2 – Unlike stresses, that change from ply to ply, the global strains have the same value for all plies.

Solving equation (4) for the global strains gives

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} 132470 & 96220 & 0 \\ 96220 & 235070 & 0 \\ 0 & 0 & 101220 \end{bmatrix}^{-1} \times \begin{bmatrix} \frac{P \times \Phi}{4 \times t} + \sigma_x \\ \frac{P \times \Phi}{2 \times t} + \sigma_y \\ \frac{2 \times T}{\pi \times \Phi^2 \times t} \end{bmatrix} \quad (5)$$

Which, upon inversion of matrix [A], becomes

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} 1.07 & -0.44 & 0 \\ -0.44 & 0.61 & 0 \\ 0 & 0 & 0.99 \end{bmatrix} \times [10^{-5}] \times \begin{bmatrix} \frac{P \times \Phi}{4 \times t} + \sigma_x \\ \frac{P \times \Phi}{2 \times t} + \sigma_y \\ \frac{2 \times T}{\pi \times \Phi^2 \times t} \end{bmatrix} \quad (6)$$

We next rotate the global strains in equation (6) to the local ply frames. For this, we use the rotation matrix for UD55 plies.

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \frac{1}{2}\gamma_{12} \end{bmatrix} = \begin{bmatrix} 0.33 & 0.67 & \pm 0.94 \\ 0.67 & 0.33 & \mp 0.94 \\ \mp 0.47 & \pm 0.47 & -0.34 \end{bmatrix} \times \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \frac{1}{2}\gamma_{xy} \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \frac{1}{2}\gamma_{12} \end{bmatrix} = \begin{bmatrix} 0.33 & 0.67 & \pm 0.94 \\ 0.67 & 0.33 & \mp 0.94 \\ \mp 0.47 & \pm 0.47 & -0.34 \end{bmatrix} \times \begin{bmatrix} 1.07 & -0.44 & 0 \\ -0.44 & 0.61 & 0 \\ 0 & 0 & 0.50 \end{bmatrix} \times [10^{-5}] \times \begin{bmatrix} \frac{P \times \Phi}{4 \times t} + \sigma_x \\ \frac{P \times \Phi}{2 \times t} + \sigma_y \\ \frac{2 \times T}{\pi \times \Phi^2 \times t} \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 0.06 & 0.26 & \pm 0.47 \\ 0.57 & -0.09 & \mp 0.47 \\ \mp 1.42 & \pm 1.00 & -0.34 \end{bmatrix} \times [10^{-5}] \times \begin{bmatrix} \frac{P \times \Phi}{4 \times t} + \sigma_x \\ \frac{P \times \Phi}{2 \times t} + \sigma_y \\ \frac{2 \times T}{\pi \times \Phi^2 \times t} \end{bmatrix} \quad (7)$$

Equation (7) gives the ply strains of standard UD55 pipes as a function of the applied loading. We recall that

P is the internal pressure.

Φ is the pipe diameter

t is the pipe wall thickness

σ_x is the average axial stress

σ_y is the average hoop stress

T is the torque.

Equation (7) is very accurate. To obtain the total ply strains, we simply add the residual thermal and hydric strains to the mechanical strains in equation (7). The load-dependent durability analysis of UD55 pipes readily follows from the unified equation and knowledge of the total ply strains.

5 – Pipe short-term strengths: Let us compute the short-term strengths of UD55 pipes.

Example 1 – ASTM D 2105 axial test: The external loading in this case consists of just the axial tensile stress. Taking $P = T = \sigma_y = 0$ in equation (7) gives

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 0.06 & 0.26 & \pm 0.47 \\ 0.57 & -0.09 & \mp 0.47 \\ \mp 1.42 & \pm 1.00 & -0.34 \end{bmatrix} \times [10^{-5}] \times \begin{bmatrix} \sigma_x \\ 0 \\ 0 \end{bmatrix} \quad (8)$$

Expanding the above, we obtain three values for the short-term axial tensile strength.

$$\begin{aligned} \varepsilon_1 &= 0.06 \times 10^{-5} \times \sigma_x \\ \varepsilon_2 &= 0.57 \times 10^{-5} \times \sigma_x \\ \gamma_{12} &= \mp 1.42 \times 10^{-5} \times \sigma_x \end{aligned}$$

The first equation refers to fiber rupture. Assuming the short-term tensile rupture strain in the fiber direction as $\varepsilon_1 = 0.03$, we have

$$\sigma_x = \frac{0.03}{0.06 \times 10^{-5}} = 50\,000 \text{ kg/cm}^2$$

The second equation refers to transverse ply failure. We have three scenarios to consider.

1. **Axial strength to infiltration.** This is the short-term axial strength at the onset of ply cracking. Entering the infiltration threshold T_i for ε_2 , we have

For polyester pipes

$$\sigma_x = \frac{0.0040}{0.57 \times 10^{-5}} = 700 \text{ kg/cm}^2$$

For vinyl ester pipes

$$\sigma_x = \frac{0.0050}{0.57 \times 10^{-5}} = 880 \text{ kg/cm}^2$$

2. **Axial strength to weep.** This is the short-term axial strength at the onset of weeping. Entering the weep threshold T_w for ε_2 , we have

For polyester pipes

$$\sigma_x = \frac{0.0045}{0.57 \times 10^{-5}} = 790 \text{ kg/cm}^2$$

For vinyl ester pipes

$$\sigma_x = \frac{0.0055}{0.57 \times 10^{-5}} = 960 \text{ kg/cm}^2$$

3. **Axial strength to rupture.** This is the usual short-term axial strength of the pipe. Entering the rupture threshold T_r for ε_2 , we have

For polyester pipes

$$\sigma_x = \frac{0.0070}{0.57 \times 10^{-5}} = 1220 \text{ kg/cm}^2$$

For vinyl ester pipes

$$\sigma_x = \frac{0.0100}{0.57 \times 10^{-5}} = 1750 \text{ kg/cm}^2$$

Finally, the third equation refers to shear failure. Assuming $\gamma_{12} = 0.02$ we have

$$\sigma_x = \frac{0.02}{1.42 \times 10^{-5}} = 1400 \text{ kg/cm}^2$$

As we see, the transverse ply failure governs the short-term axial strength of UD55 pipes.

This is a simple and direct computation. The reader is invited to compare it with the failure envelope proposed by ISO 14692.

Example 2 – Split disc hoop test: The loading in this case is the tensile hoop stress. All other stresses are equal to zero.

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 0.06 & 0.26 & \pm 0.47 \\ 0.57 & -0.09 & \mp 0.47 \\ \mp 1.42 & \pm 1.00 & -0.34 \end{bmatrix} \times [10^{-5}] \times \begin{bmatrix} 0 \\ \sigma_y \\ 0 \end{bmatrix}$$

Expanding the above, we obtain three values for the short-term hoop tensile strength.

$$\varepsilon_1 = 0.26 \times 10^{-5} \times \sigma_y$$

$$\varepsilon_2 = -0.09 \times 10^{-5} \times \sigma_y$$

$$\gamma_{12} = 1.00 \times 10^{-5} \times \sigma_y$$

The first equation refers to fiber rupture. Assuming as before $\varepsilon_1 = 0.03$, we have

$$\sigma_y = \frac{0.03}{0.26 \times 10^{-5}} = 11\,500 \text{ kg/cm}^2$$

The second equation refers to transverse ply failure. The negative – compressive – transverse strain will not cause failure and is ignored.

The third equation refers to shear failure. Assuming $\gamma_{12} = 0.02$, we have

$$\sigma_y = \frac{0.02}{1.00 \times 10^{-5}} = 2000 \text{ kg/cm}^2$$

Shear failure governs the short-term hoop strength of UD55 oil pipes

Example 3 – Torque analysis: The torque effect comes from equation (7), by taking the axial stress and the internal pressure equal to zero.

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 0.06 & 0.26 & \pm 0.47 \\ 0.57 & -0.09 & \mp 0.47 \\ \mp 1.42 & \pm 1.00 & -0.34 \end{bmatrix} \times [10^{-5}] \times \begin{bmatrix} 0 \\ 0 \\ \frac{2 \times T}{\pi \times \Phi^2 \times t} \end{bmatrix}$$

The analysis is straightforward and will not be carried out here.

Example 4 – Compute the mechanical strains in the local ply frame of a UD55 pipe operating under the following conditions.

$$\begin{aligned} P &= 1000 \text{ psi} \\ t &= 0.3 \text{ in} \\ T &= 100.0 \text{ kg.m} \\ \Phi &= 12 \text{ in} \\ \sigma_x &= 1400 \text{ psi} \\ \sigma_y &= 0 \end{aligned}$$

The first thing to do is to convert all quantities into units of kg and cm. This is required, since matrix $[A]$ is in units of kg and cm.

We have

$$\begin{aligned} P &= 70 \text{ kg/cm}^2 \\ t &= 0.76 \text{ cm} \\ T &= 10\,000 \text{ kg.cm} \\ \Phi &= 300 \text{ mm} \\ \sigma_x &= 100 \text{ kg/cm}^2 \\ \sigma_y &= 0 \end{aligned}$$

Next, we apply equation (7)

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 0,06 & 0,26 & \pm 0,47 \\ 0,57 & -0,09 & \mp 0,47 \\ \mp 1,42 & \pm 1,00 & -0,34 \end{bmatrix} \times 10^{-5} \times \begin{bmatrix} \frac{70 \times 300}{4 \times 7,6} + 100 \\ \frac{70 \times 300}{2 \times 7,6} \\ \frac{2 \times 10\,000}{\pi \times 30^2 \times 0,76} \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 0,06 & 0,26 & \pm 0,47 \\ 0,57 & -0,09 & \mp 0,47 \\ \mp 1,42 & \pm 1,00 & -0,34 \end{bmatrix} \times 10^{-5} \times \begin{bmatrix} 791 \\ 1381 \\ 9,3 \end{bmatrix}$$

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 406,5 & \pm 4,4 \\ 326,6 & \mp 4,4 \\ \pm 357,8 & - 3,2 \end{bmatrix} \times 10^{-5}$$

$$\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{bmatrix} = \begin{bmatrix} 410 \\ 330 \\ 360 \end{bmatrix} \times 10^{-5}$$

These are the mechanical strains in the ply frame, ready for the unified equation. The total strains are obtained by adding the residual thermal and hydric components to the above mechanical strains.

Appendix 2

The fatigue mechanism

Abstract. The theory of fracture mechanics emphatically prohibits the growth of subcritical cracks under static loads. How is it possible that such powerful prohibition does not apply to cyclic loadings? What is the justification for the puzzling subcritical crack growth under cyclic fatigue? The explanation for this apparent contradiction is simple and trivial for both the elastic and plastic regime.

In addition to this explanation, this paper also derives an analytical expression for the celebrated empirical Paris law. Furthermore, it also provides justification for the phenomenon of fatigue retardation, while deriving an expression for the cut-off frequency separating elastic-dominated from plastic-dominated crack growth.

1 – Introduction: The published literature does not satisfactorily explain the mechanism of fatigue crack growth. I have not been able to find a single passing mention of the mechanism explaining how repetitive loads would cause the growth of subcritical cracks. This paper is an attempt to explain this in a way consistent with the theory of fracture mechanics. Alongside with a proposal to explain a mechanism for the fatigue phenomenon, we also provide an expression for the crack extension per cycle, similar to the well-known Paris law.

In its simplest form, the Paris law states that the crack growth per cycle is a power function of the range of the stress intensity factor $\Delta K = K_{\max} - K_{\min}$.

$$\frac{da}{dN} = M(\Delta K)^Z$$

$$\frac{da}{dN} = M(K_{\max} - K_{\min})^Z$$

Entering the definition of the load ratio $R = \frac{K_{\min}}{K_{\max}}$

$$\frac{da}{dN} = M \times (1 - R)^Z \times (K_{\max})^Z$$

Where

N is the number of cycles

$\Delta K = K_{\max} - K_{\min}$ is the range of the stress intensity factor

R is the load ratio

a is the crack length

da/dN is the crack extension per cycle

M and Z are material-dependent empirical constants.

To my knowledge, there is no theoretical justification for the Paris law. In fact, this empirical law has seen strong opposition since its introduction in 1961, having survived all these years still awaiting justification. This paper is an attempt not only to explain the fatigue mechanism, but also to derive an equation similar to the Paris law.

The discussion that follows assumes the validity of a basic tenet of fracture mechanics, which denies the possibility of crack growth under stress intensity factors K less than the critical K_C . In mathematical terms, no crack ever grows ($da/dN = 0$) unless the stress intensity factor K or the energy release rate G reach their corresponding critical values K_C and G_C . Both K_C and G_C are well known material properties.

Therefore, according to fracture mechanics, subcritical cracks should remain stationary and never grow under $K < K_C$ or $G < G_C$. However, experience shows that subcritical cracks do grow under cyclic loads. An explanation is required as to why cyclic loads can grow subcritical cracks, while static loads cannot.

The questions pursued in this paper are:

- *How is it possible that cyclic loads grow subcritical cracks, while static loads do not?*
- *What drives the mechanism of fatigue crack growth?*

To answer these questions, we dive into the brittle rupture mechanism of cracked materials.

2 – Brittle versus ductile ruptures. Ductile ruptures occur in the absence of cracks, by the coalescence of voids that form in the stressed material. This is the dominant mode of material failure, observed both in static tensile and cyclic loadings. The fatigue $S - N$ curves are examples of ductile failure.

Brittle failures are the subject of fracture mechanics. According to fracture mechanics, cracked materials rupture on reaching a critical stress intensity factor K_C or a critical energy release rate G_C .

3 – Stress intensity versus energy release rate. In the following discussion, we assume a through crack of length “ $2a$ ” in an infinite slab of unit thickness.

The stress intensity factor K defines the magnitude of the stress field near the crack tips. Its mathematical expression is

$$K = Y\sigma\sqrt{\pi a}$$

Where Y is a geometry-dependent constant, $2a$ is the crack length and σ is the stress that would exist in the absence of the crack. The stress at a point located a radial distance “ r ” ahead of the crack tip is

$$\left[\text{stress at crack tips} \right] = \frac{K}{\sqrt{2\pi r}}$$

Where r is the radial distance from the crack tip to the point under consideration.

The above expression indicates the presence of high stresses at the crack tips, as r approaches zero. To avoid infinite stresses, the theory of fracture mechanics postulates the existence of plastic zones ahead of the crack tips. The stresses in the plastic zones are constant and finite. The mechanism of crack growth in the plastic zones is the same as the one observed in ductile failures, both involving the coalescence of voids. The differences are:

- *In ductile failures, the entire material develops voids when overstrained*
- *In brittle failures, the voids develop only in the overstrained material ahead of the crack tips*

Crack growth occurs with the onset of voids coalescence in the overstrained material ahead of the crack tips. This overstrained condition requires the stress intensity factor K to reach a critical level K_C .

Static cracks grow only if $K > K_C$.

Let us now describe crack growth from the perspective of the rate of release of elastic energy. The elastic energy stored at the crack tips of cracks of size $2a$ in infinite slabs of unit thickness is

$$\left[\begin{matrix} \text{stored elastic} \\ \text{energy} \end{matrix} \right] = G = \frac{\pi \sigma^2 a^2}{2E}$$

For static conditions, the stress σ is constant.

Therefore, the elastic energy released per unit area of crack growth under constant stress is

$$\frac{dG}{da} = \frac{\pi \sigma^2 a}{E}$$

This equation quantifies the elastic energy released per unit area of crack growth. Similar to the stress intensity approach, crack growth occurs when the stored elastic energy per unit area is equal to a critical level G_C .

$$G_C = \frac{\pi \sigma^2 a}{E}$$

Therefore, we have two conditions for crack growth

$$K > K_C$$

$$G > G_C$$

The relationship between K and G is

$$K^2 = EG$$

The analysis of crack growth may follow two equivalent routes. We can do it using the stress intensity factor K , or the elastic energy release rate G . The choice of route is a matter of convenience.

- *The energy release approach is convenient when the applied stress is constant, as in static strain-corrosion. Under constant stress, the elastic energy plots as a straight line against the crack size, which simplifies the analysis.*
- *The stress intensity approach is convenient when the crack size is constant, as in cyclic fatigue. When the crack sizes are constant, the stress intensity factor K plots exactly as the applied stress.*

The crack growth in fatigue situations occurs at constant crack sizes and variable stress. Therefore, since the crack sizes are constant, we choose to discuss the fatigue problem in terms of the stress intensity factor K

$$K = Y\sigma\sqrt{\pi a}$$

Our discussion considers a subcritical crack of size “ $2a$ ” in an infinite slab of unit thickness. The applied stress varies from a minimum value $\sigma_{\min}$ to a maximum value $\sigma_{\max}$ in the time interval ΔT . The crack size $2a$ is assumed constant in one cycle. As the stress grows, the stress intensity factor K also grows. In the absence of plastic zones, the infinite value of the stresses at the crack tips implies in growth of the subcritical crack under any value of K , including $K < K_C$. This is obvious and clear. The process repeats itself with a small crack extension per cycle, as long as the plastic zones never catch up with the crack tips.

From the preceding description, we see nothing special in the mechanism of subcritical crack growth in a fatigue cycle. The only requirement for such growth is a cyclic frequency high enough to prevent the plastic zones from overtaking the crack tips.

Figure 1 shows a stress waveform, assumed triangular to facilitate the analysis. Since the crack size is constant in one cycle, the stresses intensity factor K plots exactly like the stress.

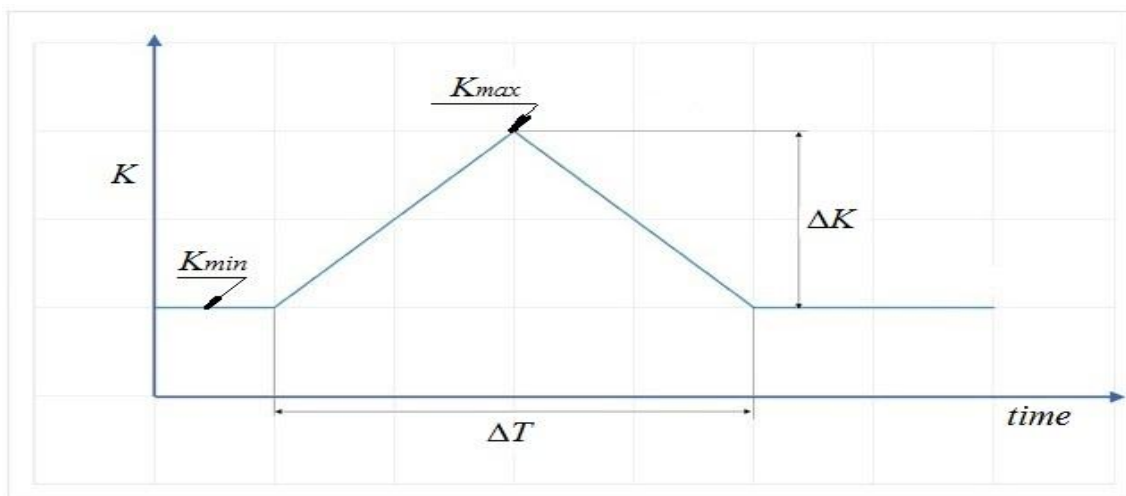


Figure 1

Triangular waveform showing the wave parameters. For constant crack sizes, the stress intensity factor K plots exactly like the stress, ranging from a minimum K_{min} to a maximum K_{max} .

4 – Elastic subcritical crack growth. The formation of the plastic zones are not instantaneous. They take some time to form. Under low frequency loadings, with long cycles, the plastic zones have plenty of time to form and prevent crack growth when $K < K_c$. However, under high frequency loadings the plastic zone may not fully form, allowing crack growth at values of $K < K_c$. This explains the subcritical crack growth for high frequency cyclic loads.

Under high frequency loadings, the plastic zone simply does not have time to form, leaving the crack tips unprotected. From this, we conclude that a condition for elastic fatigue crack growth is that the rate of crack growth be higher than the rate of growth of the plastic zone.

$$\frac{da}{dN} > \frac{dr_p}{dN}$$

Where r_p is the radius, or size, of the plastic zone.

In high frequency loadings, the fast rising stress intensity factor will cause crack growth at the tips before the plastic zone has time to develop and protect them. High frequency loadings, with short cycles, produce extremely small crack growth and negligible formation of plastic zones per cycle.

5 – The elastic Paris law: In the absence of plastic zones, the released elastic energy produces a crack of extension of Δa

$$\Delta a = \int_a^{a+\Delta a} G da$$

Taking into account the relation between the stored elastic energy and the stress intensity factor, we have

$$K = Y\sigma\sqrt{\pi a}$$

$$G_c E = K_c^2$$

Entering these expressions in the above equation, we have

$$\Delta a = \frac{1}{E} \times \int_a^{a+\Delta a} K^2 da$$

This equation gives the crack extension Δa in one cycle as a function of the stress intensity factor K . From figure 1, integrating over the time variable “t”, instead of the crack size “a”, we have

$$\Delta a = \frac{2}{E} \times \int_0^{\Delta T/2} K^2 \frac{da}{dt} dt$$

Where da/dt is the rate of crack growth and the factor 2 accounts for the integration over the half-cycle $\Delta T/2$.

From figure 1, and taking into consideration the first half cycle, we have

$$K = K_{min} + \frac{2 \times \Delta K}{\Delta T} \times t$$

Or, in terms of the loading frequency ν and the load ratio R

$$K = K_{min} + 2 \times \nu \times (1 - R) \times K_{max} \times t$$

From figure 2, we see that the rate of crack growth da/dt relates directly to the driving stress intensity factor K as

$$\frac{da}{dt} = C \times K = C \times (1 - R) \times K_{max}$$

Where C is a constant.

Entering all this in the equation to compute Δa and performing the integration, we have

$$\Delta a = \frac{7 \times C}{3 \times E \times \nu} \times (1 - R)^3 \times (K_{max})^3$$

Since Δa is the crack extension per cycle, we can write the above as

$$\frac{da}{dN} = \frac{7 \times C}{3 \times E \times \nu} \times (1 - R)^3 \times (K_{max})^3$$

This is our version of the Paris equation for elastic-dominated crack growth. It includes the loading frequency ν .

For pure cyclic loadings ($R = 0$), the above equation reduces to

$$\frac{da}{dN} = \frac{7 \times C}{3 \times E \times \nu} \times K_{max}^3$$

For pure static loadings ($R = 1$) the rate of crack growth is zero, as it should.

It is quite possible that the difference between our elastic-dominated equation and the classical Paris law lies in the fact that the latter has been empirically determined under plastic-dominated conditions. This is the topic of the next section.

Figure 2 plots the rate of crack growth da/dt and the crack extension Δa per cycle.

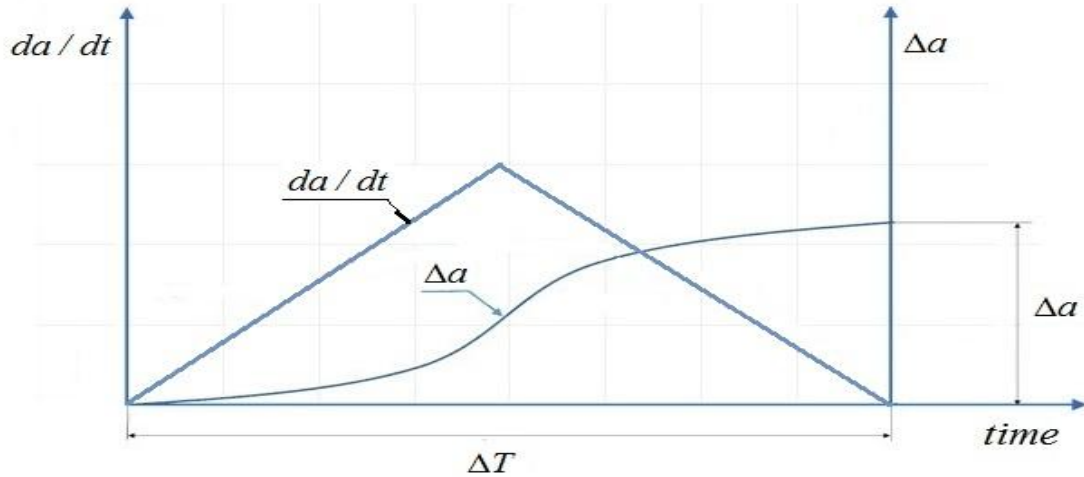


Figure 2

Δa is the crack extension per cycle in the absence of plastic zones at the crack tips. The rate of crack growth increases linearly with the stress intensity factor

6 – The plastic Paris law. The classical Paris law is most likely applicable to lower frequency loadings and plastic-dominated crack growth. Let us develop arguments in support of this statement.

In the plastic-dominated mode, it takes a block of ΔN cycles to produce a small crack increment. This is unlike the elastic-dominated case, in which we have a crack increment per cycle. The plastic-dominated crack growth, like the S – N curves, depends on the number of cycles, not on the loading frequency. The factors affecting the plastic-dominated crack growth are the stress intensity factor K and the load ratio R , which determine the strain distribution in the plastic zone. The closer the distance to the crack tips, the higher is the strain and the quickest is the coalescence.

In a first approximation, we can assume our elastic equation applicable to the plastic case as well, with the following provisos.

- The void coalescence leading to crack growth takes place after ΔN cycles. This is typical of S – N ductile fractures. The elastic cracks, on the other hand, show minute growth in every cycle.
- The rate of plastic-dominated crack growth is independent of the loading frequency

With the above in mind, we propose the following equation for the rate of plastic-dominated crack growth.

$$\frac{da}{dN} = B \times (1 - R)^3 \times (K_{max})^3$$

Where B is a constant.

For pure static loadings, $R = 1$, implying in the absence of crack growth.

For pure cyclic loadings, $R = 0$, the rate of crack growth is

$$\frac{da}{dN} = B \times K_{max}^3$$

As we see, our equation for plastic-dominated crack growth is equal to the classical Paris law with the exponent $Z = 3$. This implies in the classical Paris law being valid for low loading frequencies and plastic-dominated crack growth.

Example 1 – Let us review the differences between elastic and plastic crack growths.

The explanation for the growth of subcritical cracks in plastic zones comes directly from the well-known ductile failure described by the classical $S - N$ fatigue lines. In essence, the plastic-dominated subcritical cracks grow by a ductile cyclic fatigue process in which a high density of voids coalesce to grow the crack. Since it takes many cycles to coalesce the voids, the plastic-dominated cracks grow in jumps. By contrast, the elastic-dominated cracks grow with every cycle.

	<i>Elastic crack growth</i>	<i>Plastic crack growth</i>
<i>Loading frequency</i>	<i>high</i>	<i>low</i>
<i>Number of cycles to grow cracks</i>	<i>1</i>	<i>Many</i>
<i>Rate of crack growth</i>	<i>Fast</i>	<i>Slow</i>
<i>Plastic zone</i>	<i>Behind the crack tips</i>	<i>Ahead of the crack tips</i>
<i>Coalescence of voids</i>	<i>No</i>	<i>Yes</i>
<i>Striations on failed surface</i>	<i>No</i>	<i>Yes</i>
<i>Retardation</i>	<i>No</i>	<i>Yes</i>
<i>Fatigue limit</i>	<i>No</i>	<i>Yes</i>

Example 2. In their book on Fracture Mechanics, Janssen, Zuidema and Wanhill present a regression equation for the crack growth rate in structural steel BS4360 at room temperature and cycle frequencies 1 – 10 Hz.

$$\frac{da}{dN} = 5.01 \times 10^{-12} \times (1 - R)^{3.1} \times (K_{max})^{3.1}$$

For comparison, our proposed plastic-dominated equation is

$$\frac{da}{dN} = B \times (1 - R)^3 \times (K_{max})^3$$

7 – The cut-off frequency. In discussing the elastic-dominated crack growth, we have assumed small cycle duration and the cracks growing faster than the plastic zones. This assumption assures crack growth in the elastic domain, ahead of the plastic zone. In mathematical terms, we have

$$\frac{dr_p}{dN} < \frac{da}{dN}$$

Where r_p is the radius, or size, of the plastic zone and a is the crack size.

As the cycle duration increases, the plastic zone has time to catch up with and even surpass the crack tip. In mathematical terms, we have

$$\frac{dr_p}{dN} > \frac{da}{dN}$$

This inequality defines the condition for plastic-dominated crack growth.

The cut-off frequency, separating plastic from elastic-dominated crack growth, comes from the equation

$$\frac{dr_p}{dN} = \frac{da}{dN}$$

Entering our equation for the elastic crack growth in the above, we have

$$\frac{dr_p}{dN} = \frac{7 \times C}{3 \times E \times \Omega} \times (1 - R)^3 \times (K_{max})^3$$

$$\Omega = \frac{7 \times C}{3 \times E \times \frac{dr_p}{dN}} \times (1 - R)^3 \times (K_{max})^3$$

Where Ω is the cut-off frequency, separating the elastic from plastic crack growth mode.

Loading frequencies above Ω imply in plastic zones lagging behind the crack tips. Loading frequencies below Ω allow the plastic zones to overtake and get ahead of the crack tips.

Frequencies below the cut-off point produce plastic-dominated crack growth.

8 – Fatigue retardation and fatigue limit. Either by accident or by design, sometimes the loading may have a high magnitude spike of long duration that substantially increases the plastic zone ahead of the crack tips. In this situation, the contraction of the elastically strained deformed material places the plastic zone in compression, thereby affording extra protection against crack growth. The presence of this compressed plastic zone ahead of the crack tips explains two interesting phenomena.

- *The origin of the widely reported retardation phenomenon*
- *The existence of a fatigue limit*

The retardation phenomenon is the slowing down of the fatigue process in cracks previously submitted to long-duration load spikes. The origin of this retardation is the compressive stress in the plastic zone subtracting from the active stress. To obtain reasonable retardation, the duration of the load spike should be long enough to produce highly compressed sizeable plastic zones. Besides, their magnitude should be low enough lest they exceed the critical stress intensity factor K_{Ic} , thereby producing static failure.

The fatigue limit is a threshold stress marking the onset of crack growth. Its existence is a direct consequence of the spike stress compressing the plastic zone. To grow cracks the cyclic stress should be higher than this compressive threshold.

9 – Conclusion. An interesting and important feature of composite materials is the ability of the fibers to arrest and prevent the growth of large cracks. The failure of composite materials involves the development and coalescence of many small cracks, as the voids in metal ductile failure. This important fact implies in the Paris not being applicable to composites. The fatigue regression equations of composites are similar to the $S - N$ curves typical of ductile failure of metals.

The Paris law, describing the growth of large cracks, is not applicable to composites

Dec 29 2024

Appendix 1

The work of R. F. Regester

Introduction. In November of 1968, R. F. Regester published the best paper ever in the history of industrial composites, titled *Behavior of Fiber Reinforced Materials in Chemical Service*. In his pioneering and extremely well conducted research, Regester analyzed the behavior of typical liners and corrosion barriers immersed in sulfuric and hydrochloric acid solutions. The liner omission in a few cases allowed the study of corrosion barriers of chopped fibers in direct contact with acid solutions. The resin used was a bisphenol A polyester, the best available for chemical service in those days. The work consisted in measuring and reporting, as a function of time, the concentration profiles and the depths penetrated by the two acids.

The complete work included immersion in water and caustic solutions, not just acid. However, for the purposes of this book, an analysis of the reported results in acid solutions will suffice. For the complete work, including immersion in water and caustic, please see Ref. (16).

What follows is my interpretation of the original findings reported in 1968 by the great Mr. Regester. The published results supports all the concepts put forth in this book

1. Time of exposure. As postulated in chapter 10, the advancing front of highly aggressive chemicals is approximately flat. Furthermore, the penetrated plies remain in place, providing a protective shield that significantly lowers the acidic concentration in the laminate. The published work fully confirms these proposals. Figures 3 and 4 show typical penetrant concentrations in corrosion barriers immersed in 25% H_2SO_4 and 15% HCl solutions at 100C over a 6-month period. The reader will notice the absence of resin-rich liners in these experiments.

The figures show the rate of ion penetration decreasing with increasing exposure times. The fast penetration rate observed in the first 2 months drops off sharply after longer exposures. The high drop off in the rate of penetration indicates that saturation of the entire corrosion barrier would take several years.

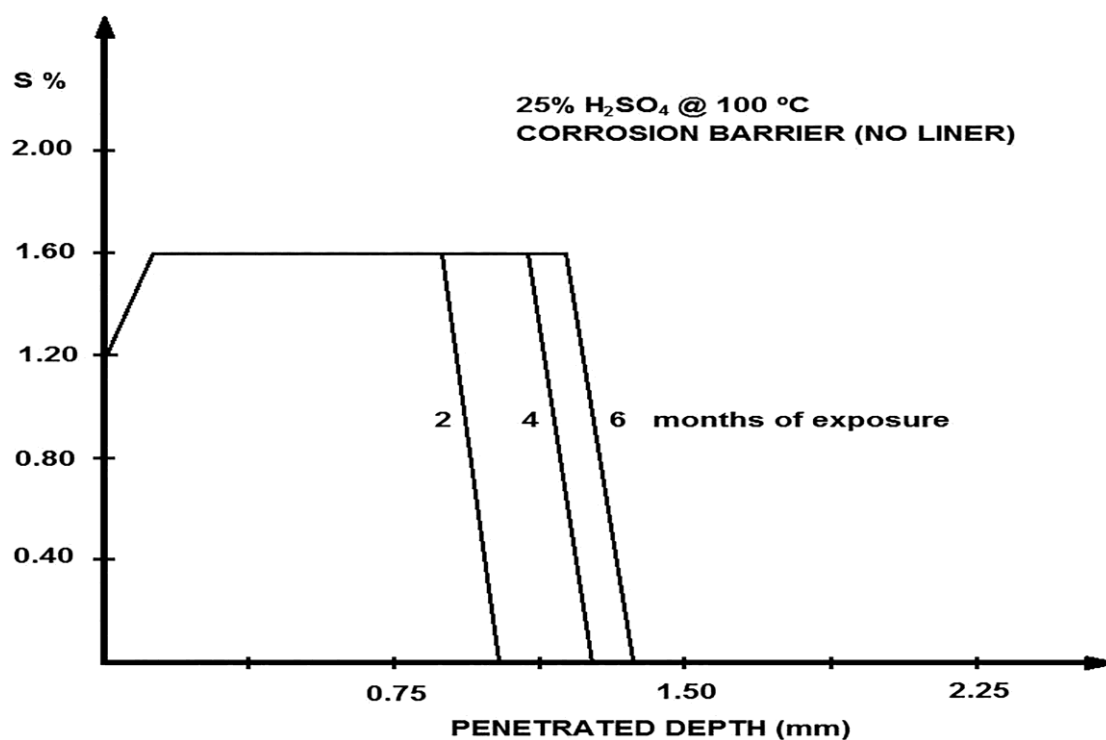


Figure 3

In the absence of a resin-rich protective liner, the corrosion barrier sees direct contact with the acid solution. The nearly flat advancing fronts of H₂SO₄ in the corrosion barrier are clearly visible after 2, 4 and 6 months of exposure. The reader will note the lower acid concentration at the thin inner resin-rich skin (1.2%) compared with the 1.6% concentration in the corrosion barrier itself. As explained in chapter 10, this results from the acid accumulated in voids and at the glass-resin interphase present in the corrosion barrier. The void-free resin-rich inner skin picks up less resin. The figure also shows the large penetration rate observed in the first 2 months dropping off sharply with time. This drop is a consequence of the shielding effect. The slow rate of penetration indicates a large service life for the corrosion barrier in 25% sulfuric acid at 100C.

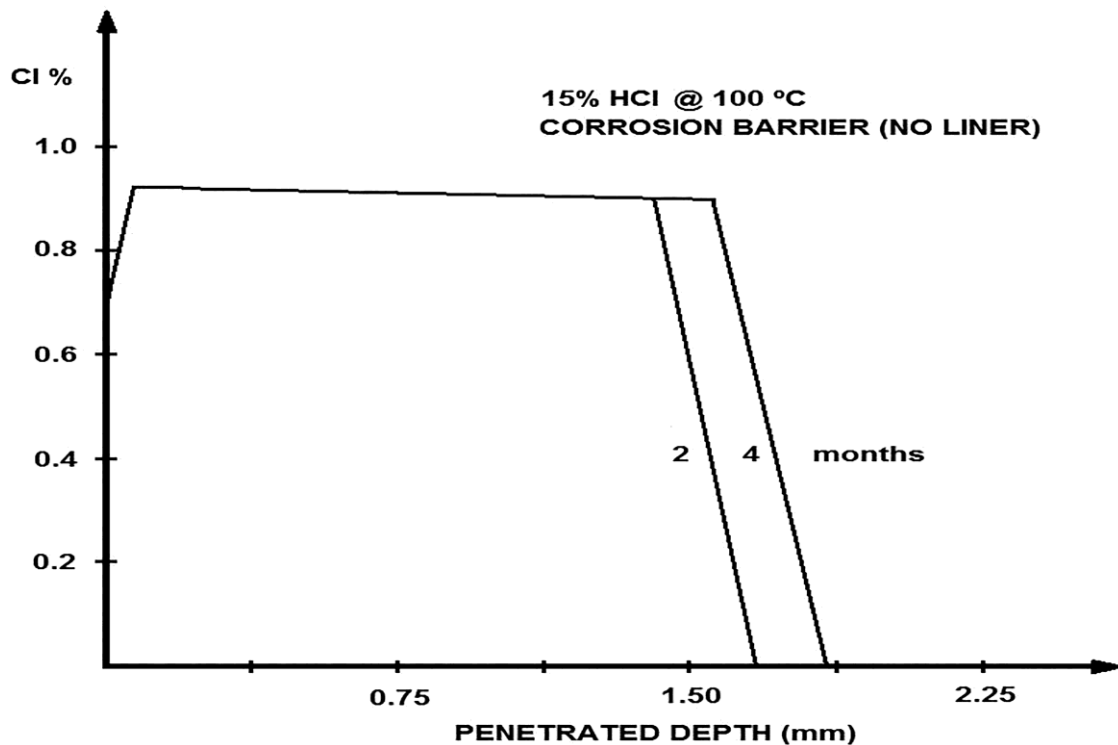


Figure 4

The penetration of HCl in corrosion barriers follows the same pattern described above for H_2SO_4 . In 4 months, the 25% sulfuric acid solution penetrates 1.25 mm, versus 2.0 mm for the 15% HCl solution. The fast penetrating HCl requires equipment with thick corrosion barriers.

2. Glass content. The penetration rates of reactive species in laminates is higher than in pure resin castings. This is a consequence of the high permeability glass-resin debonds present in laminates. Furthermore, and for the same reason, the concentration profile of penetrated chemicals is higher in laminates than in resin castings. Both the chemical concentration profile and the penetration rate increase with increments in the glass content of the laminates.

Laminates with and without resin-rich liners were exposed to 25% sulfuric acid solution at 100C. As expected, the results indicated the resin-rich liner slowing down the penetration rate. However, the concentration of SO_4 ions in the corrosion barrier and in the structural plies did not depend on the liner presence (figure 5).

Surprisingly, the resin-rich liners did not slow down the penetration of Cl ions from a 15% HCl solution at 100C. As shown in figure 6, the concentration and penetrated depth of Cl ions are independent of the presence of the resin-rich liner. The explanation for this null result is quite possibly the premature cracking of the brittle liner of bisphenol A in the presence of the 15% HCl solution at 100C. Cracked liners would certainly have no effect in preventing the free passage of chloride ions.

The foregoing discussion indicates the following hierarchy of plies with respect to chemical concentration and penetration rates.

- **Liner:** The resin-rich liner has the lowest chemical concentration and the lowest penetration rate. It is unfortunate that most liners breakdown in aggressive service.

- **Corrosion barrier:** The corrosion barrier of chopped fibers displays medium chemical concentration and medium penetration rate. The chopped fibers in the corrosion barrier are the critical plies in chemical service.
- **UD structure:** The UD plies in the laminate structure display the highest chemical concentration and penetration rate. The structural UD plies are not as good as corrosion barriers in slowing down the penetration of chemicals.

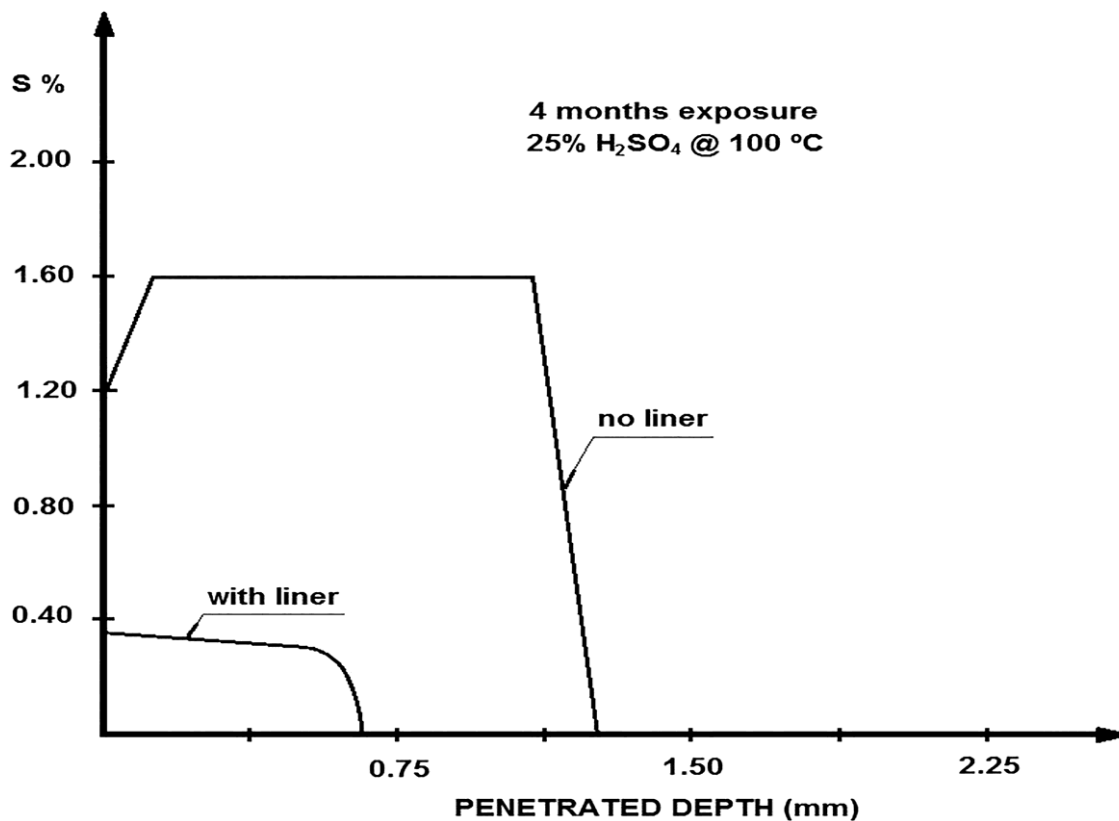


Figure 5

The concentration of sulfur atoms in the resin-rich liner is lower (0.40%) than in the corrosion barrier (1.60%). This is because the sulfuric acid accumulates in voids and debonds present at the glass-resin interface. After 4 months of exposure at 100C, the 25% sulfuric acid penetrates 1.25 mm in the unprotected corrosion barrier, versus 0.70 mm in the resin-rich liner. Intact liners have a significant effect in reducing the short-term chemical penetration in laminates. This initial protection, however, vanishes in the long-term, following the liner destruction. The long-term concentration and penetration of corrosive chemicals in laminates do not depend on the liner presence.

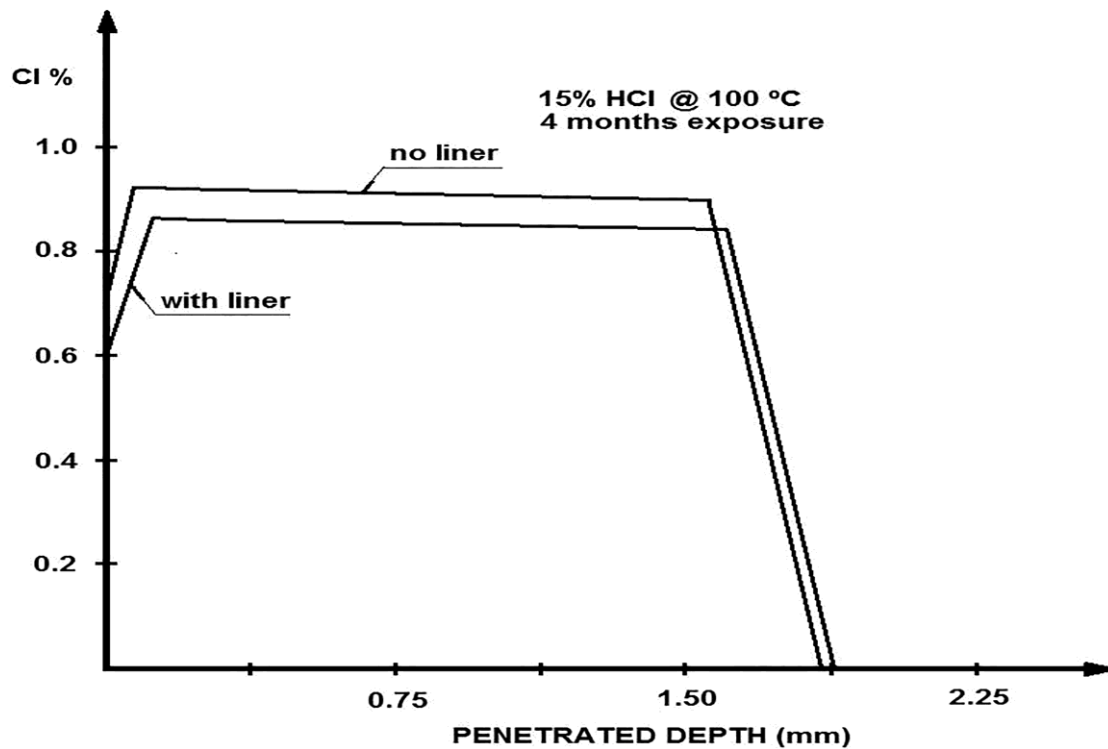


Figure 6

The resin-rich liners crack and fail shortly after contact with 15% HCl solutions at 100C. This failure neutralizes its short-term protective effect. The reader should compare these graphs with those in figure 5.

3. Acid concentration. The chemical concentration profiles in immersed laminates vary with the concentration of the aggressive solution. To confirm this, Regester tested corrosion barriers immersed in 5%, 25% and 50% solutions of sulfuric acid at 100C for 4 months. The results in figure 7 show the presence of SO_4 ions in the laminate increasing with increments in the concentration of the solution. The penetrated depth also increased with the solution concentration, although the increase was more in the concentration range of 5% to 25% than between 25% to 50%. This occurs regardless of the liner presence.

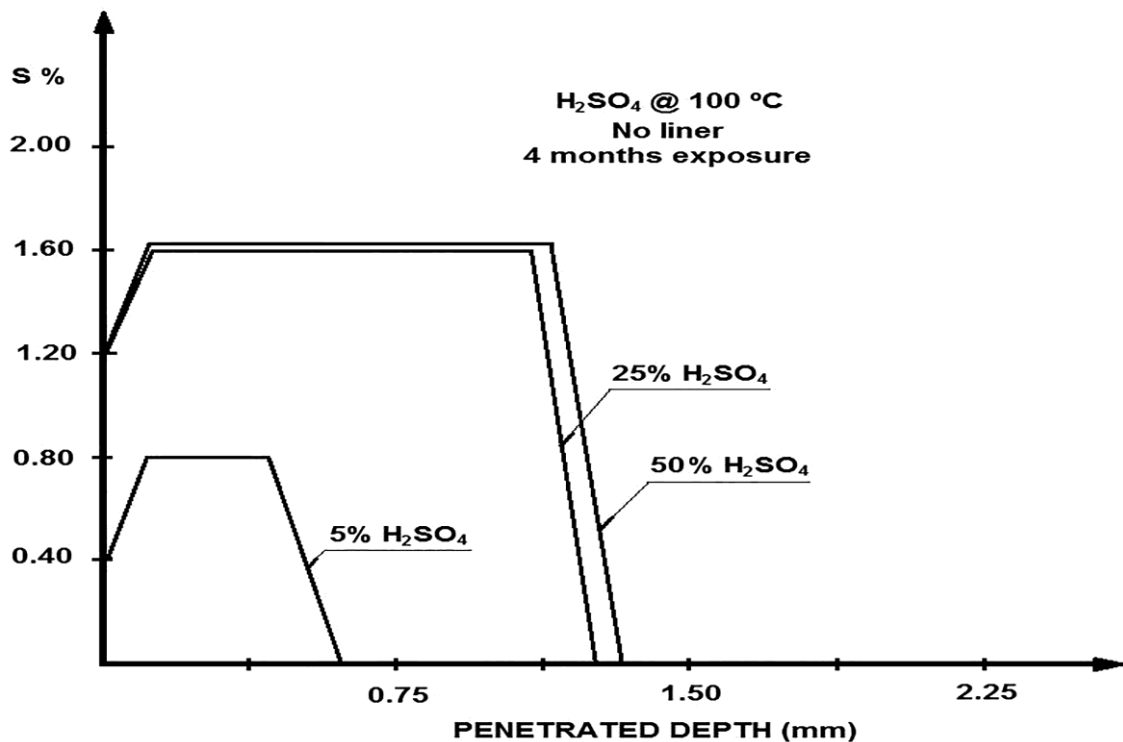


Figure 7

The lower penetrating power of the weak 5.0% sulfuric acid solution gives smaller concentrations in the corrosion barrier than solutions of higher concentrations. This explains the long chemical durability of composite pipes in urban sewer service.

4. Temperature. The effect of temperature was determined by measuring the concentration profile and the penetration rates at 100C and 115C for corrosion barriers exposed to 25% and 50% sulfuric acid solutions. Figure 8 shows a temperature increase from 100C to 125C substantially increasing the concentration profile and the penetrated depth. These results indicate that increments in the operating temperature increase both the rate of penetration and the concentration profile of the offending chemicals.

The resin suppliers provide chemical resistance guides for their products, indicating the maximum allowable operating temperature in a variety of industrial chemicals.

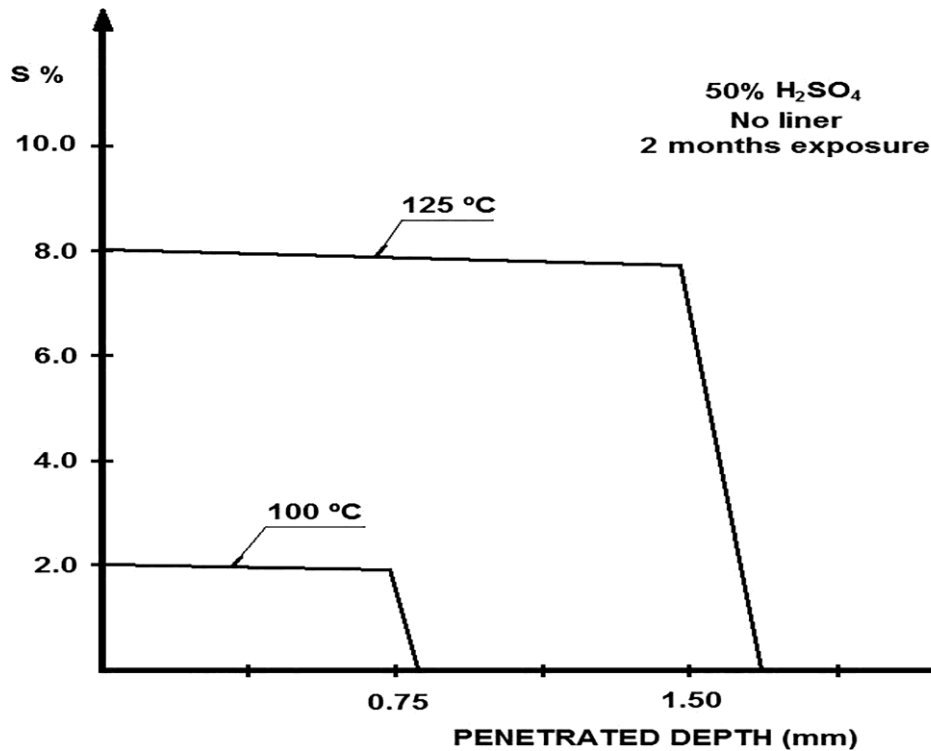


Figure 8

The operating temperature has a significant effect on the chemical concentration profile and the rate of penetration. The figure shows the dramatic difference in acid concentration and in penetrated depth in the corrosion barrier of laminates exposed to 50% solutions of sulfuric acid at 100C and 125C. The resin suppliers publish chemical resistance guides for their products, indicating the maximum allowable temperature in a variety of applications.

5. Type of cation. To examine the effect of solutions with different cations, Regester tested NaCl and HCl solutions at the same temperature and concentration of Cl anions. The experiment lasted 2 months at 100C and indicated a strong effect of cations on the penetration rate of anions. In particular, the large effective ionic diameter of the easily hydrated Na ions substantially decreases their diffusion rate, which slows down the penetration rate of the chloride ions in NaCl solutions. To maintain electrical neutrality, the slowest moving ion will determine the rate of diffusion for both. This explains the deeper and faster penetration of HCl solutions when compared with NaCl. A similar reasoning applies to sulfuric acid and their salts.

6. The work of Regester in actual service. A careful analysis of the results published by Regester leads to some interesting interpretations and conclusions. Tables 6 and 7 compare the sulfur and chloride ion concentrations in the test solutions and in the laminates. These tables give the following information

- The 25% sulfuric acid solution, with a sulfur content of 8.2%, saturates the resin-rich liner with a sulfur content of 0.3%. Assuming a zero void fraction for the neat resin in the liner, this indicates a solubility $S = 0.3/8.2 = 3.7\%$ for sulfuric acid in bisphenol A resins at 100C.
- The sulfur concentration is five times higher – $1.6/0.3 = 5$ – in corrosion barriers than in resin-rich liners. This comes from the high void content and the resin-glass debonds

present in the corrosion barrier. This result indicates the possibility of a high acid presence at the resin-glass interphase. High acid concentration at the interphase is bad for strain-corrosion. For details, see chapter 11.

- At 100C, the outside 5% sulfuric acid solution saturates the corrosion barrier at a very low – 0.7% – sulfur content. This explains the apparent absence of strain-corrosion in underground sanitation pipes in urban sewage applications. For details, see chapter 11.
- The solubility of HCl solutions in bisphenol A resins at 100C is also very low.
- The low acid solubility in resins indicate good performance of composite laminates in acidic solutions.

We next discuss the results reported by Regester in light of the hypotheses proposed in chapter 10.

- The slow penetrating aggressive chemicals advance in nearly flat concentration profiles. This agrees with the proposals in chapter 10.
- The presence of chemicals in the resin matrix is a function of the resin-chemical solubility and the concentration of the external solution. This conclusion is in full agreement with the hypotheses of chapter 10.
- The penetrated plies that remain in place provide a protective shield that slows down the rate of chemical advance and depletes the aggressive presence in the laminate. This is also in full agreement with chapter 10.
- The liner presence retards the advance of chemicals. However, this may not be significant for long-term protection. Although not explicitly reported by Regester, this conclusion is an obvious part of his work.
- The liner integrity does not last very long in harsh environments.

<i>H₂SO₄ concentration in the acid solution</i>	<i>Sulfur concentration in the acid solution</i>	<i>Sulfur concentration in the liner</i>	<i>Sulfur concentration in the corrosion barrier</i>
5%	1.6%	-----	0.7%
25%	8.2%	0.3%	1.6%
50%	16.3%	-----	1.6%

Table 6

Shows the low solubility of sulfuric acid in resin-rich liners and in corrosion barriers at 100 C. The higher concentration in corrosion barriers results from acid accumulation in resin cracks and resin-glass debonds.

<i>HCl concentration in the acid solution</i>	<i>Chlorine concentration in the acid solution</i>	<i>Chlorine concentration in the liner</i>	<i>Chlorine concentration in the corrosion barrier</i>
15%	14.6%	-----	0.8%

Table 7

Shows the low solubility of hydrochloric acid in corrosion barriers at 100 C.

7. A few words about resin-rich liners. In this section, we say a few words about resin-rich liners and their usefulness. It is wise to ignore the long-term protection of liners, even in applications that keep them intact throughout the equipment lifetime. Intact and pristine liners would at best delay the chemical penetration, while having no effect on its

final concentration profile in the laminate. It is safe and prudent to ignore the liner presence in all cases, intact or otherwise. Most aggressive chemicals quickly crack resin-rich liners, destroying their delaying effect. See figure 6 for the case of 15% HCl solution at 100C. The premature destruction of resin-rich liners should not be a cause of concern, since the chemical concentration profiles in laminates do not depend on their presence.

Let us illustrate this for underground sanitation pipes carrying acidic urban sewage. From table 6, we see that a 5% solution of sulfuric acid – simulating urban sewage – produces a very low and harmless acid concentration of 0.7% in the corrosion barrier. Given this background, we consider two scenarios.

- **Scenario 1. Sanitation pipes with no liners.** *The corrosion barriers operate in direct contact with the acidic sewer. We have the following situations:*
 - **Corrosion barriers strained below the infiltration threshold.** *The absence of cracks in this scenario assures a low acid concentration in the corrosion barrier. Corrosion barriers in direct contact with the 5.0% acid solution saturate with a low and harmless acid concentration of just 0.7%. The absence of cracks below the infiltration threshold assures a low acid concentration and a low penetration rate in the corrosion barrier. The pipe durability with corrosion barriers below the infiltration threshold, even in the absence of liners, is very high.*
 - **Corrosion barriers strained above the infiltration and below the weep threshold.** *The difference between this and the previous situation is the faster and deeper acid penetration. The 5.0% solution of sulfuric acid that fills the small infiltration cracks has no effect on the resin and does little damage to the widely dispersed chopped fibers. The pipe life, although less than in the previous case, is still high. The work of Regester implies that sanitation pipes carrying urban sewer can operate above the infiltration threshold.*
 - **Corrosion barriers strained above the weep threshold.** *The large crack densities that develop in this case are too severe, allowing the full strength 5.0% sulfuric acid solution to quickly traverse the corrosion barriers and attack the UD structural plies. The aligned UD fibers, unlike the random chopped fibers of the corrosion barrier, deteriorate fast in the presence of the full-strength acid. The strain-corrosion that ensues in this case is fast, leading to fast deterioration of the pipes.*
- **Scenario 2. Sanitation pipes with intact liners.** *In this scenario, the corrosion barriers operate fully shielded from direct contact with the 5.0% sulfuric acid solution.*
 - *The short delay in the penetration time from the liner presence is irrelevant. Everything else remains as in the previous scenario.*

There is no strain-corrosion failure below the weep threshold. The long-term strain-corrosion performance of pipes in sewer service is, for all practical purposes, independent of the liner presence. This conclusion is true of every long-term situation involving liners.

The preceding discussion explains the apparent absence of strain-corrosion in underground sewage pipes operating below the weep threshold. For details, see chapter 11.

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Biography

Antonio Carvalho is an electrical engineer with full dedication to the composites industry. Starting his professional life in early 1971, he has acquired and accumulated a considerable engineering expertise working for the global fiber and resin suppliers Owens Corning, Reichhold and Ineos, as well as a consultant for major contractors and engineering firms in South America.

His contribution to the industry includes many technical articles, papers and books on a variety of topics, all related to composites and directed to a Portuguese speaking audience.

Carvalho is an active member of the Latin America Composites Association, ALMACO, as well as a provider of engineering expertise for the composites industry.

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